Chapter 4,5 review

Sections labeled at the start of the related problems

4.1 Classify as equivalent inequalities, equivalent equations, equivalent expressions, or not equivalent.

1) $-\frac{1}{4}v \le -7$, $v \ge 28$

$(-4)\left(-\frac{1}{4}v\right) \le (-7)(-4)$ $\quad v \ge 28$

$v \ge 28$ EQUIVALENT

Inequalities are equivalent if they have the same solution set, so you need to solve each one to determine if they are equivalent.

Choose the number that is a solution of the inequality.

2) $-3n - 7 \le -4n - 18$

A) -9 B) -8 C) -10 D) -11

$-3n - 7 \le -4n - 18$

$+4n \quad +7 \quad +4n \quad +7$

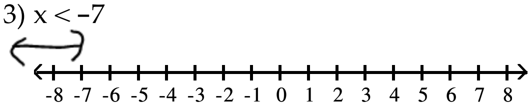
$n \le -11$

so D

Solving inequalities is similar to solving equations. The main differences are that the answers are typically infinite and so usually are given in interval notation. Also, you need to remember to switch the inequality if you multiply or divide by a negative number, as in #6 and #8.
One other time you will switch an inequality is when you have the variable on the right side and want it on the left side. Then a mirror image happens, which includes a switch in the inequality. Examples of this type are in #20 and #21.

Graph the inequality and write the solution set using both set-builder notation and interval notation.

3) $x < -7$



$\{x \mid x < -7\}$

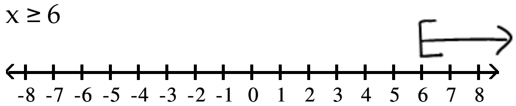
$(-\infty, -7)$

For both the graph notation and the interval notation, it is necessary to put the numbers in numerical order, as they are found on the real number line. The difference between the (or) and the [or] are whether the inequality includes an =

$\le$ Gives] $\quad$ $<$ gives)

$\ge$ Gives [$\quad$ $>$ gives (

4) $x \ge 6$

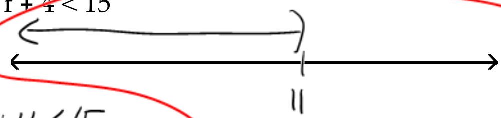


$\{x \mid x \ge 6\}$

$[6, \infty)$

Solve and graph the inequality. Write the solution set using interval notation.

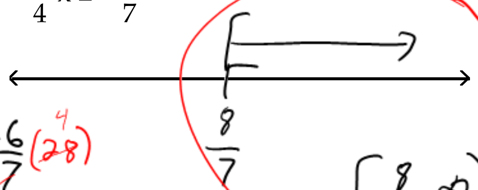
5) $f + 4 < 15$



$f + 4 < 15$
$\quad -4 \quad -4$
$f < 11$

$(-\infty, 11)$

6) $-\frac{3}{4}x \le -\frac{6}{7}$



$(28) \; -\frac{3}{4}x \le -\frac{6}{7}(28)$

$-21x \le -24$

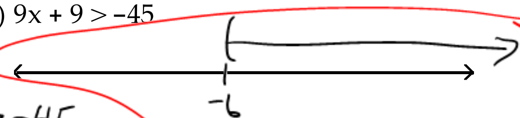
$x \ge \frac{24}{21}$

$x \ge \frac{8}{7}$

$\left[\frac{8}{7}, \infty\right)$

The steps at the beginning might include clearing fractions, which is when you may see a multiply by a negative number. Make sure that if you do you switch the inequality.

7) $9x + 9 > -45$



$9x + 9 > -45$
$\quad -9 \quad -9$

$\frac{9x}{9} > \frac{-54}{9}$

$x > -6$

$(-6, \infty)$

Solve.

8) $-30r - 5 \le -5(5r - 4)$ Put your answer in interval notation.

$-30r - 5 \le -25r + 20$
$+25r \; +5 \quad +25r \; +5$

$\frac{-5r}{-5} \le \frac{25}{-5}$

$r \ge -5$

$[-5, \infty)$

The last steps may include a divide by negative number. Make sure you switch the inequality.

Find the domain of the function. Put your answer in set-builder notation.

9) $f(x) = \sqrt{6x - 5}$

$6x - 5 \ge 0$
$\quad +5 \quad +5$

$\frac{6x}{6} \ge \frac{5}{6}$

$x \ge \frac{5}{6}$

$\left\{x \mid x \ge \frac{5}{6}\right\}$

Solve the inequality.

10) A car rental company has two rental rates. Rate 1 is $45 per day plus $.18 per mile. Rate 2 is $90 per day plus $.09 per mile. If you plan to rent for one week, how many miles would you need to drive to pay less by taking Rate 2?

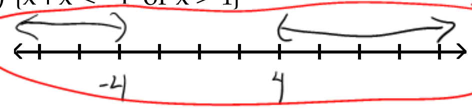
Let x = # miles driven, And for 7 days.
Then Rate 1 cost is $45 \cdot 7 + .18x$
& Rate 2 cost is $90 \cdot 7 + .09x$
Paying less for rate 2 is Rate 1 > Rate 2 so

$$315 + .18x > 630 + .09x$$
$$-315 \quad -.09x \quad -315 \quad -.09x$$
$$\frac{.09x}{.09} > \frac{315}{.09}$$
$$x > 3500$$

so $\boxed{3500 \text{ MILES}}$

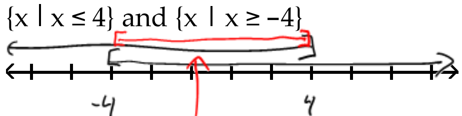
4.2 Graph the set on the number line.

11) {x | x < −4 or x > 4}



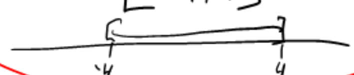
For compound inequalities, they will either give you the word OR (or union) or AND (or intersection). The way I do these is to graph each inequality on the same numberline.
Then if it is OR, I include as my final answer all of the numberline that has been covered by any of the inequalities. These may be all real numbers.
If it is AND, I include as my final answer all of the numberline that has been covered by both inequalities, so the overlap. These may have no solution.

12) {x | x ≤ 4} and {x | x ≥ −4}



OVERLAP

so $[-4, 4]$



Find the indicated intersection or union.

13) Let A = {q, s, u, v, w, x, y, z}, B = {q, s, y, z}, C = {v, w, x, y, z}, and D = {s}. List the elements in the set A ∪ B.

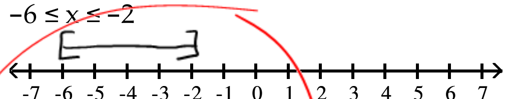
$A \cup B = \{q, s, u, v, w, x, y, z\}$

∪ means what is in one, the other or both.

∪ is union and can be interpreted as what is in one, the other, or both.
∩ is intersection and can be interpreted as what is in both.

Graph and write interval notation for the compound inequality.

14) −6 ≤ x ≤ −2



$[-6, -2]$

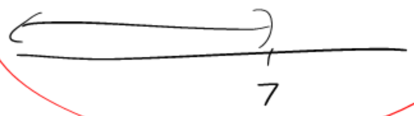
This compound inequality is in a condensed form. These type will not have the keywords OR or AND involved, though sometimes the instuctions may reference the intersection. These type are always an AND intersection problem. You will never see an OR intersection compound inequality in this condensed form.

These AND condensed form compound inequalities are the easiest type to solve. The final answer is easy to get without graphing both inequalities and looking for the overlap.

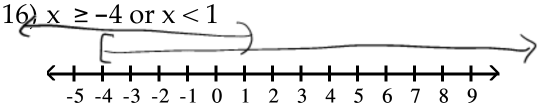
15) x < 5 or x < 7

OR: What is included in one, the other. or both.

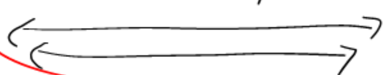
$(-\infty, 7)$



16) x ≥ –4 or x < 1

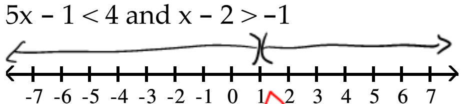


$(-\infty, \infty)$



Solve the inequality and graph the solution set.

17) 5x – 1 < 4 and x – 2 > –1



AND: What is included by both.

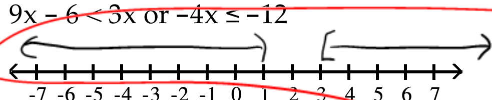
$5x - 1 < 4$ $\quad$ $x - 2 > -1$

$5x < 5$ $\quad$ $x > 1$

$x < 1$

NO OVERLAP SO $\emptyset$

18) 9x – 6 < 3x or –4x ≤ –12



$(-\infty, 1) \cup [3, \infty)$

One note about the solutions that include all of the numbers to the left or right of a number. You will use the infinity sign. The negative infinity will always go first, and the positive infinity will always go last. You are also limited to just the (or) symbol with the infinity regardless of the type of inequality you are working with.

$9x - 6 < 3x$
$-3x + 6 \quad -3x + 6$

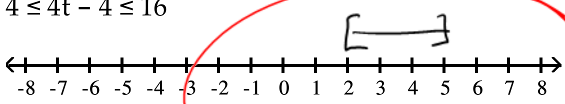
$\frac{6x}{6} < \frac{6}{6}$

$x < 1$

$-4x \le -12$
$\overline{-4} \quad \overline{-4}$

$x \ge 3$

19) $4 \le 4t - 4 \le 16$

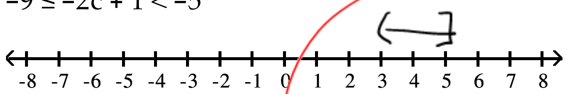


$4 \le 4t - 4 \le 16$
$+4 \quad +4 \quad +4$
$8 \le 4t \le 20$
$\frac{8}{4} \le \frac{4t}{4} \le \frac{20}{4}$
$2 \le t \le 5$

$[2, 5]$

For these condensed intersection problems, just isolate the variable in the middle. When you do a step, make sure you do that step to all THREE sides, the left, the middle and the right. If you divide or multiply by a negative number, make sure to switch both inequalities.

20) $-9 \le -2c + 1 < -5$

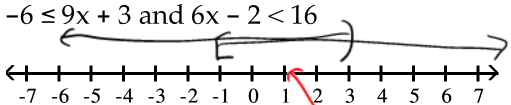


$-9 \le -2c + 1 < -5$
$-1 \quad -1 \quad -1$
$-10 \le -2c < -6$
$\frac{-10}{-2} \quad \frac{-2c}{-2} \quad \frac{-6}{-2}$
$5 \ge c > 3$
$3 < c \le 5$

$(3, 5]$

To easily jump to the final answer, the numbers need to be in numberline order, so if you have switched the inequalities, as in this example. you will have to switch them again with a mirror image switch to get the numbers in the right order.

21) $-6 \le 9x + 3$ and $6x - 2 < 16$

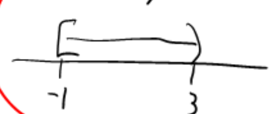


OVERLAP

$-6 \le 9x + 3$
$-3 \quad -3$
$-9 \le 9x$
$\frac{-9}{9} \le \frac{9x}{9}$
$-1 \le x$
$x \ge -1$

$6x - 2 < 16$
$+2 \quad +2$
$6x < 18$
$\frac{6x}{6} < \frac{18}{6}$
$x < 3$

SO $[-1, 3)$

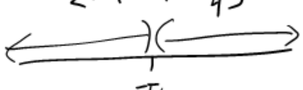


Write the domain of f in interval notation.

22) $f(x) = \dfrac{x + 1}{4x + 5}$

$4x + 5 = 0$
$-5 \quad -5$
$4x = -5$
$\frac{4x}{4} = \frac{-5}{4}$
$x = -\frac{5}{4}$

DOMAIN IS
$\{x \mid x \ne -\frac{5}{4}\}$



$\boxed{(-\infty, -\tfrac{5}{4}) \cup (-\tfrac{5}{4}, \infty)}$

The domain of a rational function is the set of all numbers that don't cause a divide by 0 error, so set the denominator = 0 to find the error and then exclude it from all real numbers to get the domain.

4.3 Classify as either true or false.

23) $|x|$ is always positive.

FALSE. $|x|$ can $= 0$ as well

One interpretation of the absolute value is all NON NEGATIVE numbers. This includes the positive numbers as well as 0. The smallest $|x|$ can be is 0.

Solve the equation.

24) $|b + 2| - 8 = 1$

$|b+2| = 9$

$b+2 = 9 \qquad b+2 = -9$

$b = 7 \qquad b = -11$

$\boxed{\{-11, 7\}}$

Check

$b = 7$

$|7+2| - 8 = 1$

$|9| - 8 = 1$

$9 - 8 = 1$ ✓

$b = -11$

$|-11+2| - 8 = 1$

$|-9| - 8 = 1$

$9 - 8 = 1$ ✓

Make sure you isolate the absolute value equation/inequality before you break it up into its two parts. For an equation, the two parts are thus:
Part one - Write without absolute values.
Part two - Write without absolute values and make the right side its opposite.
Given that there are two parts to solve, you should expect to have two solutions to an absolute value equation.

25) $|z| = -7$

$|z|$ is nonnegative, so $\emptyset$

OR $z = 7, \quad z = -7$

Check: $|7| = -7$ NO, $|-7| = -7$ NO, so $\emptyset$

Once you isolate the absolute value, if it is equal to, less than or equal to, or less than a negative number, you will have no solution to the problem.

26) Let $f(x) = |x| - 5$. Find all x for which $f(x) = 10$.

$|x| - 5 = 10$

$|x| = 15$

$x = 15, \ x = -15$

check!

$\boxed{\{15, -15\}}$

Isolate first.

27) $|5s - 1| = |s + 6|$

$5s - 1 = s + 6$

$-s + 1 \quad -s + 1$

$4s = 7$

$\frac{4s}{4} = \frac{7}{4}$

$s = \frac{7}{4}$

opposites

$5s - 1 = -s - 6$

$+s + 1 \quad +s + 1$

$6s = -5$

$\frac{6s}{6} = \frac{-5}{6}$

$s = -5/6$

Check!

$\boxed{\left\{-\frac{5}{6}, \frac{7}{4}\right\}}$

For the problems with two absolute values, isolate each on its own side. Then you will again get two parts to solve. Drop absolute values for the first part, and drop absolute values and change the right side to its opposite for the second part.

28) $|n + 5| = |2 - n|$

$n + 5 = 2 - n$

$+n - 5 \quad -5 + n$

$2n = -3$

$\frac{2n}{2} = \frac{-3}{2}$

$n = -\frac{3}{2}$

opposites

$n + 5 = -2 + n$

$-n - 5 \quad -5 - n$

$0 = -7$

$\emptyset$

Check!

$\boxed{\left\{-\frac{3}{2}\right\}}$

If one of the parts gives you no solution, you will only have the other answer as the final answer. If one of your sides gives you all real numbers, you will have all real numbers for the final answer.

Solve the absolute-value inequality.

29) $|r - 9| > 9$

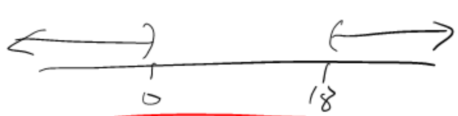
OR PROBLEM (> is greatOR)

so $r - 9 > 9$ OR $r - 9 < -9$

switch, opposite

$+9 \quad +9 \qquad +9 \quad +9$

$r > 18$ or $r < 0$



$\boxed{(-\infty, 0) \cup (18, \infty)}$

Absolute value inequalities are either OR or AND problems. Isolate the absolute value first, and then you will be able to tell which one it is. | | > is an OR, | | < is an AND.

30) $|g+8|<5$

AND (< is less thAND)

so $-5<g+8<5$

$-8 \quad -8 \quad -8$

$-13<g<-3$

$\boxed{(-13,-3)}$

The AND absolute value inequalities can be translated into a condensed AND complound inequality, which is the easiest type. Therefore I always solve them this way.

31) $|8y-5|+1<-1$

$-1 \quad -1$

$|8y-5|<-2$ AND

An $|\#|$ is non negative, so it is not possible for $|\#|<-2$

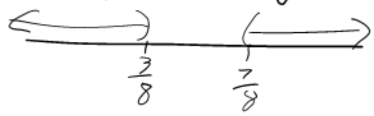
$\boxed{\varnothing}$

or

$2<8y-5<-2$

$7<8y<3$

$\frac{7}{8}<y<\frac{3}{8}$

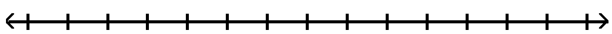


NO OVERLAP SO

$\boxed{\varnothing}$

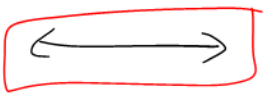
Solve and graph.

32) $|2k+2|<-7$



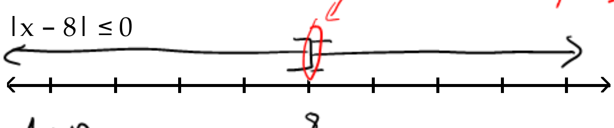
$|\#|$ cant be <-7

$\boxed{\varnothing}$



OVERLAP SO

33) $|x-8|\le 0$



AND

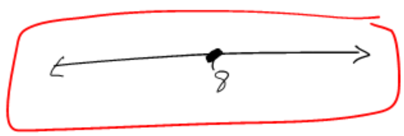
so

$-0\le x-8\le 0$

$+8 \quad +8 \quad +8$

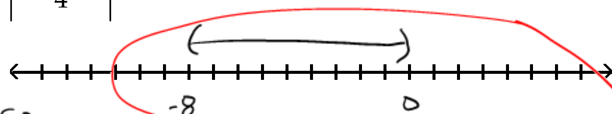
$8\le x\le 8$

$\boxed{\{8\}}$



If you have an absolute value inequality <= to 0, you will only get one number as an aswer. If you have an absolute value > 0 you will get all real numbers except one value.

34) $\left|\frac{9y+36}{4}\right|<9$



AND so

$(-8,0)$

$4\cdot(-9)<\frac{4\cdot(9y+36)}{4}<(9)\cdot 4$

$-36<9y+36<36$

$-36 \quad -36 \quad -36$

$\frac{-72}{9}<\frac{9y}{9}<\frac{0}{9}$

$-8<y<0$

35) $|7k + 6| > -9$

an $|\#|$ is always > -9, so $\boxed{(-\infty, \infty)}$

OR AN OR PROBLEM:

$7k+6 > -9$ OR $7k+6 < 9$ switch opposite

$7k > -15$ $\quad$ $7k < 3$

$k > -15/7$ $\quad$ $k < 3/7$

$-\frac{15}{7}$ $\quad$ $\frac{3}{7}$

OR: $(-\infty, \infty)$

The absolute value inequalities
| | > -#,
| | >= -#,
| | >= 0 are always true, and so are all real numbers solutions.

36) $|7k - 7| + 4 > 12$

$-4 \quad -4$

Isolate first!

$|7k-7| > 8$

OR:

$7k-7 > 8$ OR $7k-7 < -8$ switch opposite

$+7 \quad +7$ $\quad$ $+7 \quad +7$

$\frac{7k}{7} > \frac{15}{7}$ $\quad$ $\frac{7k}{7} < \frac{-1}{7}$

$k > \frac{15}{7}$ $\quad$ $k < -\frac{1}{7}$

$-\frac{1}{7}$ $\quad$ $\frac{15}{7}$

$(-\infty, -\frac{1}{7}) \cup (\frac{15}{7}, \infty)$

37) $|4x - 7| > 0$

OR:

$4x-7 > 0$ OR $4x-7 < -0$ switch opposite

$+7 \quad +7$ $\quad$ $+7 \quad +7$

$\frac{4x}{4} > \frac{7}{4}$ $\quad$ $\frac{4x}{4} < \frac{7}{4}$

$x > \frac{7}{4}$ $\quad$ $x < \frac{7}{4}$

$\frac{7}{4}$

$(-\infty, \frac{7}{4}) \cup (\frac{7}{4}, \infty)$

38) $|11 - 4x| < -1$

$|\#|$ NOT < -1 so $\emptyset$

Find the requested solution.

39) Let $f(x) = |9 - 8x|$. Find all x for which $f(x) \le 1$.

$|9-8x| \le 1$

AND so

$-1 \le 9-8x \le 1$

$-9 \quad -9 \quad -9$

$-10 \le -8x \le -8$

$\div -8 \quad \div -8 \quad \div -8$

$\frac{5}{4} \ge x \ge 1$

$1 \le x \le 5/4$

$[1, \frac{5}{4}]$

4.4 **Complete the sentence.**

40) To indicate that the boundary line is part of the solution, draw it

A) with arrows at its ends.
B) as a dashed line.
C) as a solid line.
D) with solid dots at its ends.

a boundary line is part of the solution if it's inequality is either $\geq$ or $\leq$

The $=$ makes it part of the solution

Choose the ordered pair which is a solution of the inequality.

41) $2x + 4y \geq 8$

A) (0, 1)
$2(0)+4(1)=4 \geq 8$
FALSE

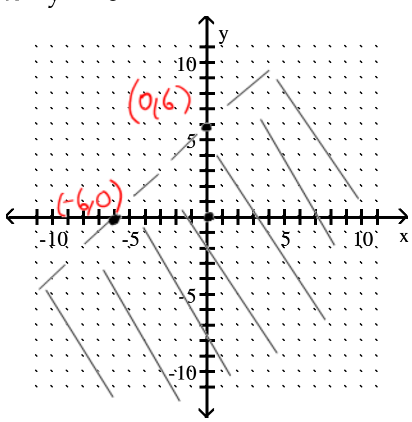
B) (2, 1)
$2(2)+4(1)=8 \geq 8$
TRUE

C) (0, 0)
$2(0)+4(0)=0 \geq 8$
FALSE

D) (1, 1)
$2(1)+4(1)=6 \geq 8$
FALSE

Graph on a plane.

42) $x - y > -6$



BOUNDARY LINE
$x-y=-6$

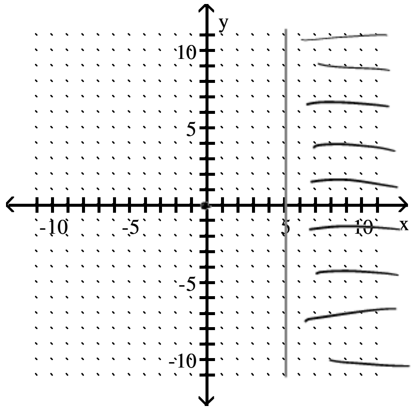
x	y
0	6
-6	0

& DASHED

TEST POINT
(0,0)
$0-0>-6$
$0>-6$
TRUE
so shade
(0,0)

Two variable linear inequalities are like 2 variable linear equations. You first get two points so you can graph the line. The differences are some times the line is solid (<= or >=) and some times it is dashed (< or >) and the final answer is the shaded region on either one side of the line or the other side of the line, determined by checking a test point. If the point works in the inequality, you shade the point side of the line. If it does not work, you shade the other side of the line.

43) x ≥ 5

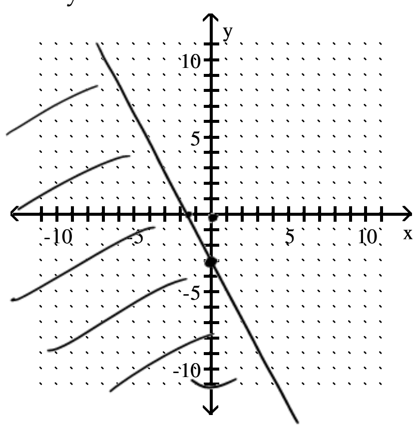


BOUNDARY LINE
IS X = 5,
CROSSES X @ 5
SO VERTICAL
& SOLID

TEST POINT
(0,0)

0 ≥ 5
False, so
dont shade
(0,0)

44) 2x + y ≤ −3



BOUNDARY LINE
2X + Y = −3

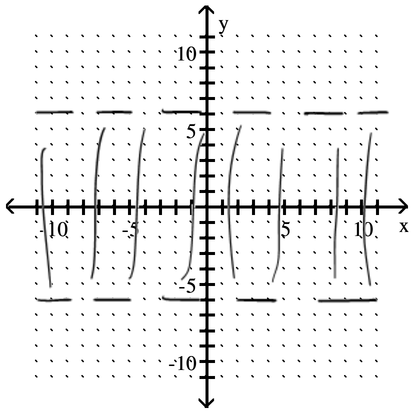
x	y
0	−3
−3/2	0

& SOLID

TEST
(0,0)

2(0)+0 = 0 ≤ −3
FALSE
SO DONT
SHADE (0,0)

45) $-6 < y < 6$



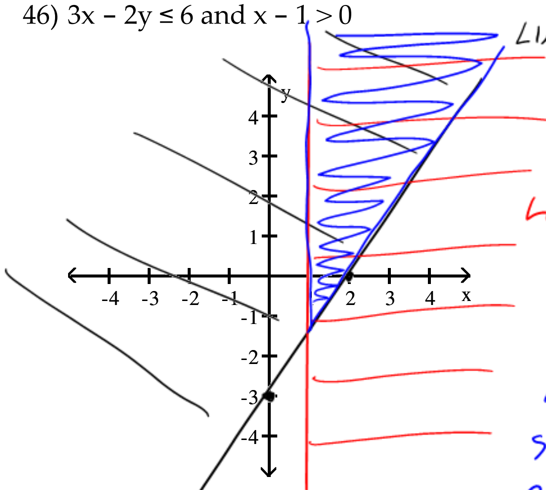
BOUNDARY LINES are
$y=6$, $y=-6$
THESE CROSS THE Y AXIS & SO ARE HORIZONTAL & DASHED

SHADING
<6 & >-6
SO IN BETWEEN

For two inequalities in two variables, our current book expects you to find the intersection of the two shadings. Other books will ask for the union as well.

Graph the system of linear inequalities.

46) $3x - 2y \leq 6$ and $x - 1 > 0$



LINE 1: BOUNDARY $3x-2y=6$ TEST $(0,0)$

x	y
0	-3
2	0

& SOLID

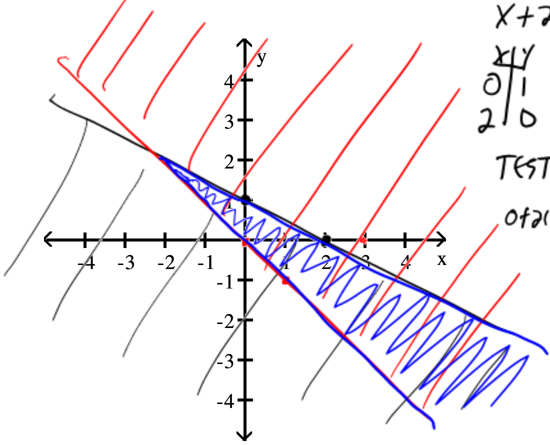
$3(0)-2(0)=0\leq 6$
TRUE, SO shade $(0,0)$

LINE 2: BOUNDARY $x-1=0$
$x=1$ VERTICAL, CROSSING @ $x=1$ & SOLID

TEST $(0,0)$
$0-1=-1>0$
FALSE
SO DON'T SHADE $(0,0)$

AND IS INTERSECTION. SO THE REGION SHADED BY BOTH, INCLUDING BOARDER

47) x + 2y ≤ 2 and x + y ≥ 0



LINE ONE

$x + 2y \le 2$

x	y
0	1
2	0

SOLID

TEST (0,0)

$0 + 2(0) = 0 \le 2$

TRUE

LINE TWO

$x + y \ge 0$

x	y
0	0
1	-1

SOLID

~~TEST (0,0)~~

TEST (3,0)

$3 + 0 = 3 \ge 0$

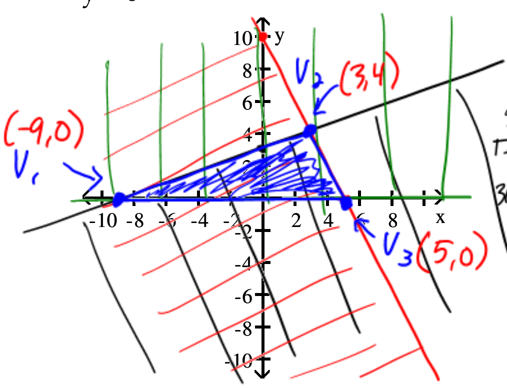
TRUE

you have to pick another x value to get two points

(0,0) is on the line so you have to pick another point to check.

Graph the system of inequalities. Find the coordinates of the vertices.

48) 3y − x ≤ 9,
y + 2x ≤ 10,
y ≥ 0



LINE 1:

$3y - x \le 9$

x	y
0	3
-9	0

SOLID

TEST (0,0)

$3(0) - 0 = 0 \le 9$

TRUE

LINE 2:

$y + 2x \le 10$

x	y
0	10
5	0

SOLID

TEST (0,0)

$0 + 2(0) = 0 \le 10$

TRUE

LINE 3:

$y \ge 0$

CROSSES y @ 0, HORIZONTAL

SOLID

& shade ≥ 0 so up.

VERTICES:

V_1 is @ (−9,0)

V_3 is @ (5,0)

TO FIND V_2, WE NEED TO SOLVE THE SYSTEM OF EQUATIONS CONSISTING OF THE LINES THAT CROSS @ V_2:

$3y - x = 9 \quad R_1$

$y + 2x = 10 \quad R_2$

$2R1 + R2$

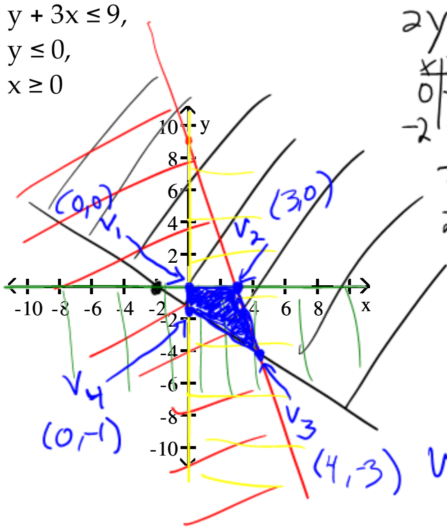
$$\begin{array}{r} 6y - 2x = 18 \\ + \quad y + 2x = 10 \\ \hline 7y = 28 \\ \frac{7y}{7} = \frac{28}{7} \\ y = 4 \end{array}$$

R2:

$$\begin{array}{r} 4 + 2x = 10 \\ -4 \qquad -4 \\ 2x = 6 \\ \frac{2x}{2} = \frac{6}{2} \\ x = 3 \end{array}$$

V_2: (3,4)

49) $2y + x \ge -2$,
$y + 3x \le 9$,
$y \le 0$,
$x \ge 0$



LINE 1.
$2y + x \ge -2$

x	y
0	-1
-2	0

SOLID

TEST (0,0)
$2(0)+0 = 0 \ge -2$
TRUE

LINE 2
$Y + 3X \le 9$

x	y
0	9
3	0

SOLID

TEST (0,0)
$0+3(0) = 0 \le 9$
TRUE

LINE 3
$y \le 0$
HORIZONTAL THROUGH 0, SOLID, SHADE BELOW

LINE 4.
$X \ge 0$
VERTICAL THROUGH 0, SOLID, SHADE RIGHT

BLUE INTERSECTION

$V_1 = (0,0)$ $V_2 = (3,0)$ $V_4 = (0,-1)$

V_3: $2y + x = -2$ R_1
$y + 3x = 9$ R_2

$-3R_1 + R_2$
$-6y - 3x = 6$
$+\ y + 3x = 9$
$-5y = 15$
$\frac{-5y}{-5} = \frac{15}{-5}$
$y = -3$

R_1
$2(-3) + x = -2$
$-6 + x = -2$
$+6 \quad +6$
$x = 4$
$(4,-3)\ V_3$

5.3 Write an equivalent expression by factoring out the greatest common factor.

50) $12wx - 20wy - 16wz$

$\boxed{4w(3x - 5y - 4z)}$

check:
$4w(3x-5y-4z)$
$= 12wx - 20wy - 16wz$ ✓

Factoring is dividing so you check by multiplying.

Write an equivalent expression by factoring out the greatest common factor.

51) $12c^5 - 60c^3$

$\boxed{12c^3(c^2 - 5)}$

check:
$12c^3(c^2-5) =$
$= 12c^5 - 60c^3$ ✓

52) $24x^9y^7 - 36x^6y^5 + 36x^4y^3$

$\boxed{12x^4y^3(2x^5y^4 - 3x^2y^2 + 3)}$

check $12x^4y^3(2x^5y^4 - 3x^2y^2 + 3) =$
$= 24x^9y^7 - 36x^6y^5 + 36x^4y^3$ ✓

Factor out a factor with a negative coefficient.

If the first term is negative factor out at least a negative 1.

53) $-2x + 6$

$\boxed{-2(x-3)}$

check: $-2(x-3) = -2x+6$ ✓

54) $-2x^2 + 4x - 12$

$\boxed{-2(x^2-2x+6)}$

Check

$-2(x^2-2x+6) = -2x^2+4x-12$ ✓

Write an equivalent expression by factoring.

If the greatest common factor is a binomial, keep it in its parentheses.

55) $3x(5x + 6) - 4(5x + 6)$

$\boxed{(5x+6)(3x-4)}$

If the polynomial has 4 terms, most of the time you will factor by grouping. Group the first two terms and the second two terms. Then factor out the GCF, and repeat with the binomial GCF. If there is no binomial GCF in the latter step, you will not be able to finish the factoring.

56) $18x^2 + 12xy + 15xy + 10y^2$

$6x(3x+2y) + 5y(3x+2y)$

$\boxed{(3x+2y)(6x+5y)}$

check!

57) $x^3 - 2x^2 + 9x - 18$

$x^2(x-2) + 9(x-2)$

$= \boxed{(x-2)(x^2+9)}$

58) $(m + 7)(a - 6) + (m + 7)(a + 1)$

$(m+7)((a-6)+(a+1))$

$= (m+7)(a-6+a+1)$

$= \boxed{(m+7)(2a-5)}$

5.4 Factor.

59) $p^2 - 7p + 10$

$\boxed{(p-2)(p-5)}$

check

FOIL:

$p^2 - 5p - 2p + 10$

$p^2 - 7p + 10$ ✓

Guaranteed:

$(p- \quad)(p- \quad)$

When factoring a trinomial, I start by setting up two binomial parentheses and then writing down the guarantees in the factoring, thinking about the FOIL process. I have put down the starting guarantees in blue for these problems.

Factor.

60) $x^2 + 3x - 18$

$\boxed{(x+6)(x-3)}$

check

F O I L

$x^2 - 3x + 6x - 18$

$x^2 + 3x - 18$ ✓

$(x+ \quad)(x+ \quad)$

61) $2x^2 - 2x - 12$

$2(x^2 - x - 6)$

$\boxed{2(x+2)(x-3)}$

check:

$2(x^2 - 3x + 2x - 6)$

$2(x^2 - x - 6)$

$2x^2 - 2x - 12$ ✓

$2(x \quad)(x \quad)$

62) $8 - 6z - 9z^2$

$\boxed{(4+3z)(2-3z)}$

Check!

$(\quad z)(\quad z)$

63) $21x^2 - 91x - 70$

$7(3x^2 - 13x - 10)$

$\boxed{7(3x+2)(x-5)}$

Check!

$7(3x \quad)(x \quad)$

64) $5x^3 + 5x^2 - 30x$

$5x(x^2 + x - 6)$

$\boxed{5x(x+3)(x-2)}$

Check!

$5x(x \quad)(x \quad)$

65) $60x^3 - 5x^2 - 30x$

$5x(12x^2 - x - 6)$

$\boxed{5x\ (3x+2)(4x-3)}$

Check!

$5x(\quad x \quad)$

66) $8x^2 - 18xy + 9y^2$

$\boxed{(4x-3y)(2x-3y)}$

check!

$(\quad - \quad)(\quad - \quad)$

67) $x^2 - x - 35$

$\cancel{(x \quad 7)(x \quad 5)}$

$\boxed{\text{PRIME}}$

$(x \quad)(x \quad)$

5.5 Factor completely.

68) $z^2 - 14z + 49$

$(z-7)(z-7)$

$\boxed{(z-7)^2}$

$(z - \quad)(z - \quad)$

If the two binomial factors end up being the same, you write them as one binomial squared.

Factor completely.

69) $y^2 - 9$

$\boxed{(y+3)(y-3)}$

Look for the difference of squares. The factoring is always guaranteed and these are some of the easiest of the factoring problems.

70) $6pq^4 - 6pr^4$

$6p(q^4 - r^4)$

$6p\ (q^2+r^2)(q^2-r^2)$

$\boxed{6p\ (q^2+r^2)(q+r)(q-r)}$

You want to keep factoring until you can go no further.

71) $\frac{1}{49} - p^2$

$$\boxed{\left(\tfrac{1}{7}+p\right)\left(\tfrac{1}{7}-p\right)}$$

72) $25 - (x + 4y)^2$

$$\left(5+(x+4y)\right)\left(5-(x+4y)\right)$$

$$\boxed{(5+x+4y)(5-x-4y)}$$

Remember to keep the binomial in parentheses until you are ready to remove them. In this case you need to distribute the negative to remove the second set of parentheses.

73) $\underbrace{r^2 + 2rs + s^2} - 16$

$$(r+s)(r+s)-16$$

$$(r+s)^2-16$$

$$\left((r+s)+4\right)\left((r+s)-4\right)$$

$$\boxed{(r+s+4)(r+s-4)}$$

This is a more rare case of a four termed polynomial. The factoring is stillo by grouping, but you group the first three terms together into a perfect square trinomial. Then you have a difference of squares. Sometimes you group the last three instead. You will know by where the - is in the problem.

5.6 Factor completely.

74) $x^3 - 343$

$$\boxed{(x-7)(x^2+7x+49)}$$

$$x^3-343$$

$$(\underset{S}{-})(\underset{O}{+}\quad\underset{AP}{+})$$

75) $27a^3 - 64b^3$

$$\boxed{(3a-4b)(9a^2+12ab+16b^2)}$$

$$(\underset{3a}{L}-\underset{4b}{R})(\underset{3a\cdot 3a}{LL}+\underset{3a\cdot 4b}{LR}+\underset{4b\cdot 4b}{RR})$$

There are three patterns that make the sum or difference of cubes easy to do.

First: The parenthese pattern is a binomial and a trinomial.

Second, the sign pattern is given by S O AP. This stands for Same, Oppoiste, and Always Positive. If the cubes are a difference, you would have - + + for the signs.
For a sum, you would have a + - + pattern.

Third: The binomial is made out of the cube roots. If you think of the binomial as the left and right part, like (L R), then the trinomial is made up of terms (LL LR RR) where LL is the left term times the left term, etc. You already have the signs determined, so just worry about the terms with this pattern.

One last note is that the resulting trinomial is PRIME if it is an x^2 type trinomial. So don't try to factor it further. It doesn't go. If it is an x^4 type trinomial, it may well factor, but it will be nearly impossible for you to get the factoring directly (LOTS of trial and error). So the trinomial is either PRIME or a waste of your time.

76) $1000s^3 + 1$

$$\boxed{(10s+1)(100s^2-10s+1)}$$

77) $p^6 - 1$

$(p^3)^2 - 1^2$

$(p^3+1)(p^3-1)$

$\boxed{(p+1)(p^2-p+1)(p-1)(p^2+p+1)}$

This problem can be factored the farthest if you first interpret the difference as a difference of squares. It can also be thought of as a difference of cubes. The blue factoring is following the difference of cubes interpretation. Notice the last trinomial is not easy to factor. In fact there is no obvious way to even start.

$(p^2)^3 - 1^3$

$(p^2-1)(p^4+p^2+1)$

$(p+1)(p-1)(p^4+p^2+1)$

? ?

5.7 Factor completely.

78) $32a^4b - 18b^3$

$2b(16a^4 - 9b^2)$

$\boxed{2b\,(4a^2+3b)(4a^2-3b)}$

79) $\underbrace{a^4 + a^3} + \underbrace{a + 1}$

$a^3(a+1) + 1(a+1)$

$(a+1)(a^3+1)$

$(a+1)(a+1)(a^2-a+1)$

$\boxed{(a+1)^2(a^2-a+1)}$

5.8 Solve the equation.

80) $x^2 - x = 72$

$-72 \quad -72$

$x^2 - x - 72 = 0$

$(x-9)(x+8) = 0$

$x - 9 = 0 \qquad x + 8 = 0$

$+9 \quad +9 \qquad -8 \quad -8$

$\boxed{x = 9, \quad x = -8}$

Check: $x = 9$

$x^2 - x = 72$

$9^2 - 9 = 72$

$81 - 9 = 72$ ✓

$x = -8$

$x^2 - x = 72$

$(-8)^2 - (-8) = 72$

$64 + 8 = 72$ ✓

For polynomial equations, you first want to get the equation = 0 by moving everyting to one side. Then factor. If you can do this, the next step is to set each factor = 0 and solve for your answers. This last step uses the Property of Zero Factors, or the Zero Factor Property.

The ZERO FACTOR PROPERTY is a nice way to remember the order of steps for these equations: First, =0 then Factor, then use the property.

Make sure to check your answers as these are equations.

Solve the equation.

81) $6y^2 + 19y + 15 = 0$

$(2y+3)(3y+5) = 0$

$2y + 3 = 0 \qquad 3y + 5 = 0$

$2y = -3 \qquad 3y = -5$

$\boxed{y = -\frac{3}{2}, \quad y = -\frac{5}{3}}$

check $y = -\frac{3}{2}$

$6\left(-\frac{3}{2}\right)^2 + 19\left(-\frac{3}{2}\right) + 15 = 0$

$\cancel{6}^{3}\left(\frac{9}{\cancel{4}_{2}}\right) + 19\left(-\frac{3}{2}\right) + 15 = 0$

$\frac{27}{2} - \frac{57}{2} + 15 = 0$

$-\frac{30}{2} + 15 = 0$

$-15 + 15 = 0$ ✓

Check $-5/3$

$6\left(-\frac{5}{3}\right)^2 + 19\left(-\frac{5}{3}\right) + 15 = 0$

$6\left(\frac{25}{9}\right) + 19\left(-\frac{5}{3}\right) + 15 = 0$

$\frac{50}{3} - \frac{95}{3} + 15 = 0$

$-\frac{45}{3} + 15 = 0$

$-15 + 15 = 0$ ✓

82) $12n^2 + 15n = 0$

$3n(4n+5) = 0$

$3n = 0 \qquad 4n + 5 = 0$

$\frac{3n}{3} = \frac{0}{3} \qquad -5 \quad -5$

$n = 0 \qquad \frac{4n}{4} = \frac{-5}{4}$

$n = -\frac{5}{4}$

$\boxed{\left\{0, -\frac{5}{4}\right\}}$

Check!

If the problem is factored before it is = 0, you will have to multiply to simplify and then pull all of it to one side to factor.

83) $(x + 7)(x - 7) = -24$

$x^2 + \cancel{7x} - \cancel{7x} - 49 = -24$

$+24 \quad +24$

$x^2 - 25 = 0$

$(x+5)(x-5)=0$

$x+5=0 \quad x-5=0$

$\boxed{x=-5, \quad x=5}$

Check!

84) $x^4 - 90x^2 + 729 = 0$

$(x^2-81)(x^2-9)=0$

$(x-9)(x+9)(x-3)(x+3)=0$

$\boxed{x=-9, 9, -3, 3}$

Check!

85) $x^3 + 10 = 10x^2 + x$

$-10x^2 - x \quad -10x^2 - x$

$\underbrace{x^3 - 10x^2} - x + 10 = 0$

$x^2(x-10) - 1(x-10) = 0$

$(x-10)(x^2-1)=0$

$(x-10)(x+1)(x-1)=0$

$\boxed{x = 10, -1, 1}$

Solve the problem.

86) Let $g(x) = 9x + x^2$. Find a so that $g(a) = -14$.

$g(a) = -14$

$9a + a^2 = -14$

$+14 \quad +14$

$a^2 + 9a + 14 = 0$

$(a+7)(a+2)=0$

$\boxed{a=-7, a=-2}$

Check:

$g(a)=-14$

$g(-7)=9(-7)+(-7)^2$

$= -63+49 = -14$ ✓

$g(-2)=9(-2)+(-2)^2$

$= -18+4 = -14$ ✓

Find the domain of the function h.

87) $h(x) = \dfrac{-1x}{-9x^2 + 324}$

$-9x^2+324=0$

$-9(x^2-36)=0$

$-9(x+6)(x-6)=0$

$-9\neq 0 \quad x=-6, x=6$

So $\boxed{\{x \mid x \neq -6, x \neq 6\}}$

Solve.

88) Find two consecutive integers such that the sum of their squares is 421.

$x, (x+1)$ are 2 consecutive integers

$x^2 + (x+1)^2 = 421$

$x^2 + (x+1)(x+1) = 421$

$x^2 + x^2 + 2x + 1 = 421$

$-421 \quad -421$

$2x^2 + 2x - 420 = 0$

$2(x^2 + x - 210) = 0$

$2(x-14)(x+15)=0$

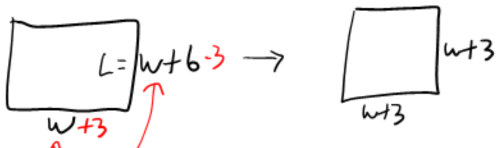
$2\neq 0 \quad x=14, \ x=-15$

For $x=14$, $x, (x+1)$ are

For $x=-15$ $x, (x+1)$ are

$\boxed{\{14, 15\} \text{ or } \{-15, -14\}}$

89) The length of a rectangle is 6 inches more than its width. If 3 inches are taken from the length and added to the width, the figure becomes a square with an area of 81 square inches. What are the dimensions of the original figure?



$A = LW$

$81 = (w+3)(w+3)$

$81 = w^2 + 6w + 9$

$0 = w^2 + 6w - 72$

$0 = (w+12)(w-6)$

$\cancel{w=-12}$, $w = 6$

$w = 6\text{in}$, $L = 6+6 = 12\text{in}$

$\boxed{6\text{in by } 12\text{in}}$

Solve the equation. Round to the nearest tenth, if necessary.

90) If an object is thrown upward from the ground with an initial velocity of 112 ft/sec, its height after t sec is given by $h = 112t - 16t^2$. Find the number of seconds before the object hits the ground.

The ground is $h = 0$ so

$0 = 112t - 16t^2$

$0 = 16t(7 - t)$

$16 \neq 0$ $t = 0$, $t = 7$

$t = 0$ is when the object is thrown.

$t = 7$ is when it hits the ground

so $\boxed{7 \text{ seconds}}$

91) A ball is dropped from a cliff that is 256 ft high. The distance S (in feet) that it falls in t seconds is given by the formula $S = 16t^2$. How many seconds will it take for the ball to hit the ground? Round to the nearest tenth of a second.

$S = 256$

$S = 16t^2$

$256 = 16t^2$

$16t^2 - 256 = 0$

$16(t^2 - 16) = 0$

$16(t+4)(t-4) = 0$

$16 \neq 0$, $t = -4$, $t = 4$

$\boxed{4 \text{ seconds}}$

Extra formulas you might need to do these story problems are the following:

Area of a rectangle
Perimeter of a rectangle

Pythagorean Theorem for a right triangle

Area of a circle
Circumference of a circle

D=RT

Representations of consecutive integers or consecutive even/odd integers.