Chapter 7 Review

Sections labeled at the start of the related problems

1.6 State whether the equation is an example of the product rule, the quotient rule, the power rule, raising a product to a power, or raising a quotient to a power.

1) $(x^5)^3 = x^{15}$

$\boxed{\text{POWER RULE}}$

2) $m^2 \cdot m^9 = m^{11}$

$\boxed{\text{PRODUCT RULE}}$

Multiply and simplify. Leave your answer in exponential notation.

3) $x^6 \cdot x^0$

$= x^{6+0} = \boxed{x^6}$ OR $x^6 \cdot 1 = x^6$

4) $(-4m^3z^4)(5m^2z^2)$ $= (-4 \cdot 5)\, m^{3+2} z^{4+2}$

$= \boxed{-20m^5z^6}$

Divide and simplify.

5) $\dfrac{-8x^8y^7}{4x^2y^5}$ $= \left(\dfrac{-8}{4}\right) x^{8-2} y^{7-5}$

$= \boxed{-2x^6y^2}$

Evaluate.

6) Evaluate $-x^0$ for $x = -2$.

$-x^0 = -(x^0) = \boxed{-1}$ OR $-(-2)^0 = -1$

7) Evaluate $(-x)^0$ for $x = -4$.

$(-x)^0 = \boxed{1}$

or $(-(-4))^0 = (4)^0 = 1$

Write an equivalent expression without a negative exponent.

8) $\frac{y^{-3}}{x^2}$ $= \boxed{\frac{1}{x^2 y^3}}$

9) $\frac{x^{-2}y^5}{z^{-7}}$ $= \boxed{\frac{y^5 z^7}{x^2}}$

10) $\frac{1}{3^{-5}}$ $= \boxed{3^5}$

11) $3a^{-2}$ $= \boxed{\frac{3}{a^2}}$

Write an equivalent expression with negative exponents.

12) $\frac{1}{7^5}$ $= \boxed{7^{-5}}$

13) $\frac{1}{(-7)^3}$ $= \boxed{(-7)^{-3}}$

14) $9x^5$ $= \boxed{\frac{9}{x^{-5}}}$

Simplify using only positive exponents. Leave the answer in exponenetial notation.

15) $(5x^{-3}y^{-4})(4xy^{-3})$

$$= \frac{(5 \cdot 4)\cancel{x}}{x^{\cancel{3}\,2} y^4 y^3} = \boxed{\frac{20}{x^2 y^7}}$$

16) $\frac{45a^{-3}b^3}{9a^{-7}b^7} = \frac{\overset{5}{\cancel{45}}\, a^{7-3}\, \cancel{b^{3}}}{\cancel{9}\, \cancel{a^{3}}\, b^{7-3}} = \boxed{\frac{5a^4}{b^4}}$

Simplify. Write the answer using only positive exponents. Leave the answer in exponenetial notation.

17) $(7^3)^{-7} = 7^{3(-7)} = 7^{-21} = \boxed{\frac{1}{7^{21}}}$

18) $(-3x^4y)^3 = (-3)^3x^{12}y^3 = \boxed{-27x^{12}y^3}$

19) $\left(\frac{-2w^7}{x^1}\right)^2 = \frac{(-2)^2w^{14}}{x^2} = \boxed{\frac{4w^{14}}{x^2}}$

Simplify. Write the answer using positive exponents only. Leave the answer in exponenetial notation.

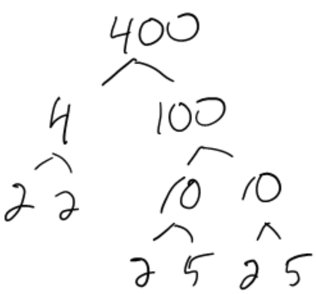
20) $\left(\frac{2x^3y^{-3}}{x^{-2}y^4}\right)^{-3} = \frac{2^{-3}x^{-9}y^{9}}{x^{6}y^{-12}} = \frac{y^{12}y^{9}}{2^3x^6x^9} = \boxed{\frac{y^{21}}{8x^{15}}}$

21) $(r^3s)^2(r^2s^2)^5 = r^6s^2r^{10}s^{10} = r^{6+10}s^{2+10} = \boxed{r^{16}s^{12}}$

7.1 Simplify.

22) $\sqrt{\frac{361}{289}} = \sqrt{\frac{19\cdot19}{17\cdot17}} = \boxed{\frac{19}{17}}$

23) $-\sqrt{400} = -\sqrt{2\cdot2\cdot2\cdot2\cdot5\cdot5} = -2\cdot2\cdot5 = \boxed{-20}$



Identify the radicand and index.

24) $2ab\sqrt[3]{b^2 - 3}$

radicand $= b^2 - 3$

INDEX $= 3$

For the given function, find the indicated function value, if it exists. If the value does not exist, answer "Does not exist".

25) For $g(x) = \sqrt{x^2 - 20}$, find $g(5)$.

$g(5) = \sqrt{(5)^2 - 20} = \sqrt{25-20} = \boxed{\sqrt{5}}$

26) For $g(x) = \sqrt{x^2 - 20}$, find $g(1)$.

$g(1) = \sqrt{(1)^2 - 20} = \sqrt{1-20} = \sqrt{-19}$

NOT REAL, DOES NOT EXIST

Simplify. Assume that variables can represent any value.

THIS IS why $|\ |$ is needed.

27) $\sqrt{16y^2} = \sqrt{16} \cdot \sqrt{y^2} = \boxed{4|y|}$

28) $-\sqrt{x^{10}} = \boxed{-|x^5|}$

Simplify. Unless otherwise specified, assume that variables can represent any number.

29) $\sqrt[3]{-512} = \sqrt[3]{-2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2} = -2\cdot2\cdot2 = \boxed{-8}$

30) $\sqrt[4]{\dfrac{81}{256}} = \sqrt[4]{\dfrac{3\cdot3\cdot3\cdot3}{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2}} = \dfrac{3}{2\cdot2} = \boxed{\dfrac{3}{4}}$

31) $\sqrt[5]{(x-4)^5} = \sqrt[5]{(x-4)(x-4)(x-4)(x-4)(x-4)} = \boxed{x-4}$

32) $-\sqrt[3]{-125x^3}$ $= -\sqrt[3]{-5\cdot5\cdot5\cdot x\cdot x\cdot x} = \boxed{+5x}$

2 negatives

Simplify. Assume all variables represent nonnegative values.

This is why | | is not needed...

33) $\sqrt{z^{10}}$ $= \sqrt{zzzzzzzzzz} = z\cdot z\cdot z\cdot z\cdot z = \boxed{z^5}$

10 z's divide into pairs 5 times $(10 \div 2 = 5)$

34) $\sqrt{9x^2 + 36x + 36}$

$= \sqrt{(3x+6)(3x+6)} = \sqrt{(3x+6)^2} = \boxed{3x+6}$

35) $\sqrt{(x - 9)^{20}}$ so $\boxed{(x-9)^{10}}$

$20 \div 2 = 10$

For the given function, find the indicated function value, if it exists. If the value does not exist, answer "Does not exist".

36) For $f(x) = \sqrt[3]{x + 1}$, find $f(-9)$.

$f(-9) = \sqrt[3]{(-9)+1} = \sqrt[3]{-8} = \boxed{-2}$

ODD ROOTS OF NEGATIVE #'S IS NEGATIVE

37) For $f(x) = \sqrt[4]{x - 3}$, find $f(-13)$.

$f(-13) = \sqrt[4]{(-13)-3} = \sqrt[4]{-16}$ $\boxed{\text{DOES NOT EXIST}}$

EVEN ROOTS OF NEGATIVE #'S DOES NOT EXIST (NOT REAL)

Determine the domain of the function. Express your answer in interval notation.

38) $f(x) = \sqrt{x - 7}$

$x - 7 \geq 0$
$+7 \quad +7$
$x \geq 7$

$\boxed{[7, \infty)}$

39) $f(x) = \sqrt[6]{x + 10}$

$x + 6 \geq 0$
$-6 \quad -6$
$x \geq -10$

$\boxed{[-10, \infty)}$

40) $f(x) = \sqrt[3]{x - 1}$

$\boxed{(-\infty, \infty)}$

ODD ROOTS DON'T HAVE DOMAIN ISSUES

7.2 Write an equivalent expression using radical notation and, if possible, simplify. Assume that even roots are of nonnegative quantities.

41) $x^{1/6} = \sqrt[6]{x}$

42) $m^{4/3} = \sqrt[3]{m^4} = \sqrt[3]{m\,m\,m\,m} = \boxed{m\sqrt[3]{m}}$

$3 \overline{\smash{)}4}$ 1 R1

Rewrite using exponential notation. Assume that even roots are of nonnegative quantities and that all denominators are nonzero.

43) $\sqrt[7]{17} = \boxed{17^{\frac{1}{7}}}$

44) $\sqrt[7]{mn} = \boxed{(mn)^{\frac{1}{7}}} = m^{\frac{1}{7}} n^{\frac{1}{7}}$

45) $\left(\sqrt[4]{5x^3y}\right)^5 = \boxed{(5x^3y)^{\frac{5}{4}}}$ ← power ← ROOT

Rewrite with positive exponents. Assume that even roots are of nonnegative quantities and that all denominators are nonzero.

→ No | | needed.

46) $x^{-4/5} = \boxed{\dfrac{1}{x^{\frac{4}{5}}}}$

47) $\dfrac{1}{9p^{-8/9}} = \boxed{\dfrac{p^{8/9}}{9}}$

Use the laws of exponents to simplify. Do not use negative exponents in the answer. Assume that even roots are of nonnegative quantities and that all denominators are nonzero.

48) $x^{1/5} \cdot x^{4/5}$ $= x^{\frac{1}{5}+\frac{4}{5}} = x^{\frac{5}{5}} = x^{1} = \boxed{x}$

49) $\dfrac{6^{6/13}}{6^{-3/13}}$ $= 6^{\frac{6}{13}} \cdot 6^{\frac{3}{13}} = 6^{\frac{6}{13}+\frac{3}{13}} = \boxed{6^{\frac{9}{13}}}$

50) $(x^{1/6})^{1/5}$ $= x^{\frac{1}{6}\cdot\frac{1}{5}} = \boxed{x^{\frac{1}{30}}}$

Use rational exponents to simplify. Do not use fraction exponents in the final answer. Assume that even roots are of nonnegative quantities.

51) $\sqrt[6]{a^2}$ $= a^{\frac{2}{6}} = a^{\frac{1}{3}} = \boxed{\sqrt[3]{a}}$

52) $\sqrt[6]{2x^{42}}$ $= (2x^{42})^{1/6} = 2^{\frac{1}{6}} x^{\frac{42}{6}} = 2^{1/6} x^{7}$

$= \boxed{x^{7}\sqrt[6]{2}}$

53) $\left(\sqrt[20]{3x}\right)^4$ $= (3x)^{\frac{4}{20}} = (3x)^{\frac{1}{5}} = \boxed{\sqrt[5]{3x}}$

54) $\sqrt[5]{\sqrt[7]{x}}$ $= \sqrt[5]{x^{\frac{1}{7}}} = (x^{\frac{1}{7}})^{\frac{1}{5}} = x^{\frac{1}{7}\cdot\frac{1}{5}} = x^{\frac{1}{35}} = \boxed{\sqrt[35]{x}}$

Solve the problem.

55) It was determined that the proper length L of the letters of a word printed on pavement is given by $L = \frac{0.000169d^{2.27}}{h}$, where d is the distance of a car from the lettering and h is the height of the eye above the surface of the road. All units are in meters. Find L to the nearest tenth of a meter when h = 1.3 m and d = 38 m.

$$L = \frac{0.000169(38)^{2.27}}{1.3} \approx \boxed{0.5 \text{ meters}}$$

DON'T FORGET A STORY ANSWER FOR A STORY PROB.

7.3 Multiply.

56) $\sqrt{2}\sqrt{5}$

$$= \sqrt{2\cdot 5} = \boxed{\sqrt{10}}$$

57) $\sqrt[3]{18p}\sqrt[3]{15q}$

$$= \sqrt[3]{18p\cdot 15q} = \boxed{\sqrt[3]{270pq}} \qquad \sqrt[3]{2\cdot 3\cdot 3\cdot 3\cdot 5\cdot p\cdot q} = \boxed{3\sqrt[3]{10pq}}$$

6 3 3 5

2 3

(BETTER ANSWER)

58) $\sqrt{\frac{x}{14}}\sqrt{\frac{y}{11}}$

$$= \sqrt{\frac{x}{14}\cdot\frac{y}{11}} = \boxed{\sqrt{\frac{xy}{154}}}$$

SHOULD BE RATIONALIZED, IN GENERAL

Simplify by factoring.

59) $-\sqrt{28}$

$$= -\sqrt{2\cdot 2\cdot 7} = \boxed{-2\sqrt{7}}$$

60) $\sqrt[3]{750}$

$$= \sqrt[3]{2\cdot 3\cdot 5\cdot 5\cdot 5} = \boxed{5\sqrt[3]{6}}$$

75 10

3 25 2 5

5 5

61) $\sqrt[3]{216x^4y^5}$

$$= \sqrt[3]{2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 3\cdot xxxx\ yyyyy} = 2\cdot 3\cdot x\cdot y\sqrt[3]{x\cdot y}$$

$$= \boxed{6xy\sqrt[3]{xy}}$$

EVEN ROOTS MAY NEED | | !

Find a simplified form of f(x). Assume that x can be any real number.

62) $f(x) = \sqrt[3]{216x^{10}}$ $= \sqrt[3]{2\cdot2\cdot2\,3\cdot3\cdot3\,x\cdot x\cdot x\,x\cdot x\cdot x\,x\cdot x\cdot x\,x} = 2\cdot3\cdot x\cdot x\cdot x\sqrt[3]{x}$

$= \boxed{6x^3\sqrt[3]{x}}$ $\left(3\overset{3\,R\,1}{\overline{)10}}\right.$ so $\left.\sqrt[3]{x^{10}} = x^3\sqrt[3]{x}\right)$

63) $f(x) = \sqrt{32(x-4)^2}$

$= \sqrt{2\cdot2\,2\cdot2\cdot2\,(x-4)(x-4)} = 2\cdot2\cdot|x-4|\sqrt{2} = \boxed{4|x-4|\sqrt{2}}$

Simplify. Assume that no radicands were formed by raising negative numbers to even powers.

64) $\sqrt[3]{512x^4y^5}$ $= \sqrt[3]{2\cdot2\cdot2\,2\cdot2\cdot2\,2\cdot2\cdot2\,x\,x\,x\,x\,y\,y\,y\,y\,y}$

$= 2\cdot2\cdot2\cdot x\cdot y\sqrt[3]{x\cdot y\cdot y} = \boxed{8xy\sqrt[3]{xy^2}}$

65) $\sqrt[3]{343\,x^4y^5}$ $= \sqrt[3]{7\cdot7\cdot7\,x\,x\,x\,x\,y\,y\,y\,y\,y} = 7\cdot x\cdot y\sqrt[3]{x\cdot y\cdot y} = \boxed{7xy\sqrt[3]{xy^2}}$

Multiply and simplify. Assume all variables represent nonnegative real numbers. Write your answer in radical notation.

66) $\sqrt{15}\sqrt{27}$ $= \sqrt{15\cdot27} = \sqrt{3\cdot5\,3\cdot3\cdot3} = 3\cdot3\sqrt{5} = \boxed{9\sqrt{5}}$

$3\overset{4\,R\,2}{\overline{)14}}$ $3\overset{6\,R\,1}{\overline{)19}}$

67) $\sqrt[3]{xy^5}\sqrt[3]{x^{13}y^{14}}$ $= \sqrt[3]{x\,y^5\,x^{13}\,y^{14}} = \sqrt[3]{x^{14}y^{19}}$ $\boxed{x^4y^6\sqrt[3]{x^2y}}$

7.4 Simplify by taking the roots of the numerator and the denominator. Assume all variables represent positive numbers.

68) $\sqrt{\frac{4}{81}}$ $= \frac{\sqrt{4}}{\sqrt{81}} = \boxed{\frac{2}{9}}$

69) $\sqrt[3]{-\frac{8}{125}}$ $= \frac{-\sqrt[3]{8}}{\sqrt[3]{125}} = \boxed{\frac{-2}{5}}$

70) $\sqrt[4]{\dfrac{256x^5}{y^{18}z^8}} = \sqrt{\dfrac{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot x\cdot x\cdot x\cdot x\cdot x}{y^4 y^4 y^4 y^4 y^2 z^2 z^2 z^2 z^2}} = \dfrac{2\cdot2\cdot x}{y^4\cdot z^2}\sqrt{\dfrac{x}{y^2}} = \boxed{\dfrac{4x}{y^4z^2}\sqrt{\dfrac{x}{y^2}}}$

$4\overline{)18}$ 4R2

Divide and, if possible, simplify. Assume all variables represent positive real numbers.

71) $\dfrac{\sqrt{14y}}{\sqrt{7y}} = \sqrt{\dfrac{\cancel{14}^{2}\cancel{y}}{\cancel{7}\cancel{y}}} = \boxed{\sqrt{2}}$

72) $\dfrac{\sqrt[3]{80x^4y^2}}{\sqrt[3]{10x^2y}} = \sqrt[3]{\dfrac{\cancel{80}^{8}x^{\cancel{4}\,2}y^{\cancel{2}}}{\cancel{10}\cancel{x^2}\cancel{y}}} = \sqrt[3]{8x^2y} = \sqrt[3]{2\cdot2\cdot2\cdot x^2\cdot y} = \boxed{2\sqrt[3]{x^2y}}$

73) $\dfrac{\sqrt[5]{486x^{16}y^{13}}}{\sqrt[5]{2xy^{-2}}} = \sqrt[5]{\dfrac{\cancel{486}^{243}x^{16-1}y^{13+2}}{\cancel{2}\cancel{x}\cancel{y^{-2}}}} = \sqrt[5]{243x^{15}y^{15}}$

$= \sqrt[5]{3\cdot3\cdot3\cdot3\cdot3\cdot x^{15}y^{15}} = \boxed{3x^3y^3}$

$5\overline{)15}$ 3

74) $\dfrac{\sqrt{360mn}}{3\sqrt{5}} = \dfrac{1}{3}\sqrt{\dfrac{\cancel{360}^{72}mn}{\cancel{5}}} = \dfrac{1}{3}\sqrt{72mn} = \dfrac{1}{3}\sqrt{2\cdot2\cdot2\cdot3\cdot3\cdot m\cdot n}$

$= \dfrac{1}{\cancel{3}}\cdot2\cdot\cancel{3}\sqrt{2mn} = \boxed{2\sqrt{2mn}}$

Rationalize the denominator. Assume all variables represent positive numbers.

75) $\sqrt[3]{\dfrac{4}{5}}$

$$= \frac{\sqrt[3]{4}}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5\cdot 5}}{\sqrt[3]{5\cdot 5}} = \frac{\sqrt[3]{4\cdot 5\cdot 5}}{\sqrt[3]{5\cdot 5\cdot 5}} = \boxed{\frac{\sqrt[3]{100}}{5}}$$

76) $\sqrt{\dfrac{50}{x}}$

$$= \frac{\sqrt{2\cdot 5\cdot 5}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{2\cdot 5\cdot 5\cdot x}}{\sqrt{x\cdot x}} = \frac{5\sqrt{2x}}{x}$$

77) $\sqrt{\dfrac{7}{54xy^2}}$

$$= \frac{\sqrt{7}}{\sqrt{2\cdot 3\cdot 3\cdot 3\cdot x\cdot y\cdot y}} \cdot \frac{\sqrt{2\cdot 3\cdot x}}{\sqrt{2\cdot 3\cdot x}} = \frac{\sqrt{2\cdot 3\cdot 7\cdot x}}{\sqrt{2\cdot 2\cdot 3\cdot 3\cdot 3\cdot 3\cdot x\cdot x\cdot y\cdot y}} = \boxed{\frac{\sqrt{42x}}{18xy}}$$

7.5 Add or subtract. Simplify by combining like radical terms, if possible. Assume all variables and radicands represent nonnegative numbers.

78) $4\sqrt{7} + 5\sqrt{7}$

A) 63 B) $20\sqrt{7}$ C) $9\sqrt{7}$ D) $9\sqrt{14}$

$4\sqrt{7} + 5\sqrt{7} = \boxed{9\sqrt{7}}$ OR $4\sqrt{7} + 5\sqrt{7} = (4+5)\sqrt{7} = 9\sqrt{7}$

79) $5\sqrt{200} - 2\sqrt{8}$

$2\cdot 2\cdot 2\cdot 5\cdot 5$ $\quad 2\cdot 2\cdot 2$

$$= 5\cdot 2\cdot 5\sqrt{2} - 2\cdot 2\sqrt{2} = 50\sqrt{2} - 4\sqrt{2}$$
$$= \boxed{46\sqrt{2}}$$

80) $\sqrt{6a} - 4\sqrt{54a} - 4\sqrt{216a}$

$2\cdot 3\cdot 3\cdot 3\cdot a$ $\quad 2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 3\cdot a$

$$= 1\sqrt{6a} - 12\sqrt{6a} - 24\sqrt{6a} = \boxed{-35\sqrt{6a}}$$

81) $13\sqrt[3]{2} - 3\sqrt[3]{54}$

$2\cdot 3\cdot 3\cdot 3$

$$= 13\sqrt[3]{2} - 9\sqrt[3]{2} = \boxed{4\sqrt[3]{2}}$$

82) $4\sqrt[3]{4} - 7\sqrt{6} + 3\sqrt[3]{4} + 5\sqrt{6}$

$$= \boxed{7\sqrt[3]{4} - 2\sqrt{6}}$$

Multiply. Assume that all variables represent nonnegative real numbers.

83) $6\sqrt{5}(\sqrt{11} + \sqrt{5})$

$= 6\sqrt{5}\cdot\sqrt{11} + 6\sqrt{5}\cdot\sqrt{5} = 6\sqrt{5\cdot 11} + 6\sqrt{5\cdot 5} = 6\sqrt{55} + 30$

84) $(\sqrt{11} + 2)(\sqrt{11} - 2)$

F O I L

$= \sqrt{11}\cdot\sqrt{11} \cancel{- 2\sqrt{11}} \cancel{+ 2\sqrt{11}} - 4 = 11 - 4 = \boxed{7}$

$\sqrt{11}\cdot\sqrt{11} = \sqrt{11\cdot 11} = 11$

$\sqrt{58}\cdot\sqrt{58} = \sqrt{58\cdot 58} = 58$ ETC

85) $(\sqrt{5} + 4)(\sqrt{6} - 7)$

$= \sqrt{5}\cdot\sqrt{6} - 7\sqrt{5} + 4\sqrt{6} - 28 = \boxed{\sqrt{30} - 7\sqrt{5} + 4\sqrt{6} - 28}$

86) $(\sqrt[3]{9} + 4)(\sqrt[3]{3} - 6)$

$= \sqrt[3]{9}\cdot\sqrt[3]{3} - 6\sqrt[3]{9} + 4\sqrt[3]{3} - 24$

$= \sqrt[3]{3\cdot 3\cdot 3} - 6\sqrt[3]{3\cdot 3} + 4\sqrt[3]{3} - 24 = 3 - 6\sqrt[3]{9} + 4\sqrt[3]{3} - 24$

$= \boxed{-21 - 6\sqrt[3]{9} + 4\sqrt[3]{3}}$

87) $(2 + \sqrt{7})^2$

$(2+\sqrt{7})(2+\sqrt{7}) = 4 + 2\sqrt{7} + 2\sqrt{7} + 7 = \boxed{11 + 4\sqrt{7}}$

88) $\sqrt[3]{xy^2}\left(\sqrt{xy} - \sqrt[5]{x^3y}\right)$

$\sqrt[3]{xy^2}\sqrt{xy} - \sqrt[3]{xy^2}\sqrt[5]{x^3y} = (xy^2)^{\frac{1}{3}}\cdot(xy)^{\frac{1}{2}} - (xy^2)^{\frac{1}{3}}(x^3y)^{\frac{1}{5}} = x^{\frac{1}{3}}y^{\frac{2}{3}}x^{\frac{1}{2}}y^{\frac{1}{2}} - x^{\frac{1}{3}}y^{\frac{2}{3}}x^{\frac{3}{5}}y^{\frac{1}{5}} = x^{\frac{1}{3}+\frac{1}{2}}y^{\frac{2}{3}+\frac{1}{2}} - x^{\frac{1}{3}+\frac{3}{5}}y^{\frac{2}{3}+\frac{1}{5}} = x^{\frac{5}{6}}y^{\frac{7}{6}} - x^{\frac{14}{15}}y^{\frac{13}{15}} =$

$\sqrt[6]{x^5}\,\sqrt[6]{y^7} - \sqrt[15]{x^{14}}\,\sqrt[15]{y^{13}} = \sqrt[6]{x^5}\,y\sqrt[6]{y} - \sqrt[15]{x^{14}}\,\sqrt[15]{y^{13}} = \boxed{y\sqrt[6]{x^5y} - \sqrt[15]{x^{14}y^{13}}}$

yyyyyyy

Rationalize the denominator. Assume all variables represent positive numbers.

89) $\frac{2}{8 - \sqrt{5}}$

$\frac{2}{(8-\sqrt{5})}\cdot\frac{(8+\sqrt{5})}{(8+\sqrt{5})} = \frac{2(8+\sqrt{5})}{64 \cancel{+ 8\sqrt{5}} \cancel{- 8\sqrt{5}} - 5} = \boxed{\frac{16+2\sqrt{5}}{59}}$

90) $\frac{10 - \sqrt{7}}{10 + \sqrt{7}}$

$\frac{(10-\sqrt{7})}{(10+\sqrt{7})}\cdot\frac{(10-\sqrt{7})}{(10-\sqrt{7})} = \frac{100 - 10\sqrt{7} - 10\sqrt{7} + 7}{100 - 10\sqrt{7} + 10\sqrt{7} - 7} =$

$= \boxed{\frac{107 - 20\sqrt{7}}{93}}$

91) $\dfrac{3\sqrt{x}}{(\sqrt{x}+3\sqrt{y})} \cdot \dfrac{(\sqrt{x}-3\sqrt{y})}{(\sqrt{x}-3\sqrt{y})} = \dfrac{3\sqrt{x}(\sqrt{x}-3\sqrt{y})}{x-\cancel{3\sqrt{xy}}+\cancel{3\sqrt{xy}}-9y} = \boxed{\dfrac{3\sqrt{x}(\sqrt{x}-3\sqrt{y})}{x-9y}}$

$= \dfrac{3x-9\sqrt{xy}}{x-9y}$

Multiply and simplify. Assume all variables represent nonnegative real numbers. Write your answer in radical notation.

92) $\sqrt{x^2y^3}\,\sqrt[3]{xy^4} = (x^2y^3)^{\frac{1}{2}} \cdot (xy^4)^{\frac{1}{3}} = x^{\frac{2}{2}}y^{\frac{3}{2}} \cdot x^{\frac{1}{3}}y^{\frac{4}{3}}$

$= x^{\frac{2}{2}+\frac{1}{3}}\,y^{\frac{3}{2}+\frac{4}{3}} = x^{\frac{4}{3}\cdot\frac{2}{2}}\,y^{\frac{17}{6}} = x^{\frac{8}{6}}y^{\frac{17}{6}} = \sqrt[6]{x^8}\,\sqrt[6]{y^{17}}$

$= \sqrt[6]{x^8y^{17}} = \boxed{xy^2\sqrt[6]{x^2y^5}}$

Divide and, if possible, simplify. Assume all variables represent positive real numbers.

93) $\dfrac{\sqrt[3]{y^2}}{\sqrt[4]{y}} = \dfrac{y^{\frac{2}{3}}}{y^{\frac{1}{4}}} = y^{\frac{2}{3}-\frac{1}{4}} = y^{\frac{5}{12}} = \boxed{\sqrt[12]{y^5}}$

94) $\dfrac{\sqrt[5]{a^4b^2}}{\sqrt[3]{ab^2}} = \dfrac{(a^4b^2)^{\frac{1}{5}\cdot\frac{3}{3}}}{(ab^2)^{\frac{1}{3}\cdot\frac{5}{5}}} = \dfrac{(a^4b^2)^{\frac{3}{15}}}{(ab^2)^{\frac{5}{15}}} = \dfrac{\sqrt[15]{(a^4b^2)^3}}{\sqrt[15]{(ab^2)^5}} = \sqrt[15]{\dfrac{a^{12-5}\cancel{b^6}}{\cancel{a^5}b^{10-6}}} = \boxed{\sqrt[15]{\dfrac{a^7}{b^4}}}$

Solve the problem. Assume all variables represent nonnegative real numbers.

95) For $f(x) = \sqrt[4]{x^2}$ and $g(x) = \sqrt[4]{6x^{11}} - \sqrt[4]{x^{30}}$, find $(f \cdot g)(x)$.

$(f\cdot g)(x) = \sqrt[4]{x^2}\cdot\left(\sqrt[4]{6x^{11}} - \sqrt[4]{x^{30}}\right) = \sqrt[4]{6x^{13}} - \sqrt[4]{x^{32}} = \boxed{x^3\sqrt[4]{6x} - x^8}$

$4\overline{)13}^{\,3R1}$ $4\overline{)32}^{\,8}$

Solve the problem.

96) For $f(x) = x^2$, find $f(7 - \sqrt{11})$

$f(7-\sqrt{11}) = (7-\sqrt{11})^2 = (7-\sqrt{11})(7-\sqrt{11}) = 49 - 7\sqrt{11} - 7\sqrt{11} + 11$

$= \boxed{60 - 14\sqrt{11}}$

7.6 Solve.

97) $\left(\sqrt{8q-7}\right)^2 = (7)^2 \Rightarrow 8q - 7 = 49 \Rightarrow 8q = 56 \Rightarrow \boxed{q = 7}$

$+7 \quad +7 \qquad \frac{8q}{8} = \frac{56}{8}$

Check $\sqrt{8(7)-7} = \sqrt{56-7} = \sqrt{49} = 7$ ✓

98) $\sqrt{4x} + 5 = 9$

$-5 \quad -5$

$\Rightarrow \left(\sqrt{4x}\right)^2 = (4)^2 \Rightarrow \frac{4x}{4} = \frac{16}{4} \Rightarrow \boxed{x = 4}$

Check: $\sqrt{4(4)} + 5 = \sqrt{16} + 5 = 4 + 5 = 9$ ✓

99) $\left(\sqrt[3]{x+2}\right)^3 = (5)^3 \Rightarrow x + 2 = 125 \Rightarrow \boxed{x = 123}$

$-2 \quad -2$

Check: $\sqrt[3]{(123)+2} = \sqrt[3]{125} = 5$ ✓

100) $\left(3\sqrt{y}\right)^2 = (y)^2 \Rightarrow 9y = y^2 \Rightarrow y^2 - 9y = 0 \Rightarrow y(y-9) = 0$

$-9y \quad -9y$

$\Rightarrow \boxed{y = 0, y = 9}$

Check $y=0$: $3\sqrt{(0)} = (0) \Rightarrow 0 = 0$ ✓

Check $y=9$: $3\sqrt{(9)} = (9) \Rightarrow 3 \cdot 3 = 9$ ✓

101) $x = \sqrt{x+13} + 7$

$-7 \qquad -7$

$\Rightarrow (x-7)^2 = \left(\sqrt{x+13}\right)^2 \Rightarrow x^2 - 14x + 49 = x + 13 \Rightarrow x^2 - 15x + 36 = 0$

FOIL $\qquad -x \quad -13 \qquad -x \quad -13$

$\Rightarrow (x-12)(x-3) = 0 \Rightarrow x = 3, 12$

Check $x=3$; $(3) = \sqrt{(3)+13} + 7 \Rightarrow 3 = \sqrt{16} + 7 \Rightarrow 3 = 4 + 7$ NO

Check $x=12$ $(12) = \sqrt{(12)+13} + 7 \Rightarrow 12 = \sqrt{25} + 7 \Rightarrow 12 = 5 + 7$ ✓

so $\boxed{x = 12}$

102) $\sqrt[4]{y-4}+8=0$
$-8 \quad -8$

$\Rightarrow \left(\sqrt[4]{y-4}\right)^4 = (-8)^4$ $\quad$ OR $\Rightarrow \boxed{\varnothing}$ since EVEN ROOT $\neq$ − #

$\Rightarrow y-4 = 4096$
$+4 \quad +4$

$\Rightarrow y = 4100$

Check: $\sqrt[4]{(4100)-4}+8 = \sqrt[4]{4096}+8 =$
$8+8 \neq 0$ NO so $\boxed{\varnothing}$

103) $\left(\sqrt{5a-7}\right)^2 = \left(\sqrt{2a+9}\right)^2$

$\Rightarrow 5a-7 = 2a+9 \Rightarrow \frac{3a}{3} = \frac{16}{3} \Rightarrow \boxed{a = \frac{16}{3}}$
$-2a \; +7 \quad -2a \; +7$

Check: $\sqrt{5\left(\frac{16}{3}\right)-7} = \sqrt{2\left(\frac{16}{3}\right)+9}$

$\Rightarrow \sqrt{\frac{59}{3}} = \sqrt{\frac{59}{3}}$ ✓

104) $\sqrt{2x+3}-\sqrt{x+1}=1$
$+\sqrt{x+1} \quad +\sqrt{x+1}$

FOIL

$\Rightarrow \left(\sqrt{2x+3}\right)^2 = \left(\sqrt{x+1}+1\right)^2 \Rightarrow 2x+3 = x+1+2\sqrt{x+1}+1$
$-x \; -1 \quad -x-1 \quad -1$

FOIL

$\Rightarrow (x+1)^2 = \left(2\sqrt{x+1}\right)^2 \Rightarrow x^2+2x+1 = 4(x+1) \Rightarrow x^2+2x+1 = 4x+4 \Rightarrow x^2-2x-3=0 \Rightarrow$
$-4x \; -4 \quad -4x \; -4$

$\Rightarrow (x-3)(x+1)=0 \Rightarrow \boxed{x=-1,3}$

Check $x=-1$: $\sqrt{2(-1)+3}-\sqrt{(-1)+1} = \sqrt{1}-\sqrt{0} = 1-0 = 1$ ✓

Check $x=3$: $\sqrt{2(3)+3}-\sqrt{(3)+1} = \sqrt{9}-\sqrt{4} = 3-2 = 1$ ✓

105) $\sqrt{x+6}+\sqrt{2-x}=4$
$-\sqrt{2-x} \quad -\sqrt{2-x}$

FOIL

$\Rightarrow \left(\sqrt{x+6}\right)^2 = \left(4-\sqrt{2-x}\right)^2 \Rightarrow x+6 = 16-8\sqrt{2-x}+2-x \Rightarrow$
$+x \; -16 \; -2 \quad -16 \quad -2 \; +x$

FOIL

$\Rightarrow \frac{2x}{2}-\frac{12}{2} = \frac{-8}{2}\sqrt{2-x} \Rightarrow (x-6)^2 = \left(-4\sqrt{2-x}\right)^2 \Rightarrow x^2-12x+36 = 16(2-x) \Rightarrow$

$\Rightarrow x^2-12x+36 = 32-16x \Rightarrow x^2+4x+4=0 \Rightarrow (x+2)(x+2)=0 \Rightarrow \boxed{x=-2}$
$+16x \; -32 \quad -32 \; +16x$

Check: $\sqrt{(-2)+6}+\sqrt{2-(-2)} = \sqrt{4}+\sqrt{4} = 2+2 = 4$ ✓

106) $(x-6)^{1/2} = -2$ $\Rightarrow \left(\sqrt{x-6}\right)^2 = (-2)^2$ $\Rightarrow \boxed{\varnothing}$ even root $\neq$ −#.

$x-6 = 4 \Rightarrow x = 10$
$+6 \quad +6$

check $\left((10)-6\right)^{1/2} = -2$

$\Rightarrow (4)^{\frac{1}{2}} = -2 \Rightarrow 2 = -2$ NO so $\boxed{\varnothing}$

$\sqrt{4}$

Solve the problem.

107) If $f(x) = \sqrt[3]{5x + 4} + 2$, find a such that $f(a) = 5$

A) $23\frac{4}{5}$ B) 1 C) $\sqrt[3]{29}$ D) $4\frac{3}{5}$

$f(a) = 5 \Rightarrow \sqrt[3]{5(a)+4} + 2 = 5 \Rightarrow \sqrt[3]{5a+4} \underset{-2}{+2} = \underset{-2}{5} \Rightarrow \left(\sqrt[3]{5a+4}\right)^3 = (3)^3$

$\Rightarrow 5a \underset{-4}{+4} = \underset{-4}{27} \Rightarrow \frac{5a}{5} = \frac{23}{5} \Rightarrow a = \frac{23}{5} = \boxed{4\frac{3}{5} \quad D}$

Check: $f\left(\frac{23}{5}\right) = \sqrt[3]{5\left(\frac{23}{5}\right)+4} + 2 = \sqrt[3]{27} + 2 = 3 + 2 = 5$ ✓

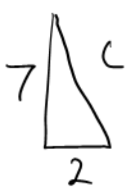
108) The distance d in miles that can be seen on the surface of the ocean is given by $d = 1.6\sqrt{h}$, where h is the height in feet above the surface. How high (to the nearest foot) would a platform have to be to see a distance of 19.5 miles?

$d = 1.6\sqrt{h} \Rightarrow \frac{19.5}{1.6} = \frac{1.6\sqrt{h}}{1.6} \Rightarrow (12.1875)^2 = (\sqrt{h})^2$

$\Rightarrow h = \underset{\text{ROUND}}{148.535} \Rightarrow \boxed{h = 149 \text{ ft}}$

7.7 Find the length of the missing side of the right triangle. Round to three decimal places, if necessary. The legs of the right triangle are represented by a and b, and the hypotenuse is represented by c.

109) $a = 2$, $b = 7$



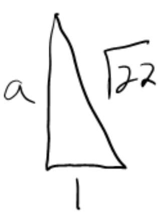
$a^2 + b^2 = c^2 \Rightarrow 2^2 + 7^2 = c^2 \Rightarrow 4 + 49 = c^2$

$\Rightarrow \sqrt{c^2} = \sqrt{53} \Rightarrow c = \pm\sqrt{53} \Rightarrow c = \sqrt{53}$

$\Rightarrow \boxed{c \approx 7.280}$

No $-\sqrt{}$ for distance

110) $b = 1$, $c = \sqrt{22}$

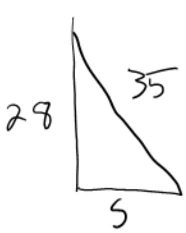


$a^2 + b^2 = c^2 \Rightarrow a^2 + 1^2 = (\sqrt{22})^2 \Rightarrow a^2 \underset{-1}{+1} = \underset{-1}{22} \Rightarrow \sqrt{a^2} = \sqrt{21}$

$\Rightarrow a = \sqrt{21} \Rightarrow \boxed{a \approx 4.583}$

Solve the problem. If necessary, round to the nearest tenth.

111) On a sunny day, a tree and its shadow form the sides of a right triangle. If the hypotenuse is 35 m long and the tree is 28 m tall, how long is the shadow?



$s^2 + 28^2 = 35^2 \Rightarrow s^2 + \underset{-784}{784} = \underset{-784}{1225} \Rightarrow$

$\sqrt{s^2} = \pm\sqrt{441} \Rightarrow \boxed{s = 21 \text{ m}}$

112) A car dealer advertised a big sale by stretching a string of banners from the top of the building to the edge of the driveway. If the building is 29 m high and the driveway is 44 m from the building, how long is the string of banners?

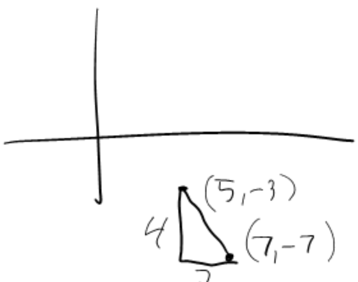
$29^2+44^2=B^2 \Rightarrow 841+1936=B^2$

$\Rightarrow \sqrt{B^2}=\sqrt{2777} \Rightarrow B=\sqrt{2777}$

$\Rightarrow B\approx \boxed{52.7\text{ m}}$

Find the distance between the pair of points. Give your answer in exact form and where appropriate find an approximation to three decimal places.

113) (5, −3) and (7, −7)



$2^2+4^2=d^2 \Rightarrow$

$4+16=d^2 \Rightarrow$

$\sqrt{20}=\sqrt{d^2} \Rightarrow$

$\sqrt{20}=\boxed{d=2\sqrt{5}\approx 4.472}$

OR

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(7-5)^2+(-7-{-3})^2}$

$=\sqrt{2^2+(-4)^2}$

$=\sqrt{4+16}$

$=\sqrt{20}=2\sqrt{5}\approx 4.472$

114) $\left(\frac{9}{11}, \frac{1}{22}\right)$ and $\left(\frac{1}{9}, \frac{19}{22}\right)$

$d=\sqrt{\left(\frac{1}{9}-\frac{9}{11}\right)^2+\left(\frac{19}{22}-\frac{1}{22}\right)^2}=\sqrt{\left(\frac{-70}{99}\right)^2+\left(\frac{\cancel{18}^{\,9}}{\cancel{22}_{\,11}}\right)^2}=\sqrt{\left(\frac{-70}{99}\right)^2+\left(\frac{9}{11}\cdot\frac{9}{9}\right)^2}=\sqrt{\frac{4900}{9801}+\frac{6561}{9801}}$

$=\sqrt{\frac{11461}{9801}}=\boxed{\frac{\sqrt{11461}}{99}\approx 1.081}$ Yuck!

115) $(-\sqrt{6}, \sqrt{23})$ and $(\sqrt{26}, -\sqrt{13})$

$d=\sqrt{(\sqrt{26}-{-\sqrt{6}})^2+(-\sqrt{13}-\sqrt{23})^2}=\sqrt{(\sqrt{26}+\sqrt{6})^2+(-\sqrt{13}-\sqrt{23})^2}=\sqrt{26+2\sqrt{26\cdot 6}+6+13+2\sqrt{13\cdot 23}+23}$

$2\cdot 13\cdot 2\cdot 3$

$=\boxed{\sqrt{68+4\sqrt{39}+2\sqrt{299}}\approx 11.294}$

Find the midpoint of the segment with the given endpoints.

116) (3, −9) and (−1, 8)

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\Rightarrow\left(\frac{3+-1}{2}, \frac{-9+8}{2}\right)=\left(\frac{2}{2}, \frac{-1}{2}\right)=\boxed{\left(1, -\frac{1}{2}\right)}$

117) $\left(-\frac{5}{2}, -\frac{3}{2}\right)$ and $\left(\frac{3}{2}, \frac{5}{2}\right)$

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\Rightarrow\left(\frac{-\frac{5}{2}+\frac{3}{2}}{2}, \frac{-\frac{3}{2}+\frac{5}{2}}{2}\right)=\left(\frac{\frac{-2}{2}}{2}, \frac{\frac{2}{2}}{2}\right)=\boxed{\left(\frac{-1}{2}, \frac{1}{2}\right)}$

118) $(\sqrt{7}, 7)$ and $(\sqrt{10}, 6)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \Rightarrow \left(\frac{\sqrt{7}+\sqrt{10}}{2}, \frac{7+6}{2}\right) = \boxed{\left(\frac{\sqrt{7}+\sqrt{10}}{2}, \frac{13}{2}\right)}$$

7.8 Express in terms of i.

119) $\sqrt{-9} = \sqrt{3\cdot 3\cdot -1} = \boxed{3i}$ $\quad \sqrt{-1} = i$ & $i^2 = -1$

120) $\sqrt{-189} = \sqrt{-1\cdot 3\cdot 3\cdot 3\cdot 7} = \boxed{3i\sqrt{21}}$

121) $-\sqrt{-216} = -\sqrt{-2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 3} = -2\cdot 3\cdot i\sqrt{2\cdot 3} = \boxed{-6i\sqrt{6}}$

Perform the indicated operation and simplify. Write the answer in the form a + bi.

122) $(6 - 6i) + (4 + 3i)$

$6 - 6i + 4 + 3i \Rightarrow \boxed{10 - 3i}$

123) $(14 - 9i) - (1 - 4i) \Rightarrow 14 - 9i - 1 + 4i \Rightarrow \boxed{13 - 5i}$

STANDARD FORM

124) $2i(5 - 9i) \Rightarrow 10i - 18i^2 \Rightarrow 10i - 18(-1) \Rightarrow 10i + 18$

$= \boxed{18 + 10i}$

125) $\sqrt{-14}\cdot\sqrt{-19} = i\sqrt{14}\cdot i\sqrt{19} = i^2\sqrt{266} = \boxed{-\sqrt{266}}$

Pull out i first!

FOIL

126) $(14 + 18i)(14 - 18i) \Rightarrow 196 - \cancel{252i} + \cancel{252i} - 324i^2$

$= 196 - 324(-1) = 196 + 324 = \boxed{520}$

127) $(-9 + 2i)^2 \Rightarrow (-9+2i)(-9+2i) \Rightarrow 81 - 18i - 18i + 4i^2$

FOIL

$\Rightarrow 81 - 36i + 4(-1) \Rightarrow 81 - 36i - 4 \Rightarrow \boxed{77 - 36i}$

128) $\left(\dfrac{2}{5 + i}\right) \cdot \dfrac{(5-i)}{(5-i)} \Rightarrow \dfrac{2(5-i)}{25 - \cancel{5i} + \cancel{5i} - i^2} \Rightarrow \dfrac{10 - 2i}{25 - (-1)} \Rightarrow \dfrac{10-2i}{25+1}$

$\Rightarrow \dfrac{10-2i}{26} \Rightarrow \dfrac{\cancel{10}^{5}}{\cancel{26}_{13}} - \dfrac{\cancel{2}^{1}i}{\cancel{26}_{13}} \Rightarrow \boxed{\dfrac{5}{13} - \dfrac{1}{13}i}$

BREAK APART
FOR STANDARD FORM

129) $\dfrac{7}{5i} \cdot \dfrac{i}{i}$

$\Rightarrow \dfrac{7i}{5i^2} \Rightarrow \dfrac{7i}{-5} \Rightarrow \boxed{-\dfrac{7}{5}i}$

i ON RIGHT
FOR standard form

130) $\left(\dfrac{8 + 9i}{9 - 3i}\right) \cdot \dfrac{(9+3i)}{(9+3i)} \Rightarrow \dfrac{72 + 24i + 81i + 27i^2}{81 + \cancel{27i} - \cancel{27i} - 9i^2} = \dfrac{72 + 105i - 27}{81 + 9}$

FOIL
FOIL

$\Rightarrow \dfrac{45 + 105i}{90} \Rightarrow \dfrac{45}{90} + \dfrac{105}{90}i \Rightarrow \boxed{\dfrac{1}{2} + \dfrac{7}{6}i}$

Find the power of i.

131) i^4

$i^4 = i^2 \cdot i^2 = (-1)(-1) = \boxed{1}$

($i^4 = 1$ is great to use for higher powers like THE NEXT ONE)

132) $i^{15} = i^4 \cdot i^4 \cdot i^4 \cdot i^2 \cdot i^1 = 1 \cdot 1 \cdot 1 \cdot (-1) \cdot i = \boxed{-i}$

OR $4\overline{)15}$ = 3 R3 $\rightarrow i^3$ so $i^{15} = i^3 = i^2 \cdot i = -i$

133) $(-i)^{10}$ $= (-1 \cdot i)^{10} = (-1)^{10} \; i^{10} = 1 \cdot i^4 \cdot i^4 \cdot i^2$

$= 1 \cdot 1 \cdot 1 \cdot (-1) = \boxed{-1}$

i^{10} : $4\overline{)10}$ 2 R 2 $\to i^2$ so $i^{10} = i^2 = -1$

134) $i^{64} + i^{945}$

$4\overline{)64}$ 16 R 0 $\to i^0$ $\qquad$ $4\overline{)945}$ 236 R 1 $\to i^1$ $\qquad$ so $i^{64} + i^{945} = \boxed{1 + i}$

$i^{64} = i^0 = 1$ $\qquad$ $i^{945} = i^1 = i$