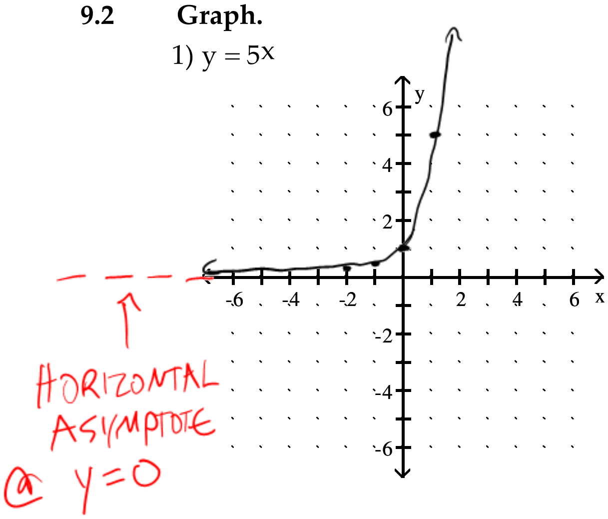
Chapter 9–10 Review Key

Sections labeled at the start of the related problems

9.2 **Graph.**

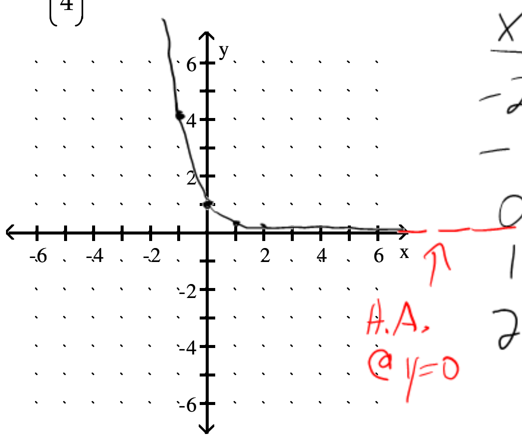
1) $y = 5^x$



x	5^x	y
-2	5^{-2}	1/25
-1	5^{-1}	1/5
0	5^0	1
1	5^1	5
2	5^2	25

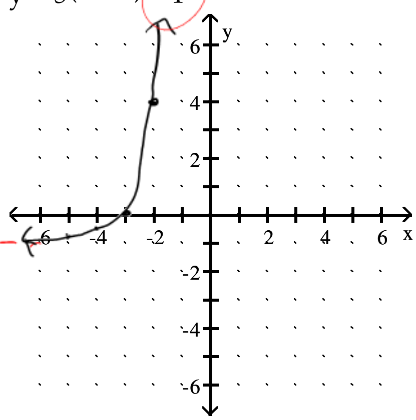
All exponential functions have a horizontal asymptote. You can get this equation by noticing what constant value is added to the exponential term. In problem 1, 2, and 4 the value is not visible because it is 0, so the asymptote is y=0. For #3, the value is -1 so the asymptote is y=-1.

2) $y = \left(\frac{1}{4}\right)^x$



x	$\left(\frac{1}{4}\right)^x$	y
-2	$\left(\frac{1}{4}\right)^{-2}$	16
-1	$\left(\frac{1}{4}\right)^{-1}$	4
0	$\left(\frac{1}{4}\right)^{0}$	1
1	$\left(\frac{1}{4}\right)^{1}$	1/4
2	$\left(\frac{1}{4}\right)^{2}$	1/16

3) $y = 5^{(x + 3)} - 1$



H.A.
@ $y=-1$

$x+3=0$

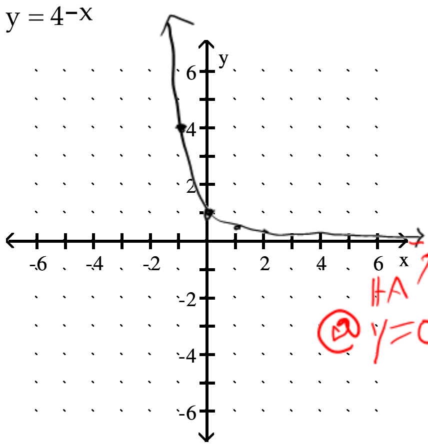
$x=-3$

x	$5^{(x+3)}-1$	y
-5	$5^{-2}-1$	$-\frac{24}{25}$
-4	$5^{-1}-1$	$-4/5$
-3	$5^{0}-1$	0
-2	$5^{1}-1$	4
-1	$5^{2}-1$	24

I started this problem by determining what x value gave me an exponent of zero. In this case x=-3 was the value. Then I pick points to plot based on a couple of x's to the left and right of x=-3. This is how I got to -5, -4, -3, -2, and-1 for the x values I have used.

This same process can be used to explain why I had -2, -1, 0, 1, and 2 for the chosen x values on the other graphs in this section.

4) $y = 4^{-x}$



HA
@ $y=0$

x	4^{-x}	y
-2	$4^{-(-2)}$	16
-1	$4^{-(-1)}$	4
0	4^{-0}	1
1	4^{-1}	$1/4$
2	4^{-2}	$1/16$

Solve the problem.

5) A computer is purchased for $3400. Its value each year is about 76% of the value the preceding year. Its value, in dollars, after t years is given by the exponential function

$$V(t) = 3400(0.76)^t$$

Find the value of the computer after 3 years.

$$V(3) = 3400(0.76)^3 = \boxed{\$1492.52}$$

If you are not sure which button is the one you need on your calculator, play around with 3^2 = 9. You will get the correct button this way. Also, make sure you use the parentheses for negative numbers. For example, (-1)^2 = 1, but -1^2 = -1, and 2^-3 = 0.125.

CALCULATOR:

EITHER 3400 * 0.76 ^ 3 =

OR 3400 * 0.76 y^x 3 =

OR 3400 * 0.76 x^y 3 =

FOR THE REST OF THIS KEY, I WILL USE ^ FOR THE EXPONENT BUTTON.

9.6 **Solve. Where appropriate, include approximations to the nearest thousandth. If no solution exists, state this.**

6) $4^x = 256$

$4^x = 4^4 \Rightarrow \boxed{x = 4}$

The key to these problems is to get the bases on each side of the equation to be the same. Then you can reduce to the equation exponent = exponent.

7) $3^x = \frac{1}{27}$

$3^x = 3^{-3} \Rightarrow \boxed{x = -3}$

8) $2^{(2x+1)} = 8$

$2^{2x+1} = 2^3 \Rightarrow 2x + 1 = 3$

$\quad\quad\quad\quad -1 \quad -1$

$\Rightarrow \frac{2x}{2} = \frac{2}{2} \Rightarrow \boxed{x = 1}$

9.3 **Simplify.**

9) $\log_2 \frac{1}{2}$

$\log_2 \frac{1}{2} = \cancel{\log_2 2}^{-1} = \boxed{-1}$

Logs and exponentials of the same base cancel each other out.

10) $\log_8 32 = x$

$\cancel{8^{\log_8}} 32 = 8^x \Rightarrow 32 = 8^x \Rightarrow 2^5 = (2^3)^x$

$\Rightarrow 2^5 = 2^{3x} \Rightarrow \frac{5}{3} = \frac{3x}{3} \Rightarrow x = \boxed{5/3}$

If you are not able to get to the same base easily, you can set the whole expression = x and then convert to an exponential equation where you can more easily get to the same base.

11) $\log_{10} 10$

$\cancel{\log_{10} 10}^{1} = \boxed{1}$

log base c of c is always 1, regardless of the base c. However no negative numbers, 0 or 1 will be the base of a log.

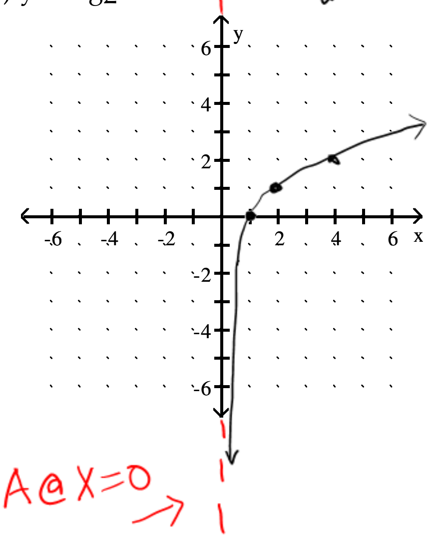
12) $\cancel{\log_9 9}^3 = \boxed{3}$

The main difference between #12 and #13 is the order of the log and exponential. Regardless of order, they still cancel. It is like square rooting a square or squaring a square root. They both cancel.

13) $9^{\log_9(6)} = \boxed{6}$

Graph.

14) $y = \log_2 x$ $= \log_2 x$



x	$\log_2 x$	y
0	$\log_2 0$	UNDEFINED, SO VERTICAL ASYMPTOTE
1	$\log_2 1$	0
2	$\log_2 2$	1
4	$\log_2 4$	2

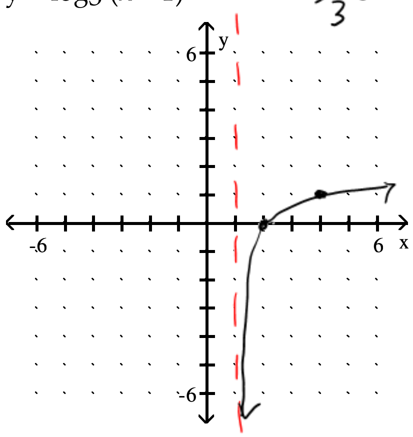
Find the value of x that leads to having a log of 0. This will be your vertical asymptote.

Then find the value of x that leads to having log of 1. This is equal to 0.

Then find the value of x that leads to having log of the base, which always equals 1.

Following this plan you can graph logs with three x values.

15) $y = \log_3 (x - 1)$ $= \log_3 (x-1)$



x	$\log_3 (x-1)$	y
1	$\log_3 0$	VA.
2	$\log_3 1$	0
4	$\log_3 3$	1
10	$\log_3 9$	2

Rewrite as an equivalent exponential equation. Do not solve.

16) $t = \log_7 49$

$7^{t} = 7^{\log_7 49} \Rightarrow \boxed{7^t = 49}$

To get rid of a log, use the big base of both sides. This is really taking the exponential base of both sides. The base should be big and to the lower left of the sides to make the original equation look like exponents.

17) $\log_5 1 = 0$ $\Rightarrow$ $\cancel{5}^{\cancel{\log_5} 1} = 5^0$ $\Rightarrow$ $\boxed{5^0 = 1}$

18) $\cancel{w}^{\log_w Q} = w^{10}$ $\Rightarrow$ $\boxed{w^{10} = Q}$

Rewrite as an equivalent logarithmic equation. Do not solve.

19) $2^3 = 8$

$\cancel{\log_2 2}^3 = \log_2 8$ $\Rightarrow$ $\boxed{\log_2 8 = 3}$

20) $5^{-3} = \frac{1}{125}$ $\cancel{\log_5 5}^{-3} = \log_5\left(\frac{1}{125}\right)$ $\Rightarrow$ $\boxed{\log_5\left(\frac{1}{125}\right) = -3}$

To get rid of a base, use a log of the same base on both sides.

You might find yourself needing to solve for something in an exponent. This is when you need to get rid of the base with a log.

On the other hand, if what you are trying to solve for is in the base, you need to get rid of the exponent, using a radical.

21) $16^{1/4} = 2$ $\cancel{\log_{16} 16}^{\frac{1}{4}} = \log_{16} 2$ $\Rightarrow$ $\boxed{\log_{16} 2 = \frac{1}{4}}$

22) $y^z = 9$

$\cancel{\log_y y}^z = \log_y 9$ $\Rightarrow$ $\boxed{\log_y 9 = z}$

Solve the problem.

23) $\log_3 x = 4$

$\cancel{3}^{\cancel{\log_3} x} = 3^4$ $\Rightarrow$ $x = 3^4 = \boxed{81}$

24) $\log_x 4 = 1$

$\cancel{\log_x} 4 = x^1 \Rightarrow x^1 = \boxed{4}$

25) $\cancel{\log_x} 125 = \cancel{3}$

$\Rightarrow \sqrt[3]{x^3} = \sqrt[3]{125} \Rightarrow \boxed{x = 5}$

4 DECIMALS

9.5 Use a calculator to find an approximation to the nearest ten–thousandth.

26) log 2.67

$\log 2.67 = \boxed{0.4265}$

CALCULATOR:

EITHER log (2.67) =

or 2.67 log =

27) $10^{-1.08872}$ $\boxed{= 0.0815}$

EITHER 10 ^ (− 1.08872) =

OR 10 y^x 1.08872 +/− =

28) ln 74 $= \boxed{4.3041}$

EITHER ln (74) =

OR 74 ln =

29) $e^{-1.9}$ $= \boxed{0.1496}$

EITHER 2nd ln (−1.9) =

OR 1.9 +/− 2nd (INV) ln =

Find the logarithm using the change–of–base formula. Round to the nearest ten–thousandth.

30) $\log_6 71.97$

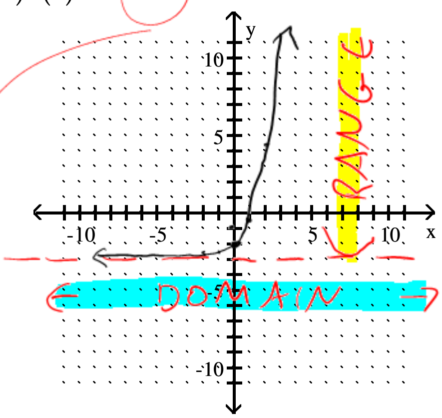
$$\log_6 71.97 = \frac{\ln 71.97}{\ln 6} \text{ or } \frac{\log 71.97}{\log 6} = \boxed{2.3866}$$

The change of base formula can also be used to change the base to any other possible base. The base 10 and e are the ones you change to if you want to use your calculator to get an approximation.

Graph and state the domain and the range of the function.

31) $f(x) = e^x - 3$

H.A.
@ $x = -3$

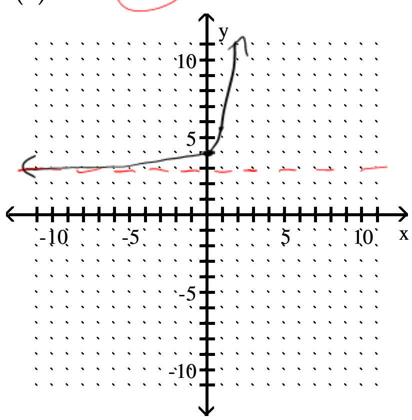


X	$e^x - 3$	Y
-2	$e^{-2} - 3$	-2.86
-1	$e^{-1} - 3$	-2.63
0	$e^0 - 3$	-2
1	$e^1 - 3$	-0.28
2	$e^2 - 3$	4.39

DOM $(-\infty, \infty)$
RAN $(-3, \infty)$

H. A. @ $X = 3$

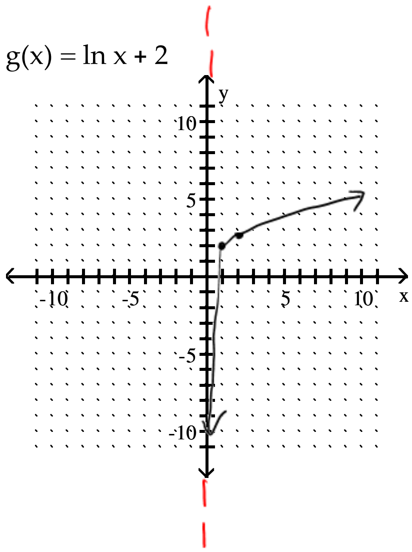
32) $f(x) = e^{-x} + 3$



X	$e^{-x} + 3$	Y
0	$e^{-0} + 3$	4
-1	$e^{-1} + 3$	5.78

DOMAIN $(-\infty, \infty)$
RANGE $(3, \infty)$

33) $g(x) = \ln x + 2$



VA @ $x=0$

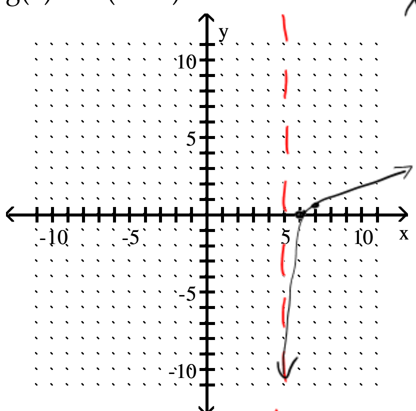
x	$\ln x + 2$	y
0	$\ln 0 + 2$	VA
1	$\ln 1 + 2$	2
2	$\ln 2 + 2$	2.69

For graphing common log (base 10) or natural log ln (base e), you still find the x value that gives log of zero to find the vertical asymptote, and find the x that gives log 1 which = 0. The third x value can be anything that makes sense. You can use your calculator to get the approximation so you don't need to be fussy on the third point.

DOMAIN $(0, \infty)$
RANGE $(-\infty, \infty)$

34) $g(x) = \ln(x - 5)$

$x - 5 = 0$
$x = 5$



VA @ $x=5$

x	$\ln(x-5)$	y
5	$\ln 0$	VA
6	$\ln 1$	0
7	$\ln 2$	0.69

DOMAIN $(5, \infty)$
RANGE $(-\infty, \infty)$

10.1 Find an equation of the circle satisfying the given conditions.

35) Center at (0, 0), radius 11

$(x-h)^2 + (y-k)^2 = r^2$

$\Rightarrow (x-0)^2 + (y-0)^2 = 11^2 \Rightarrow \boxed{x^2 + y^2 = 121}$

36) Center at (−7, 1), radius 4

$(x - -7)^2 + (y-1)^2 = 4^2$

$\boxed{(x+7)^2 + (y-1)^2 = 16}$

37) Center at (−5, −7), radius $\sqrt{11}$

$(x - {}^-5)^2 + (y - {}^-7)^2 = (\sqrt{11})^2 \Rightarrow \boxed{(x+5)^2 + (y+7)^2 = 11}$

Find the center and the radius of the circle.

38) $x^2 + y^2 = 16$

WHAT MAKES $x^2 = 0$? $x = 0$

WHAT MAKES $y^2 = 0$? $y = 0$

$\sqrt{16} = 4$

$\boxed{\text{center } (0,0), \text{ radius } 4}$

39) $(x - 2)^2 + (y + 8)^2 = 6$

WHAT MAKES $(x-2)^2 = 0$? $x = 2$

WHAT MAKES $(y+8)^2 = 0$? $y = -8$

$\sqrt{6} = \sqrt{6}$

$\boxed{\text{center } (2,-8), \ r = \sqrt{6}}$

40) $x^2 + y^2 - 10x - 16y + 80 = 0$

$-80 \quad -80$

$x^2 - 10x + 25 + y^2 - 16y + 64 = -80 + 25 + 64$

$\frac{-10}{2} = -5$

$(-5)^2 = 25$

$\frac{-16}{2} = -8$

$(-8)^2 = 64$

$(x-5)^2 + (y-8)^2 = 9$

$\boxed{\text{center } (5,8), \ r = 3}$

Make sure to add to both sides when you complete the square!

GRAPH: (IF YOU NEED TO)