Chapter 6 review

Sections labeled at the start of the related problems

6.1 Simplify by removing a factor equal to 1.

1) $\frac{2y^6}{6y^{12}}$

$\frac{2 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}{2 \cdot 3 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y} = \frac{1}{3y^6}$

OR

$\frac{2y^6}{6y^{12}} = \frac{1}{3y^6}$

Simplify by removing a factor equal to 1.

2) $\frac{10x - 5}{25}$

$\frac{10x-5}{25} = \frac{5(2x-1)}{25} = \frac{2x-1}{5}$

3) $\frac{3x + 2}{6x^2 + 19x + 10}$

$\frac{3x+2}{(3x+2)(2x+5)} = \frac{1}{2x+5}$

If you reduce and nothing is left, you are actually left with 1, so don't forget to leave it there. In this case, it is all that is left in the denominator.

4) $\frac{m^2 - 9m}{9 - m}$

$\frac{m(m-9)}{-1(m-9)} = -m$

If your factors are exact opposites, like m-9 and 9-m, or 4-x+y and -4+x-y, they will cancel and leave a -1. The -1 can either be left in the numerator or the denominator. Just make sure that you don't leave it in both, or they would cancel and this is not correct.

5) $\frac{x^3 - 125}{x^2 + 5x + 25}$

$\frac{(x-5)(x^2+5x+25)}{(x^2+5x+25)} = x-5$

When you factor out a sum or diffference of cubes, the resulting trinomial factor is either prime or very difficult to factor, so if you are trying to reduce, look for another trinomial factor that is the same to reduce.

Write in simplified form and list all restrictions on the domain.

6) $f(x) = \frac{x^2 + 7x + 6}{x^2 - 4x - 5}$

$f(x) = \frac{(x+1)(x+6)}{(x+1)(x-5)}$ $\quad f(x) = \frac{x+6}{x-5}$

DOMAIN $= \{x \mid x \neq -1, 5\}$

Make sure you find the domain restrictions before you simplify factors or you will not get all of the exceptions to the demain.

Multiply and simplify.

7) $\frac{7p - 7}{p} \cdot \frac{5p^2}{8p - 8}$

$\frac{7(p-1)}{p} \cdot \frac{5p^2}{8(p-1)} = \frac{35p}{8}$

8) $\frac{k^2 + 9k + 20}{k^2 + 11k + 28} \cdot \frac{k^2 + 7k}{k^2 + 13k + 40}$

$\frac{(k+4)(k+5)}{(k+4)(k+7)} \cdot \frac{k(k+7)}{(k+5)(k+8)} = \boxed{\frac{k}{k+8}}$

Divide and simplify.

9) $\frac{5x - 15}{x} \div \frac{x - 3}{x^3}$

$\frac{5x-15}{x} \cdot \frac{x^3}{x-3}$

$\frac{5(x-3)}{x} \cdot \frac{x^{3-2}}{x-3} = \boxed{5x^2}$

10) $\frac{y^3 - 10y}{y^2 - 100} \div \frac{y^2 - 14y + 45}{y^2 + 5y - 50} = \frac{y^3-10y}{y^2-100} \cdot \frac{y^2+5y-50}{y^2-14y+45} =$

$= \frac{y(y^2-10)}{(y+10)(y-10)} \cdot \frac{(y+10)(y-5)}{(y-9)(y-5)} = \boxed{\frac{y(y^2-10)}{(y-10)(y-9)}}$

Perform the indicated operations and, if possible, simplify. Recall that multiplications and divisions are performed in order from left to right.

11) $\frac{3x^2 - 4x - 4}{y^2 + 2y - 3} \cdot \frac{y^2 + 3y - 4}{9x^2 + 9x + 2} \div \frac{3x^2 - 5x - 2}{6x^2 - 7x - 3} = \frac{3x^2-4x-4}{y^2+2y-3} \cdot \frac{y^2+3y-4}{9x^2+9x+2} \cdot \frac{6x^2-7x-3}{3x^2-5x-2}$

$= \frac{(3x+2)(x-2)}{(y+3)(y-1)} \cdot \frac{(y+4)(y-1)}{(3x+1)(3x+2)} \cdot \frac{(2x-3)(3x+1)}{(3x+1)(x-2)}$

$= \boxed{\frac{(y+4)(2x-3)}{(y+3)(3x+1)}}$

12) $\frac{r^2 - s^2}{(r-s)^2} \div \frac{r^2 - 2rs + s^2}{r^2 - rs + s^2} \cdot \frac{(r-s)^4}{r^3 + s^3}$ $= \frac{r^2-s^2}{(r-s)^2} \cdot \frac{r^2-rs+s^2}{r^2-2rs+s^2} \cdot \frac{(r-s)^4}{r^3+s^3}$

$= \frac{(r+s)(r-s)}{(r-s)(r-s)} \cdot \frac{r^2-rs+s^2}{(r-s)(r-s)} \cdot \frac{(r-s)(r-s)(r-s)(r-s)}{(r+s)(r^2-rs+s^2)}$

$= r-s$

6.2 Perform the indicated operation and simplify.

13) $\frac{x + 4y}{x + y} + \frac{2x - y}{x + y}$

$= \frac{x+4y+2x-y}{x+y} = \frac{3x+3y}{x+y} = \frac{3(x+y)}{x+y} = 3$

14) $\frac{2x + 5}{x^2 + 7x + 10} - \frac{x + 3}{x^2 + 7x + 10}$ $= \frac{2x+5-(x+3)}{x^2+7x+10}$

$= \frac{2x+5-x-3}{x^2+7x+10} = \frac{x+2}{x^2+7x+10} = \frac{x+2}{(x+2)(x+5)}$

$= \frac{1}{x+5}$

Find the simplified form for f(x) and list all restrictions on the domain.

15) $f(x) = \frac{4x - 1}{x^2 + 2x - 24} - \frac{x - 1}{x^2 - 1}$

$(x+6)(x-4) \quad (x+1)(x-1)$

LCD: $(x+6)(x-4)(x+1)$

The domain restrictions come from before any reducing factors happened.

$\frac{(x+1)}{(x+1)}\frac{(4x-1)}{(x+6)(x-4)} - \frac{1}{(x+1)}\frac{(x+6)(x-4)}{(x+6)(x-4)}$

$= \frac{4x^2+3x-1}{(x+1)(x+6)(x-4)} - \frac{(x^2+2x-24)}{(x+1)(x+6)(x-4)} = \frac{4x^2+3x-1-x^2-2x+24}{(x+1)(x+6)(x-4)} = \frac{3x^2+x+23}{(x+1)(x+6)(x-4)}$

$x \neq -6, 4, -1, 1$

Perform the indicated operation and simplify.

16) $\dfrac{6x}{x^2 - 16} - \dfrac{x}{x - 4}$

$\dfrac{6x}{(x+4)(x-4)} - \dfrac{x}{(x-4)}\dfrac{(x+4)}{(x+4)} = \dfrac{6x}{(x+4)(x-4)} - \dfrac{x^2+4x}{(x+4)(x-4)}$

$= \dfrac{6x - x^2 - 4x}{(x+4)(x-4)} = \boxed{\dfrac{-x^2+2x}{(x+4)(x-4)}}$

Be very careful when subtracting terms. Make sure the negative sign gets properly distributed into the entire numerator.

17) $\dfrac{2}{y^2 - 3y + 2} + \dfrac{5}{y^2 - 1}$

$(y-2)(y-1) \quad (y+1)(y-1)$

$= \dfrac{2}{(y-2)(y-1)}\dfrac{(y+1)}{(y+1)} + \dfrac{5}{(y+1)(y-1)}\dfrac{(y-2)}{(y-2)}$

$= \dfrac{2y+2+5y-10}{(y-2)(y-1)(y+1)} = \boxed{\dfrac{7y-8}{(y-2)(y-1)(y+1)}}$

18) $\dfrac{4x + 5}{x^2 + 2x - 48} - \dfrac{x + 1}{x^2 - 64}$

$(x+8)(x-6) \quad (x+8)(x-8)$

$= \dfrac{(4x+5)}{(x+8)(x-6)}\cdot\dfrac{(x-8)}{(x-8)} - \dfrac{(x+1)}{(x+8)(x-8)}\cdot\dfrac{(x-6)}{(x-6)}$

$= \dfrac{4x^2-27x-40}{(x+8)(x-6)(x-8)} - \dfrac{x^2-5x-6}{(x+8)(x-8)(x-6)} = \dfrac{4x^2-27x-40-x^2+5x+6}{(x+8)(x-6)(x-8)}$

$= \boxed{\dfrac{3x^2-22x-34}{(x+8)(x-6)(x-8)}}$

Perform the indicated operation.

19) $\dfrac{x}{2x - 3y} - \dfrac{y}{3y - 2x}\dfrac{(-1)}{(-1)} = \dfrac{x}{2x-3y} + \dfrac{y}{2x-3y}$

$= \boxed{\dfrac{x+y}{2x-3y}}$

If you have exact opposite denominators, they are basically already in common. All you need to do is multiply one of them by -1/-1.

Perform the indicated operations and simplify.

20) $\dfrac{8}{4-5x} - \dfrac{2}{5x-4} + \dfrac{x-4}{5x^2+16x-16}$

$(5x-4)(x+4)$

$= \dfrac{-8}{5x-4} - \dfrac{2}{5x-4} + \dfrac{x-4}{(5x-4)(x+4)}$ $\quad \dfrac{-10}{5x-4}$

$= \dfrac{-10}{(5x-4)}\dfrac{(x+4)}{(x+4)} + \dfrac{x-4}{(5x-4)(x+4)} = \dfrac{-10x-40}{(5x-4)(x+4)} + \dfrac{x-4}{(5x-4)(x+4)}$

$= \dfrac{-10x-40+x-4}{(5x-4)(x+4)} = \boxed{\dfrac{-9x-44}{(5x-4)(x+4)}}$

6.3 Simplify.

21) $\dfrac{4+\frac{2}{x}}{\frac{x}{4}+\frac{1}{8}}$

LCD of MINI FRACTIONS is $8x$, so

$\dfrac{8x\cdot 4 + \frac{2}{x}\cdot 8x}{\frac{x}{4}\cdot 8x + \frac{1}{8}\cdot 8x} = \dfrac{32x+16}{2x^2+x} = \dfrac{16(2x+1)}{x(2x+1)} = \boxed{\dfrac{16}{x}}$

The method I use is the 2nd method taught in the book, and is the fastest method I have found to simplify these complex rational expressions.

Make sure you multiply the NUMERATORS of ALL of the individual terms in the entire fraction by the same LCD.

22) $\dfrac{m^{-1}+z^{-1}}{m^{-1}-z^{-1}} = \dfrac{\frac{1}{m}+\frac{1}{z}}{\frac{1}{m}-\frac{1}{z}}$

LCD is mz, so

$\dfrac{mz\cdot\frac{1}{m}+\frac{1}{z}\cdot mz}{mz\cdot\frac{1}{m}-\frac{1}{z}\cdot mz} = \boxed{\dfrac{z+m}{z-m}}$

A problem with negative exponents should first be rewritten in a complex rational fraction form, and then simplified. Make sure you apply the negative exponent only on its base. The 4 in the second numerator term is not part of the base of the -2 exponent, and so doesn't get flipped or squared.

23) $\dfrac{x^{-2}-4y^{-2}}{8y-16x} = \dfrac{\frac{1}{x^2}-\frac{4}{y^2}}{8y-16x}$

LCD is x^2y^2 so

$\dfrac{x^2y^2\cdot\frac{1}{x^2}-\frac{4}{y^2}\cdot x^2y^2}{x^2y^2\cdot 8y-16x\cdot x^2y^2} = \dfrac{y^2-4x^2}{8x^2y^3-16x^3y^2} = \dfrac{(y+2x)(y-2x)}{8x^2y^2(y-2x)} = \boxed{\dfrac{y+2x}{8x^2y^2}}$

24) $\dfrac{\frac{25s^2 - 49t^2}{st}}{\frac{5}{t} - \frac{7}{s}}$ LCD is st, so

$\dfrac{st\cdot(25s^2-49t^2)}{st} = \dfrac{25s^2-49t^2}{5s-7t} = \dfrac{(5s+7t)(5s-7t)}{5s-7t}$

$st\cdot\frac{5}{t} - \frac{7}{s}\cdot st$

$= \boxed{5s+7t}$

25) $\dfrac{\frac{2}{x-2} + \frac{5}{x+5}}{\frac{5}{x+5} - \frac{2}{x-5}}$ LCD is $(x-2)(x+5)(x-5)$ so

$\dfrac{\frac{2}{x-2}\cdot(x-2)(x+5)(x-5) + \frac{5}{x+5}\cdot(x-2)(x+5)(x-5)}{\frac{5}{x+5}\cdot(x-2)(x+5)(x-5) - \frac{2}{x-5}\cdot(x-2)(x+5)(x-5)} = \dfrac{2(x^2-25)+5(x^2-7x+10)}{5(x-2)(x-5)-2(x-2)(x+5)} =$

$= \dfrac{2x^2-50+5x^2-35x+50}{5x^2-35x+50-2x^2-6x+20} = \boxed{\dfrac{7x^2-35x}{3x^2-41x+70}}$

26) $\dfrac{\frac{x}{x^2+7x+10} + \frac{5}{x^2+7x+10}}{\frac{x}{x^2+5x+6} - \frac{6}{x^2+5x+6}} = \dfrac{\frac{x+5}{x^2+7x+10}}{\frac{x-6}{x^2+5x+6}} =$

$= \dfrac{x+5}{x^2+7x+10}\cdot\dfrac{x^2+5x+6}{x-6} = \dfrac{(x+5)}{(x+5)(x+2)}\cdot\dfrac{(x+2)(x+3)}{(x-6)} = \boxed{\dfrac{x+3}{x-6}}$

6.4 Solve.

27) $\frac{4}{x} + \frac{5}{6} = 1$

LCD is $6x$, so

$6x \cdot \frac{4}{x} + 6x \cdot \frac{5}{6} = 1 \cdot 6x \Rightarrow 24 + 5x = 6x$

$-5x \quad -5x$

$x = 24$

Check: $\frac{4}{24} + \frac{5}{6} = \frac{1}{6} + \frac{5}{6} = 1$ ✓

Rational equations can be checked as all equations can. The minimum checking you should do is to make sure none of the answers you got cause a divide by zero error in any of the fractions of the initial equation. If they do, throw them out. If you are left with no answers after you do this, the final result is NO SOLUTION.

28) $\frac{2}{t} = \frac{t}{5t - 12}$

LCD is $t(5t-12)$

$\frac{2}{t} \cdot t(5t-12) = \frac{t}{5t-12} \cdot t(5t-12) \Rightarrow 2(5t-12) = t^2$

$\Rightarrow 10t - 24 = t^2 \Rightarrow t^2 - 10t + 24 = 0$

$\Rightarrow (t-6)(t-4) = 0 \Rightarrow t = 4, 6$

Check: $t = 4$

$\frac{2}{4} = \frac{4}{5(4)-12}$

$\frac{1}{2} = \frac{4}{20-12}$

$\frac{1}{2} = \frac{4}{8}$ ✓

$t = 6$

$\frac{2}{6} = \frac{6}{5(6)-12}$

$\frac{1}{3} = \frac{6}{30-12}$

$\frac{1}{3} = \frac{6}{18}$ ✓

29) $\frac{6}{x-8} + \frac{6}{x} = \frac{-48}{x^2 - 8x}$

$x^2 - 8x = x(x-8)$

LCD is $x(x-8)$

$\frac{6}{x-8} \cdot x(x-8) + \frac{6}{x} \cdot x(x-8) = \frac{-48}{x(x-8)} \cdot x(x-8)$

$\Rightarrow 6x + 6x - 48 = -48$

$\Rightarrow 12x - 48 = -48$

$+48 \quad +48$

$\Rightarrow \frac{12x}{12} = \frac{0}{12} \Rightarrow x = 0$

Check: $x = 0$

$\frac{6}{0-8} + \frac{6}{0} = \frac{-48}{0(0-8)}$ No! Don't ÷ by 0

$\emptyset$

This problem has an extraneous solution x=0 that causes the second and third fraction to have a divide by zero error. This is the reason it is tossed out, leaving no solution.

30) $\frac{x}{2x+2} = \frac{-2x}{4x+4} + \frac{2x-3}{x+1}$

$2(x+1) \quad 4(x+1)$

LCD is $4(x+1)$

$\frac{x}{2(x+1)} \cdot 2(x+1) = \frac{-2x}{4(x+1)} \cdot 4(x+1) + \frac{2x-3}{x+1} \cdot 4(x+1)$

$\Rightarrow 2x = -2x + 8x - 12$

$\Rightarrow 2x = 6x - 12$

$+12 \quad -2x \quad -2x \quad +12$

$\frac{12}{4} = \frac{4x}{4}$

$x = 3$

Check: $x = 3$

$\frac{3}{2(3)+2} = \frac{-2(3)}{4(3)+4} + \frac{2(3)-3}{3+1}$

$\frac{3}{8} = \frac{-6}{16} + \frac{3}{4}$

$\frac{3}{8} = -\frac{3}{8} + \frac{3}{4} = \frac{3}{8}$ ✓

Find all values of a for which f(a) is the indicated value.

31) $f(x) = \frac{x-1}{x-8}$; $f(a) = \frac{1}{5}$

$f(a) = \frac{a-1}{a-8}$, $f(a) = \frac{1}{5} \Rightarrow \frac{a-1}{a-8} = \frac{1}{5}$ LCD is $5(a-8)$

$\frac{(a-1)}{a-8}\cdot 5(a-8) = \frac{1}{5}\cdot 5(a-8) \Rightarrow 5a-5 = a-8$

$-a+5 \quad -a+5$

$\Rightarrow 4a = -3 \Rightarrow \boxed{a = -\frac{3}{4}}$

Check $a = -\frac{3}{4}$

$f\left(-\frac{3}{4}\right) = \frac{-\frac{3}{4}-1}{-\frac{3}{4}-8} = \frac{-\frac{7}{4}}{-\frac{35}{4}} = \frac{-7}{-35} = \frac{1}{5}$ ✓

For the pair of functions f and g, find all values of a for which f(a) = g(a).

32) $f(x) = \frac{x+3}{110x}$, $g(x) = \frac{1}{x^2+2x}$

$\frac{x+3}{110x} = \frac{1}{x^2+2x}$ LCD is $110x(x+2)$

$x(x+2)$

$\frac{(x+3)}{110x}\cdot 110x(x+2) = \frac{1}{x(x+2)}\cdot 110x(x+2) \Rightarrow (x+3)(x+2) = 110$

$\Rightarrow x^2+5x+6 = 110 \Rightarrow x^2+5x-104 = 0$

$-110 \quad -110$

$\Rightarrow (x+13)(x-8) = 0 \Rightarrow \boxed{x = -13, 8}$

Check: $x = -13$ $x = 8$

$f(-13) = g(-13)$ $f(8) = g(8)$

6.5 Solve.

33) The sum of a number and its reciprocal is −2. Find the number.

$x + \frac{1}{x} = -2$ LCD is x so

$x\cdot x + \frac{1}{x}\cdot x = -2\cdot x \Rightarrow x^2+1 = -2x \Rightarrow x^2+2x+1 = 0$

$+2x \quad +2x$

$\Rightarrow (x+1)(x+1) = 0 \Rightarrow \boxed{x = -1}$

Solve.

34) One maid can clean the house three times faster than another. Working together they can clean the entire house in 3 hours. How long would it take the faster maid cleaning alone?

(faster) Maid 1 TIME $= x$

MAID 2 TIME $= 3x$

$\frac{1}{x} + \frac{1}{3x} = \frac{1}{3}$ LCD is $3x$ so

$3x\cdot\frac{1}{x} + \frac{1}{3x}\cdot 3x = \frac{1}{3}\cdot 3x \Rightarrow 3+1 = x \Rightarrow x = 4$

Faster MAID $\boxed{4 \text{ hours}}$

Work problems like this are always solved in the form 1/individual time + 1/individual time = 1/total time.

35) Frank can type a report in 4 hours and James takes 5 hours. How long will it take the two of them typing together?

$\frac{1}{4}+\frac{1}{5}=\frac{1}{T}$ LCD is $4\cdot5\cdot T$

$\frac{1}{4}\cdot 4\cdot5\cdot T+\frac{1}{5}\cdot 4\cdot5\cdot T=\frac{1}{T}\cdot 4\cdot5\cdot T \Rightarrow 5T+4T=20$

$\Rightarrow \frac{9T}{9}=\frac{20}{9} \Rightarrow T=\boxed{\frac{20}{9}\text{ hrs}}$

36) Jeff takes 5 hr longer to build a fence than it takes Bill. When they work together, it takes them 6 hours. How long would it take Bill to do the job alone?

Bill time $= x$

Jeff time $= x+5$

$\frac{1}{x}+\frac{1}{x+5}=\frac{1}{6}$ LCD is $6x(x+5)$

$\frac{1}{x}\cdot 6x(x+5)+\frac{1}{x+5}\cdot 6x(x+5)=\frac{1}{6}\cdot 6x(x+5) \Rightarrow 6(x+5)+6x=x(x+5) \Rightarrow 6x+30+6x=x^2+5x$

$\Rightarrow 12x+30=x^2+5x \Rightarrow x^2-7x-30=0 \Rightarrow (x-10)(x+3)=0$

$\Rightarrow x=10$ or $x=-3$ (no neg time) so $\boxed{10\text{ hours is Bill's time}}$

37) A loaded moving truck is traveling 20 mph faster than a freight train. In the time it takes the train to travel 90 miles, the truck travels 150 miles. Find the speed of the truck.

I solve these type of distance problems by setting up the time statement. In this problem, it states SAME TIME.

$x=$ speed of truck : since $D=RT$, $T=\frac{D}{R}$

$x-20=$ speed of train

$T_{truck}=T_{train} \Rightarrow \frac{D_{truck}}{R_{truck}}=\frac{D_{train}}{R_{train}} \Rightarrow \frac{150}{x}=\frac{90}{x-20}$ LCD is $x(x-20)$

$\Rightarrow \frac{150}{x}\cdot x(x-20)=\frac{90}{x-20}\cdot x(x-20) \Rightarrow 150(x-20)=90x \Rightarrow 150x-300=90x \Rightarrow 60x=300$

$-90x \quad +300 \quad -90x \quad +300 \quad \frac{60}{60} \quad \frac{300}{60}$

$\Rightarrow x=\boxed{50\text{ mph truck}}$

38) The speed of a stream is 5 mph. If a boat travels 48 miles downstream in the same time that it takes to travel 24 miles upstream, what is the speed of the boat in still water?

$x=$ speed of boat

$5=$ speed of stream

$\Rightarrow x+5=$ speed downstream

$x-5=$ speed upstream, $T=\frac{D}{R}$

SAME TIME

$T_{up}=T_{down} \Rightarrow \frac{D_{up}}{R_{up}}=\frac{D_{down}}{R_{down}} \Rightarrow \frac{24}{x-5}=\frac{48}{x+5}$, LCD is $(x+5)(x-5)$

$\frac{24}{(x-5)}\cdot(x+5)(x-5)=\frac{48}{(x+5)}\cdot(x+5)(x-5) \Rightarrow 24(x+5)=48(x-5) \Rightarrow 24x+120=48x-240$

$-24x \quad +240 \quad -24x \quad +240$

$\Rightarrow \frac{360}{24}=\frac{24x}{24} \Rightarrow x=\boxed{15\text{ mph boat}}$

39) A jet plane traveling at a constant speed goes 1200 miles with the wind, then turns around and travels for 1000 miles against the wind. If the speed of the wind is 50 mph and the total flight took 4 hours, find the speed of the plane in still air.

x = speed of plane, 50 = speed of wind $\Rightarrow$ $x+50$ = speed with wind ; $x-50$ = speed against wind ; $T = \frac{D}{R}$; Total time $= T_{with} + T_{against}$

TOTAL TIME=T1+T2

$T_{with} + T_{against} = 4 \Rightarrow \frac{D_{with}}{R_{with}} + \frac{D_{against}}{R_{against}} = 4 \Rightarrow \frac{1200}{x+50} + \frac{1000}{x-50} = 4$, LCD $(x+50)(x-50)$

$\Rightarrow \frac{1200}{x+50}\cdot(x+50)(x-50) + \frac{1000}{x-50}\cdot(x+50)(x-50) = 4(x+50)(x-50)$

$\Rightarrow 1200x - 60000 + 1000x + 50000 = 4x^2 - 10000 \Rightarrow 4x^2 - 2200x = 0$

$\Rightarrow 4x(x-550) = 0 \Rightarrow x = 0$ or $x = 550$ mph plane

6.6 Divide.

40) $(15x^5y^4 - 12x^2y^2 - 9x^3y) \div (-3x^2y)$

$\frac{15x^5y^4}{-3x^2y} + \frac{12x^2y^2}{-3x^2y} + \frac{9x^3y}{-3x^2y} = 5x^3y^3 + 4y + 3x$

Dividing Polynomials comes in two main cases. This problem represents the case of dividing by a monomial. Just distribute the division and simplify and you are done.

41) $(x^2 + 13x + 31) \div (x + 9)$

$x+4$

$x+9 \;\overline{)\; x^2 + 13x + 31}$

$-x^2 - 9x$

$0 + 4x + 31$

$-4x - 36$

-5

so $x + 4 - \frac{5}{x+9}$

If you are dividinng by a polynomial with at least 2 terms, you perform the division by long division.

42) $(x^2 - 10x + 16) \div (x - 8)$

$x-2$

$x-8 \;\overline{)\; x^2 - 10x + 16}$

$-x^2 + 8x$

$-2x + 16$

$+2x - 16$

0

$x-2$

or $\frac{x^2 - 10x + 16}{x-8} = \frac{(x-8)(x-2)}{x-8} = x-2$

THIS WORKS IF NO REMAINDER & YOU CAN FACTOR

43) $(25y^4 + 10y^3 + 2y - 1) \div (5y^2 + 1)$

$5y^2+0y+1 \;\overline{)\; 25y^4+10y^3+0y^2+2y-1}$

$5y^2+2y-1$

$-25y^4+0y^3+5y^2$

$10y^3-5y^2+2y$

$-10y^3+0y+2y$

$-5y^2+0y-1$

$\pm 5y^2 \mp 0y \pm 1$

0

so $5y^2+2y-1$

In long division, make sure to use place holders if any of the powers of x are missing. This will allow your work to line up nicely and ends up being much more consistent and easy to do.

Find a simplified expression for F(x) if F(x) = (f/g)(x).

44) $f(x) = 20x^2 - 46x - 10$, $g(x) = 5x + 1$

$F(x) = \frac{20x^2-46x-10}{5x+1} = \frac{2(10x^2-23x-5)}{5x+1} = \frac{2(5x+1)(2x-5)}{5x+1} = 2(2x-5)$

$= 4x-10$

or

$5x+1 \;\overline{)\; 20x^2-46x-10}$

$4x-10$

$-20x^2+4x$

$-50x-10$

$\pm 50x \pm 10$

0

so $4x-10$

6.8 Solve the formula for the specified letter.

45) $A = \frac{1}{2}h(B + b)$ for b LCD is 2

$2 \cdot A = 2 \cdot \frac{1}{2} h(B+b)$

$2A = h(B+b)$

$\Rightarrow 2A = hB + hb$

$-hB \quad -hB$

$\Rightarrow \frac{2A-hB}{h} = \frac{hb}{h}$

$b = \frac{2A-hB}{h}$

Solve the formulas for a variable the same way you solve a linear equation of one variable. That is
1- Clear Fractions
2- Distribute into parentheses
3- Isolate the object you are trying to solve for.

46) $P = \frac{A}{1 + rt}$ for r LCD is $1+rt$

$P(1+rt) = \frac{A}{1+rt}(1+rt) \Rightarrow P(1+rt) = A$

$\Rightarrow P + Prt = A \Rightarrow \frac{Prt}{Pt} = \frac{A-P}{Pt}$

$-P \quad -P$

$\Rightarrow r = \frac{A-P}{Pt}$

If after step 1 and 2 you find more than one term with the object you want to solve for, you get all of these terms on the same side and the rest on the other side. Then you should be able to factor out the wanted object. Then divide by the remaining stuff and you are done.

47) $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$ for c LCD is abc

$\frac{1}{a}\cdot abc+\frac{1}{b}\cdot abc=\frac{1}{c}\cdot abc$

$\Rightarrow bc+ac=ab$

$\Rightarrow c(b+a)=ab$

$\Rightarrow c=\frac{ab}{b+a}=\frac{ab}{a+b}$

Solve the problem.

48) The formula

$$A=\frac{2Tt+Qq}{2T+Q}$$ LCD is $2T+Q$

gives a student's average A after T tests and Q quizzes, where each test counts as 2 quizzes, t is the test average and q is the quiz average. Solve for T.

A) $T=\frac{Qq-QA}{2A-2t}$

B) $T=\frac{2At+QA-Qq}{2t}$

C) $T=\frac{2t+Qq-QA}{2A}$

D) $T=\frac{Qq-A}{2A-2t}$

$A\cdot(2T+Q)=\frac{2Tt+Qq}{2T+Q}\cdot(2T+Q)$

$\Rightarrow A(2T+Q)=2Tt+Qq$

$2AT+AQ=2Tt+Qq$

$-2Tt-AQ \quad -2Tt \quad -AQ$

$2AT-2Tt=Qq-AQ$

$T(2A-2t)=Qq-AQ$

$2A-2t \quad 2A-2t$

$T=\frac{Qq-AQ}{2A-2t}$

Stick to the same steps of 1-fractions, 2-parentheses and these problems get easy.