Chapter 8 Review

Sections labeled at the start of the related problems

8.1 **Solve.**

1) $x^2 + 2 = 11$

$-2 \quad -2$

$\sqrt{x^2} = \pm\sqrt{9}$

$\boxed{x = \pm 3}$

Problems 1 through 4 are all of a form where there is only one instance of the variable x. For this type of equation you need only isolate for that x.

If you take the square root of both sides of an equation to simplify a square power, you must include a +- sign so you get both roots. If you forget, you will only get one of the two answers to the equation.

2) $(x - 5)^2 - 4 = 0$

$+4 \quad +4$

$\sqrt{(x-5)^2} = \pm\sqrt{4}$

$x - 5 = \pm 2$

$+5 \quad +5$

$x = 5 \pm 2 = \boxed{7, 3}$

3) $\sqrt{(x - 1)^2} = \pm\sqrt{-16}$

$x - 1 = \pm 4i$

$+1 \quad +1$

$x = \boxed{1 \pm 4i}$

4) Let $f(x) = (x - 4)^2$. Find x so that $f(x) = 12$.

$f(x) = 12$

$\sqrt{(x-4)^2} = \pm\sqrt{12}$

$x - 4 = \pm 2\sqrt{3}$

$+4 \quad +4$

$x = \boxed{4 \pm 2\sqrt{3} = 4 + 2\sqrt{3},\ 4 - 2\sqrt{3}}$

Complete the square. Then write the trinomial square in factored form.

5) $x^2 - 14x$

$x^2 - 14x + 49 = (x - 7)^2$

$\frac{-14}{2} = -7$

$(-7)^2 = 49$

Notice on this problem there is an x^2 and an x, so two different instances of x. For a problem like this, you are not able to isolate for x. So we need to complete the square, which rewrites it as the previous type, with only one instance of x.

6) $x^2 + 3x + \frac{9}{4} = \left(x+\frac{3}{2}\right)^2$

$\left(\frac{+3}{2}\right)^2 = \frac{9}{4}$

7) $x^2 - \frac{2}{5}x + \frac{1}{25} = \left(x - \frac{1}{5}\right)^2$

$\left(-\frac{2}{5}\cdot\frac{1}{2}\right) \Rightarrow \left(-\frac{1}{5}\right)^2 = \frac{1}{25}$

Solve by completing the square.

8) $a^2 - 4a - 32 = 0$

$\frac{-4}{2} = -2$

$(-2)^2 = 4$

$a^2 - 4a + 4 = 32 + 4$

$\sqrt{(x-2)^2} = \pm\sqrt{36}$

$x - 2 = \pm 6$

$+2 \quad +2$

$x = 2 \pm 6 = \boxed{8, -4}$

9) $2x^2 + 7x + 3 = 0$

$-3 \quad -3$

$\frac{7}{2}\cdot\frac{1}{2} = \frac{7}{4}$

$\left(\frac{7}{4}\right)^2 = \frac{49}{16}$

$\frac{2x^2}{2} + \frac{7x}{2} = \frac{-3}{2}$

$x^2 + \frac{7}{2}x + \frac{49}{16} = -\frac{3}{2} + \frac{49}{16}$

$\sqrt{\left(x+\frac{7}{4}\right)^2} = \pm\sqrt{\frac{25}{16}}$

$x + \frac{7}{4} = \pm\frac{5}{4}$

$-\frac{7}{4} \quad -\frac{7}{4}$

$x = -\frac{7}{4} \pm \frac{5}{4}$

$x = \frac{-2}{4}, \frac{-12}{4} = \boxed{-\frac{1}{2}, -3}$

10) $x^2 + 8x = 7$

$\frac{8}{2} = 4$

$4^2 = 16$

$x^2 + 8x + 16 = 7 + 16$

$\sqrt{(x+4)^2} = \pm\sqrt{23}$

$x + 4 = \pm\sqrt{23} \quad \Rightarrow \quad x = \boxed{-4 \pm \sqrt{23}}$

$-4 \quad -4$

11) $x^2 + 4x + 40 = 0$

$\frac{4}{2} = 2$

$2^2 = 4$

$x^2 + 4x + 4 = -40 + 4$

$\sqrt{(x+2)^2} = \pm\sqrt{-36}$

$x + 2 = \pm 6i$

$-2 \quad -2$

$x = \boxed{-2 \pm 6i}$

Complete the square to find the x-intercepts of the function.

12) $f(x) = x^2 + 4x - 9$

If a question asks for intercepts, make sure you answer with a point, not just a value.

X-INT WHEN $y=0$, SO

$0 = x^2 + 4x - 9$
$+9 \qquad +9$

$\Rightarrow x^2 + 4x + 4 = 9 + 4$

$\sqrt{(x+2)^2} = \pm\sqrt{13}$

$x + 2 = \pm\sqrt{13}$
$-2 \quad -2$

$\Rightarrow x = -2 \pm \sqrt{13}$

$\Rightarrow \boxed{(-2+\sqrt{13}, 0) \ \& \ (-2-\sqrt{13}, 0)}$

ARE THE X-INTERCEPTS

The formula $s = 16t^2$ is used to approximate the distance s, in feet, that an object falls freely (from rest) in t seconds. Use this formula to solve the problem. (Round answer to the nearest tenth.)

13) How long would it take an object to fall freely from a bridge 915 ft above the water?

$S = 16t^2$ & $S = 915$ so

$\frac{16t^2}{16} = \frac{915}{16}$

$\sqrt{t^2} = \pm\sqrt{\frac{915}{16}} \Rightarrow t = \pm\sqrt{\frac{915}{16}} \approx \pm 7.56$

so $\boxed{7.6 \text{ seconds}}$

8.2 Solve using the Quadratic Formula.

14) $2x^2 + 12x = -1$

$2x^2 + 12x + 1 = 0$ (a, b, c)

$b^2 - 4ac = (12)^2 - 4(2)(1) = 144 - 8 = 136$

so $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-12 \pm \sqrt{136}}{4} = \frac{-12 \pm 2\sqrt{34}}{4} = \boxed{\frac{-6 \pm \sqrt{34}}{2}}$

Solve using the Quadratic Formula.

15) $\frac{6}{x} + \frac{6}{x+9} = 1$

$x(x+9)\frac{6}{x} + x(x+9)\frac{6}{x+9} = 1 \cdot x(x+9)$

$6(x+9) + x \cdot 6 = x(x+9)$

$6x + 54 + 6x = x^2 + 9x$

$12x + 54 = x^2 + 9x$
$-12x - 54 \qquad -12x - 54$

$\Rightarrow x^2 - 3x - 54 = 0$

$(x-9)(x+6) = 0$

$\Rightarrow \boxed{x = 9, -6}$

check $x = 9$: $\frac{6}{9} + \frac{6}{18} = 1$ ✓

check $x = -6$: $\frac{6}{-6} + \frac{6}{3} = 1$ ✓

16) $x^2 - 12x + 45 = 0$

$b^2 - 4ac = 144 - 4(45) = -36$

$x = \frac{12 \pm \sqrt{-36}}{2} = \frac{12 \pm 6i}{2} = \boxed{6 \pm 3i}$

17) 5x(x + 2) + 21 = 4x(x + 5)

$5x^2 + 10x + 21 = 4x^2 + 20x$
$-4x^2 \quad -20x \qquad -4x^2 \quad -20x$

$x^2 - 10x + 21 = 0$

$(x-7)(x-3) = 0$

$x = \boxed{3, 7}$

OR $x^2 - 10x + 21 = 0$

$b^2 - 4ac = 100 - 84 = 16$

$x = \dfrac{10 \pm \sqrt{16}}{2} = \dfrac{10 \pm 4}{2}$

$= \dfrac{14}{2}, \dfrac{6}{2} = 7, 3$

Solve.

18) $x^3 - 8 = 0$

$(x-2)(x^2 + 2x + 4) = 0$

so $x - 2 = 0$ OR $x^2 + 2x + 4 = 0$

$\boxed{x = 2}$

$b^2 - 4ac = 4 - 16 = -12$

$x = \dfrac{-2 \pm \sqrt{-12}}{2} = \dfrac{-2 \pm 2\sqrt{3}\, i}{2} = \boxed{-1 \pm \sqrt{3}\, i}$

19) Let $f(x) = 2x^2 - 5x - 1$. Find x so that $f(x) = 0$.

$f(x) = 0$

$2x^2 - 5x - 1 = 0$

$b^2 - 4ac = 25 + 8 = 33$

$x = \boxed{\dfrac{5 \pm \sqrt{33}}{4}}$

8.3 Use the discriminant to determine whether the following equations have solutions that are: two different rational solutions; two different irrational solutions; exactly one rational solution; or two different imaginary solutions.

20) $v^2 - 8v + 3 = 0$

$b^2 - 4ac$ is the discriminant.

$b^2 - 4ac = 64 - 12 = 52$

Since this is positive, we

will get two different irrational solutions

21) $w^2 + 5w + 8 = 0$

$b^2 - 4ac = 25 - 32 = -7$

since this is negative,

we will get two different imaginary solutions

(Actually, these are classified as complex & NOT REAL)

Write a quadratic equation having the given numbers as solutions.

22) −5, −10

$x = -5$, $x = -10$
$+5 \quad +5$, $+10 \quad +10$

$x + 5 = 0$ $\quad x + 10 = 0$

$(x+5)(x+10) = 0$
FOIL

$\boxed{x^2 + 15x + 50 = 0}$

23) $\frac{5}{3}, \frac{3}{5}$

$3 \cdot x = \frac{5}{3} \cdot 3$, $5x = \frac{3}{5} \cdot 5$

$3x = 5$, $5x = 3$
$-5 \quad -5$, $-3 \quad -3$

$3x - 5 = 0$, $5x - 3 = 0$

$(3x-5)(5x-3) = 0$
FOIL

$\boxed{15x^2 - 34x + 15 = 0}$

24) 4i, −4i

$x = 4i$, $x = -4i$
$-4i \quad -4i$, $+4i \quad +4i$

$x - 4i = 0$, $x + 4i = 0$

$(x-4i)(x+4i) = 0$
FoIL

$x^2 \cancel{+4ix} \cancel{-4ix} - 16i^2 = 0$ $\quad (-1)$

$\boxed{x^2 + 16 = 0}$

25) $6 - \sqrt{10}$, $6 + \sqrt{10}$

$x = 6 - \sqrt{10}$, $x = 6 + \sqrt{10}$
$-6 + \sqrt{10} \quad -6 \quad +\sqrt{10}$, $-6 - \sqrt{10} \quad -6 \quad -\sqrt{10}$

$x - 6 + \sqrt{10} = 0$, $x - 6 - \sqrt{10} = 0$

$(x - 6 + \sqrt{10})(x - 6 - \sqrt{10}) = 0$

$x^2 - 6x \cancel{-\sqrt{10}x} - 6x + 36 \cancel{+6\sqrt{10}} \cancel{+\sqrt{10}x} \cancel{-6\sqrt{10}} - 10 = 0$

$\boxed{x^2 - 12x + 26 = 0}$

8.4 Solve the problem.

26) Working together, Rick and Juanita can complete a job in 6 hours. It would take Rick 9 hours longer than Juanita to do the job alone. How long would it take Juanita alone?

Juanita time $= x$
Rick time $= x + 9$
Together $= 6$

so $\frac{1}{x} + \frac{1}{x+9} = \frac{1}{6}$ $\quad x(x+9)\cdot 6$

$6(x+9) + 6x = x(x+9)$

$6x + 54 + 6x = x^2 + 9x$

$12x + 54 = x^2 + 9x$
$-12x - 54 \qquad -12x - 54$

$x^2 - 3x - 54 = 0$

$(x-9)(x+6) = 0$

$x = 9, -6$ NO − TIME

$\boxed{9 \text{ hours}}$

Solve the problem.

27) Sue rowed her boat across Lake Bend and back in 3 hours. If her rate returning was 2 mph less than the rate going, and if the distance each way was 7 miles, find her rate going.

$D=RT \Rightarrow T=\frac{D}{R}$. Let

LET $x=$ rate to,

so $x-2=$ rate back.

TOTAL TIME $=3$

$T_{TO}+T_{back}=3$

$$\frac{D_{TO}}{R_{TO}}+\frac{D_{back}}{R_{back}}=3$$

$$x(x-2)\frac{7}{x}+x(x-2)\frac{7}{x-2}=3x(x-2)$$

$$7(x-2)+7x=3x(x-2)$$

$$7x-14+7x=3x^2-6x$$

$$14x-14=3x^2-6x$$

$-14x+14 \qquad -14x+14$

$$3x^2-20x+14=0$$

$$b^2-4ac=400-4(42)$$
$$=232$$

$$x=\frac{20\pm\sqrt{232}}{6}\approx\frac{20\pm15.2}{6}$$

$$=\frac{35.2}{6}, \frac{4.8}{6}$$

TOO Small

$$=\boxed{5.9 \text{ mph}}$$

28) The distance traveled by an object moving in a straight line is given by $s=t^2-8t$, where s is in feet and t is the time in seconds the object has been in motion. How long (to the nearest tenth) will it take the object to move 16 feet?

$\rightarrow s=16$

$$s=t^2-8t$$

$$16=t^2-8t$$

$-16 \qquad -16$

$$t^2-8t-16=0$$

$$b^2-4ac=64+64=128$$

$$t=\frac{8\pm\sqrt{128}}{2}\approx 9.7, \cancel{-1.7}$$

$$\boxed{9.7 \text{ seconds}}$$

Solve the formula for the indicated letter. Assume that all variables represent nonnegative numbers.

29) $v^2=2as$ for v

$$\sqrt{v^2}=\pm\sqrt{2as}$$

$$v=\cancel{\pm}\sqrt{2as}$$

$$\boxed{v=\sqrt{2as}}$$

30) $rm=t^2-mt$, for t

$$rm=t^2-mt$$

$-rm \qquad -rm$

$$t^2-mt-rm=0$$

1: a, b, c

$$b^2-4ac=m^2-4(-rm)$$
$$=m^2+4rm$$

$$\boxed{t=\frac{m\pm\sqrt{m^2+4rm}}{2}}$$

Answer the question. You will need to use the formula $4.9t^2 + v_0 t = s$.

31) A ball is thrown downward at a speed of 20 meters per second from an altitude of 656 meters. Approximately how long does it take to reach the ground?

$V_0 = 20$

$s = 656$

$4.9t^2 + V_0 t = s$

$4.9t^2 + 20t = 656$

$4.9t^2 + 20t - 656 = 0$

$b^2 - 4ac = 400 - 4(-3214.4)$

$= 13257.6$

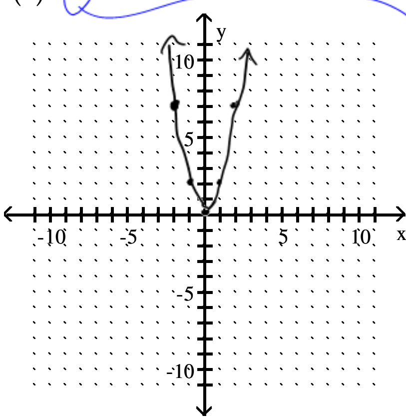
$t = \dfrac{-20 \pm \sqrt{13257.6}}{9.8} \approx 9.71, \cancel{-13.79}$

$\boxed{t = 9.71 \text{ seconds}}$

8.6 Graph.

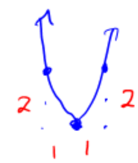
32) $f(x) = 2x^2$

x	y
-2	8
-1	2
0	0
1	2
2	8

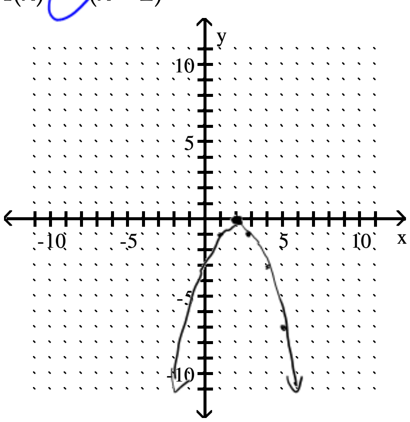


FROM THE VERTEX (0,0), YOU can go ALWAYS OVER 1, &

+2 up two



33) $f(x) = -(x - 2)^2$



vertex (2,0), over 1, down 1

This technique of over one and up or down the coefficient of the x^2 term is useful only once. If you were to keep going over 1 and down 1, as in this problem, you would end up with a line with slope -1.

Without graphing, find the vertex.

34) $f(x) = (x + 5)^2 - 1$

what makes the square term zero? $(x+5)^2 = 0$ @ $x=-5$

what is left if that happens? -1

$(-5, -1)$

35) $f(x) = 2(x - 18)^2 - 9$

$(18, -9)$

OR THINK OPPOSITE IN X, SAME IN Y

$x - 18 \Rightarrow 18$

$-9 \Rightarrow -9$

36) $f(x) = 5\left(x + \frac{1}{7}\right)^2 + 14$

$\left(-\frac{1}{7}, 14\right)$

Find the axis of symmetry of the graph of the parabola.

37) $f(x) = -\frac{19}{5}(x + 2)^2 - 2$

vertex $(-2, -2)$

AXIS OF SYMMETRY IS THE X VALUE OF THE VERTEX, SO $\boxed{x = -2}$

you must always include the $x =$ for the axis of symmetry

Without graphing, find the maximum value or minimum value.

38) $f(x) = -(x - 1)^2 + 1$

THE MINIMUM / MAXIMUM VALUE IS THE Y VALUE OF THE VERTEX,

$x^2 \to$

$-x^2 \to$

SO MAX @ $\boxed{1}$

39) $f(x) = 1.18(x + 1)^2 - 3$

MIN @ $\boxed{-3}$

Write the equation for the function having a graph that meets all of the specified conditions.

40) Has the same shape as the graph of $g(x) = 2x^2$ or $h(x) = -2x^2$ and has a maximum value at (4, 7).

max val $\Rightarrow -2x^2$

vertex (4,7) $\Rightarrow -2(x-4)^2+7$

so $\boxed{f(x) = -2(x-4)^2+7}$

8.7 Complete the square to write the function in the form $f(x) = a(x - h)^2 + k$.

41) $f(x) = x^2 + 2x - 4$

$f(x) = \underbrace{x^2+2x+1} - 4 - 1$

$\boxed{f(x) = (x+1)^2 - 5}$

$\leftarrow +1 \ -1 = 0$ so we added zero to the right side, leaving an equivalent expression.

42) $f(x) = 9x^2 + 4x + 4$

$f(x) = 9x^2 + 4x \quad + 4$

$f(x) = 9\left(\underbrace{x^2 + \frac{4}{9}x + \frac{4}{81}}\right) + 4 - \frac{4}{9}$

$\boxed{f(x) = 9\left(x + \frac{2}{9}\right)^2 + \frac{32}{9}}$

It looks like we added $\frac{4}{81}$, but since we added inside the parentheses, we actually added $9\left(\frac{4}{81}\right) = \frac{4}{9}$, making the $-\frac{4}{9}$ necessary to keep the right side in balance and equivalent.

Find the vertex.

43) $f(x) = 3x^2 + 18x + 26$

$x = \frac{-b}{2a} = \frac{-18}{6} = -3$

$y = f(-3) = 3(-3)^2 + 18(-3) + 26$
$= -1$

vertex $\boxed{(-3, -1)}$

You can use this vertex to work backwards seeing $f(x) = 3(x+3)^2 - 1$. This method is an alternative to the method of # 41, 42.

44) $f(x) = 4x^2 + 40x + 102$

$x = \frac{-b}{2a} = \frac{-40}{8} = -5$

$y = f(-5) = 4(-5)^2 + 40(-5) + 102$
$= 100 - 200 + 102 =$

vertex $\boxed{(-5, 2)}$ ($\Leftrightarrow$ $f(x) = 4(x+5)^2 + 2$)

Find the line of symmetry.

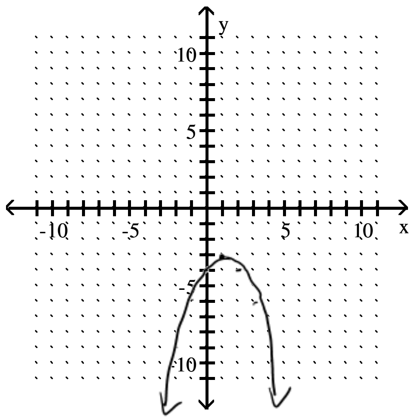
45) $f(x) = 2x^2 - 16x + 33$

$x = \frac{-b}{2a} = \frac{16}{4} = 4$

$\boxed{x = 4}$

Graph.

46) $f(x) = -x^2 + 2x - 4$



VERTEX:

$x = \frac{-b}{2a} = \frac{-2}{-2} = 1$

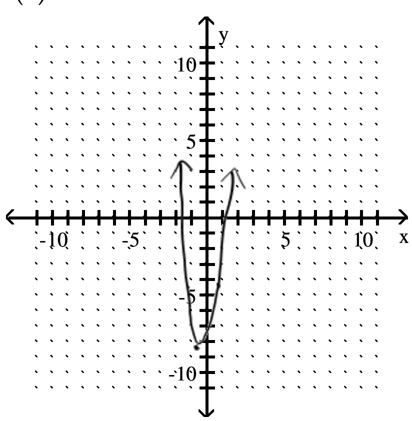
$\Rightarrow$ vertex $(1, -3)$

$y = -(1)^2 + 2(1) - 4$

$= -3$

over 1, down 1

47) $f(x) = 4x^2 + 2x - 8$



Vertex:

$x = \frac{-2}{8} = -\frac{1}{4}$

$y = 4\left(-\frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}\right) - 8$

$= \frac{1}{4} - \frac{2}{4} - 8 = \frac{-33}{4}$

$\left(-\frac{1}{4}, -\frac{33}{4}\right)$

over 1 & up 4

Find the x– and y–intercepts. If no x–intercepts exist, state so.

48) f(x) = $4x^2 + 10x + 2$

x-int $y=0$

$0 = 4x^2 + 10x + 2$

$b^2 - 4ac = 100 - 4(8) = 68$

$x = \frac{-10 \pm \sqrt{68}}{8} = \frac{-10 \pm 2\sqrt{17}}{8} = \frac{-5 \pm \sqrt{17}}{4}$

x int

so $\boxed{\left(\frac{-5 \pm \sqrt{17}}{4}, 0\right)}$

Yint $x=0$

$f(0) = 4(0)^2 + 10(0) + 2$

$= 2$

y int

so $\boxed{(0, 2)}$

49) f(x) = $-x^2 + 19x - 90$

xint $y=0$

$0 = -x^2 + 19x - 90$

$0 = -(x^2 - 19x + 90)$

$0 = -(x-9)(x-10)$

$x = 9, 10$

x int

so $\boxed{(9,0), (10,0)}$

yint $x=0$

$f(0) = -0^2 + 19(0) - 90$

$= -90$

yint

$\boxed{(0, -90)}$

vertex

8.8 Solve.

50) Which of the pairs of numbers whose sum is 74 has the largest product?

Let x & y represent the numbers.

Sum is 74: $x + y = 74$

Product: $P = xy$

since $x + y = 74$, $y = 74 - x$

so $P = x(74 - x) = -x^2 + 74x$

$x = \frac{-b}{2a} = \frac{-74}{-2} = 37$

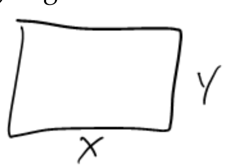
$y = 74 - 37 = 37$

so $\boxed{37 \text{ \& } 37}$

Solve.

51) A gardener is fencing off a rectangular area with a fixed perimeter of 40 ft. What is the maximum area?

vertex



$2x + 2y = 40 \Rightarrow y = 20 - x$

$A = xy = x(20 - x) = -x^2 + 20x$

$x = \frac{-b}{2a} = \frac{-20}{-2} = 10$

$A = -x^2 + 20x = -10^2 + 20(10)$

$= -100 + 200 =$

$\boxed{100 ft^2}$

52) A projectile is thrown upward so that its distance above the ground after t seconds is h = $-13t^2 + 312t$. After how many seconds does it reach its maximum height?

vertex

$t = \frac{-b}{2a} = \frac{-312}{-26} = \boxed{12 \text{ seconds}}$

$\left(h(12) = -13(12)^2 + 312(12) = 1872 \right)$ → if they were to ask for the max height

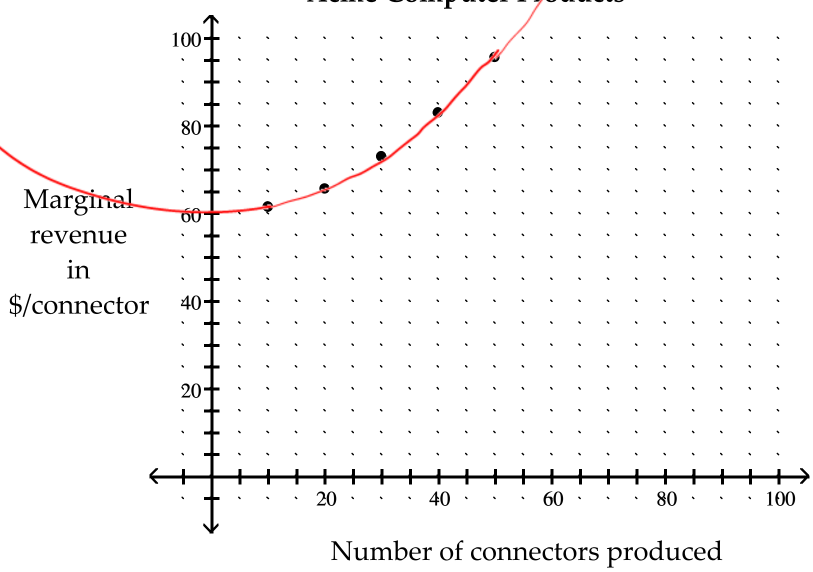
53) Bob owns a watch repair shop. He has found that the cost of operating his shop is given by $c = 4x^2 - 264x + 85$, where c is the cost in dollars, and x is the number of watches repaired. How many watches must he repair to have the lowest cost?

vertex

$$x = \frac{-b}{2a} = \frac{264}{8} = \boxed{33 \text{ watches}}$$

State (Yes or No) whether the given graph appears to represent a quadratic equation.

54) **Acme Computer Products**



YES, APPEARS QUADRATIC

Find the quadratic function that fits the set of data points.

55) (−5, −2), (−4, −1), (−3, 2)

For $y = ax^2 + bx + c$,

$(-5,-2) \Rightarrow -2 = a(-5)^2 + b(-5) + c$

$\Rightarrow 25a - 5b + c = -2$

$(-4,-1) \Rightarrow -1 = a(-4)^2 + b(-4) + c$

$\Rightarrow 16a - 4b + c = -1$

$(-3,2) \Rightarrow 2 = a(-3)^2 + b(-3) + c$

$\Rightarrow 9a - 3b + c = 2$

so our system is

$25a - 5b + c = -2$
$16a - 4b + c = -1$
$9a - 3b + c = 2$

$-R_1 + R_2$:
$-25a + 5b - c = 2$
$+\ 16a - 4b + c = -1$
$-9a + b = 1$

$-R_1 + R_3$:
$-25a + 5b - c = 2$
$+\ 9a - 3b + c = 2$
$-16a + 2b = 4$

$-9a + b = 1$
$-16a + 2b = 4 \Rightarrow -2R4 + R5$

$18a - 2b = -2$
$+\ -16a + 2b = 4$
$2a = 2$
$\boxed{a = 1}$

R_4:
$-9(1) + b = 1$
$b = 10$

$R1$:
$25(1) - 5(10) + c = -2$
$-25 + c = -2$
$c = 23$

so $f(x) = 1 \cdot x^2 + 10x + 23$

or $\boxed{f(x) = x^2 + 10x + 23}$