Chapter 1,2,3 review

Sections labeled at the start of the related problems

1.3 **Classify the following as either a pair of equivalent equations or a pair of equivalent expressions.**

1) $7x - 42$, $7(x - 6)$

$\boxed{7x-42}$; $7(x-6)$

$\boxed{7x-42}$

$\boxed{\text{EQUIVALENT}}$

To determine equivalency of expressions you need to simplify each one until you see that they are the same terms.

2) $2x + 8 = 14$, $2(x + 4) = 14$

$2x+8=14$
$-8 \quad -8$
$\frac{2x}{2} = \frac{6}{2}$
$\boxed{x=3}$

$2(x+4)=14$
$2x+8=14$
$-8 \quad -8$
$\frac{2x}{2} = \frac{6}{2}$
$\boxed{x=3}$

To determine equivalency of equations you need to solve each one and see if

$\boxed{\text{EQUIVALENT}}$

Solve the equation.

3) $\frac{1}{5}f - 3 = 1$

To solve a linear equation in one variable, start by clearing fractions, then distribute

$5 \cdot \frac{1}{5}f \overset{5}{-} 3 = 1 \cdot 5$

$f - 15 = 5$
$+15 \quad +15$

$\boxed{f = 20}$

Check

$\frac{1}{5}(20) - 3 = 1$

$4 - 3 = 1$ ✓

4) $8x - 5 + 4x = 6x - 6 - 3x$

Same as above, with the addition of the following. Move all the variables to one side

$$8x - 5 + 4x = 6x - 6 - 3x$$
$$12x - 5 = 3x - 6$$
$$-3x \quad +5 \qquad -3x \quad +5$$
$$\frac{9x}{9} = \frac{-1}{9}$$
$$\boxed{x = -\frac{1}{9}}$$

CHECK

$$8\left(-\tfrac{1}{9}\right) - 5 + 4\left(-\tfrac{1}{9}\right) = 6\left(-\tfrac{1}{9}\right) - 6 - 3\left(-\tfrac{1}{9}\right)$$
$$-\frac{8}{9} - 5\cdot\frac{9}{9} - \frac{4}{9} = -\frac{6}{9} - 6\cdot\frac{9}{9} + \frac{3}{9}$$
$$-\frac{8}{9} - \frac{45}{9} - \frac{4}{9} = -\frac{6}{9} - \frac{54}{9} + \frac{3}{9}$$
$$-\frac{57}{9} = -\frac{57}{9} \checkmark$$

Solve.

5) $3x - (8 - x) = 4[5 - (7 + 2x - 2)]$

I like to start with the innermost parenthese set and work out when they are nested like this.

$$3x - (8 - x) = 4[5 - (7 + 2x - 2)]$$
$$3x - 8 + x = 4[5 - (5 + 2x)]$$
$$4x - 8 = 4[5 - 5 - 2x]$$
$$4x - 8 = 4(-2x)$$
$$4x - 8 = -8x$$
$$+8x \quad +8 \qquad +8x \quad +8$$
$$\frac{12x}{12} = \frac{8}{12}$$
$$\boxed{x = \frac{8}{12} = \frac{2}{3}}$$

Decide whether the equation is conditional, an identity, or a contradiction. Give the solution set.

6) $2(x - 7) + (3x) = 5(x - 8) - 3$

Note: Problems 2-5 are all conditional equations, where you can solve for the variable.

$$2(x - 7) + (3x) = 5(x - 8) - 3$$
$$2x - 14 + 3x = 5x - 40 - 3$$
$$5x - 14 = 5x - 43$$
$$-5x \quad +14 \qquad -5x \quad +14$$
$$0 = -29$$

$$\boxed{\text{CONTRADICTION}}$$

When all of the variables in an equation cancel to zero, the resulting equation is either true or false. In this case it is false, and so the equation is a contradiction. Had the equation been true, like 5=5 or 0=0, the equation would have been true and would have been an identity equation.

7) 2(2g – 7) – 4g + 14 = 0

$4g - 14 - 4g + 14 = 0$

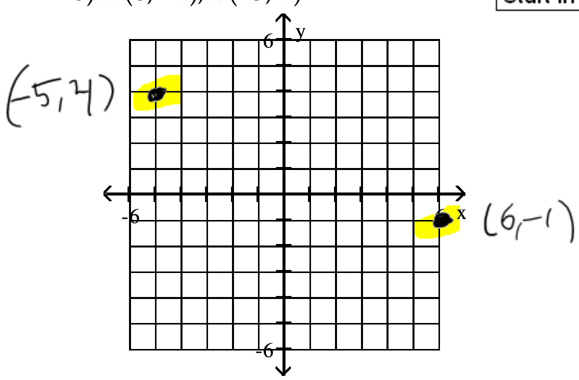
$0 = 0$

$\boxed{\text{IDENTITY}}$

2.1 **Plot the points with the given coordinates.**

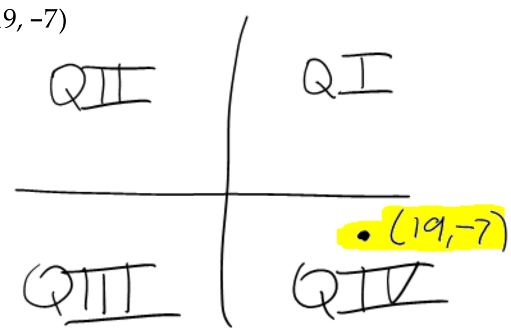
8) A(6, –1), B(–5, 4)

Start in the x direction, left or right, and then in the y direction, up or



Name the quadrant, if any, in which the point is located.

9) (19, –7)



$\boxed{\text{QIV}}$

Determine if the ordered pair is a solution of the equation. Remember to use alphabetical order for substitution.

10) (-3, 1); 2x + 7y = 1

Any time I do a substitution, as in this problem, I start by placing parentheses around the variable(s) to be substituted into. This keeps the order of operations in tact.

$2(x) + 7(y) = 1$

$2(-3) + 7(1) = 1$

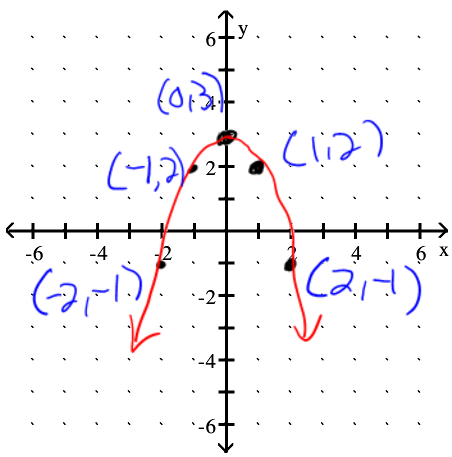
$-6 + 7 = 1$

$1 = 1$ ✓

YES, SOLUTION

Graph.

11) $y = 3 - x^2$

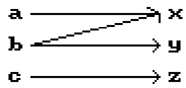


$y = 3 - (x)^2$

x	$y = 3 - (x)^2$	
0	$3 - (0)^2 = 3 - 0 = 3$	(0, 3)
1	$3 - (1)^2 = 3 - 1 = 2$	(1, 2)
2	$3 - (2)^2 = 3 - 4 = -1$	(2, -1)
-1	$3 - (-1)^2 = 3 - 1 = 2$	(-1, 2)
-2	$3 - (-2)^2 = 3 - 4 = -1$	(-2, -1)

2.2 **Is the following correspondence a function?**

12)



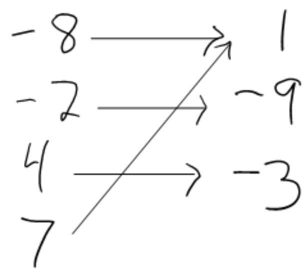
NO, b is related to multiple outputs (x & y)

For the given correspondence, write the domain and the range. Then determine whether the correspondence is a function.

13) {(−8, 1), (−2, −9), (4, −3), (7, 1)}

DOMAIN {−8, −2, 4, 7}

RANGE {−9, −3, 1}

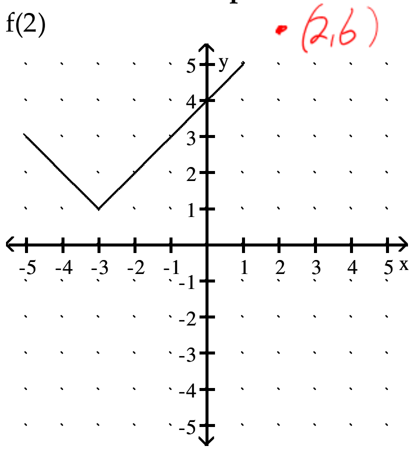


$\boxed{\text{FUNCTION}}$

NO INPUT IS RELATED TO MULTIPLE OUTPUTS.

The graph of a function f is provided. Determine the requested function value.

14) f(2)

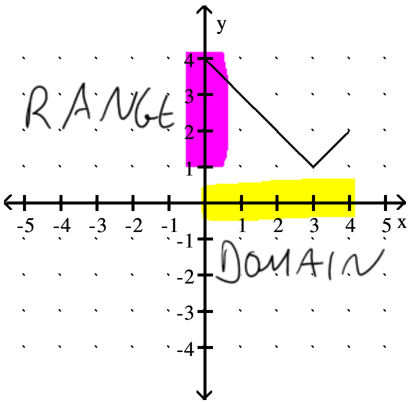


$f(2) = 6$

This graph didn't extend far enough to the right, forcing you to estimate. Since the graph is piecewise linear, it is clear where the point would have been.

For the function represented in the graph, determine the domain or range, as requested.

15) Find the domain.



The domain goes from the smallest x value to the largest x value.

The range goes from the smallest y value to the largest y value.

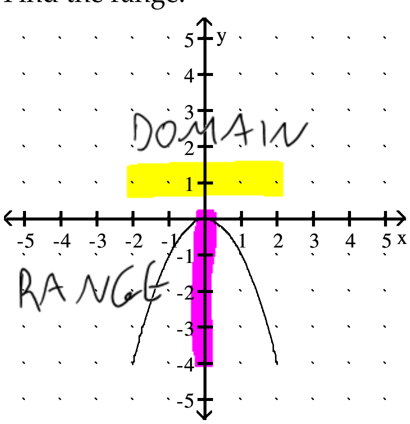
Notice that the graph does not end with an arrow on either side. This indicates that the graph stops.

DOMAIN: $[0,4]$ or $0 \le x \le 4$

RANGE: $[1,4]$ or $1 \le y \le 4$

OR $\{x \mid 0 \le x \le 4\}$

16) Find the range.



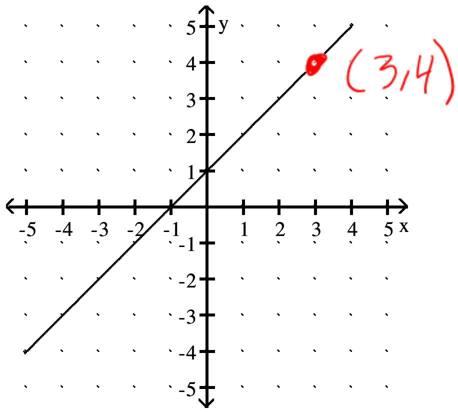
DOMAIN $[-2,2]$

RANGE $[-4,0]$

or $\{y \mid -4 \le y \le 0\}$

A function of x is depicted in the graph. Find any input values that produce the indicated output.

17) $f(x) = 4$

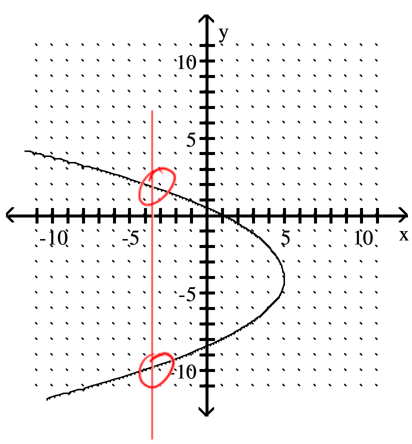


$f(x) = 4$

$y = 4$ when $\boxed{x=3}$

Determine whether the graph is the graph of a function.

18)



The vertical line test is used. Since there is a place on the graph where a vertical line crosses more than once (in this case twice) the graph is not of a function. The vertical line is a visual of the fact that

NOT FUNCTION

Find the function value.

19) Find f(3) when $f(x) = \frac{x - 6}{5x + 2}$.

$f(x) = \frac{(x) - 6}{5(x) + 2}$

$f(3) = \frac{(3) - 6}{5(3) + 2}$

$f(3) = \frac{-3}{15 + 2}$

$\boxed{f(3) = \frac{-3}{17}}$

20) Find f(x - 2) when $f(x) = \frac{2x - 5}{3x + 4}$.

$f(x) = \frac{2(x) - 5}{3(x) + 4}$

$f(x-2) = \frac{2(x-2) - 5}{3(x-2) + 4}$

$f(x-2) = \frac{2x - 4 - 5}{3x - 6 + 4}$

$\boxed{f(x-2) = \frac{2x - 9}{3x - 2}}$

Find the domain of f(x).

21) $f(x) = \frac{8}{x + 4}$

For a rational function (variables in the denominator) you need to worry about not dividing by zero. Set the denominator = 0 to find the

$x + 4 = 0$
$-4 \quad -4$
$x = -4$

$\boxed{\text{DOMAIN } \{x \text{ REAL} \mid x \neq -4\}}$

22) $f(x) = \frac{7}{-2 - x}$

$-2 - x = 0$
$+2 \qquad +2$

$-x = 2$
$\overline{-1} \quad \overline{-1}$

$x = -2$

$\boxed{\text{DOMAIN} \quad \{x \text{ REAL} \mid x \neq -2\}}$

Spoken "such that"

Solve the problem.

23) The function A described by $A(r) = 4\pi r^2$ gives the surface area of a sphere with radius r. Find the area when the radius is 4 in.

$A(r) = 4\pi (r)^2$

$A(4) = 4\pi (4 \text{ in})^2$

$A(4) = 4\pi (16 \text{ in}^2)$

$\boxed{A(4) = 64\pi \text{ in}^2}$

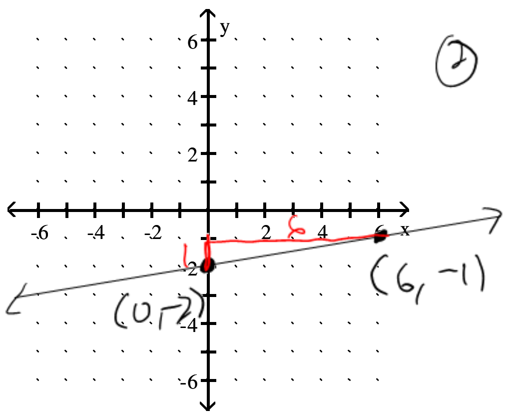
$\boxed{\approx 201.06 \text{ in}^2}$

2.3 **Graph.**

24) $f(x) = \frac{1}{6}x - 2$

① y-intercept is $(0,-2)$

② slope is $\frac{1}{6} = \frac{\text{rise}}{\text{run}}$



Find the slope of the line containing the two given points.

25) (9, −5) and (2, 5)

(x_1, y_1), (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - {}^-5}{2 - 9} = \frac{5 + 5}{-7} = \boxed{\frac{10}{-7}}$$

Find a linear function whose graph has the given slope and y–intercept.

26) Slope $-\frac{5}{3}$, y–intercept (0, 7)

$m = -\frac{5}{3}$ $\quad$ $b = 7$

Usually I will use point slope form to find an equation, but in this case, since the point given is exactly the y intercept, I can jump straight to slope intercept form. Note that the

$y = mx + b$

$y = -\frac{5}{3}x + 7$

$\boxed{f(x) = -\frac{5}{3}x + 7}$

This model is of the form f(x) = mx + b. Determine what m and b signify.

27) The cost, in dollars, of cellular phone service with Econo–phone is given by $C(x) = 0.31x + 35.90$, where x is the number of minutes used in one month.

m = .31. Slope is the change in y over the change in x, so we need the units of y and the units of x. These are dollar cost and monthly minutes, so the slope interpretation would be $0.31 per monthly minute.

b = 35.90. The y intercept is interpreted as the initial y value. In this case it is the initial monthly dollar cost, so the interpretation is that the service costs $35.90 per month initially.

The overall interpretation is that this plan is $35.90 per month and $0.31 per minute

2.4 **Find the slope of the line.**

28) $3x - 5y = 26$

The easiest form to determine the slope of a line is slope intercept. To get to this form, solve for y. A bonus of this form is that you also get the y intercept easily.

$3x - 5y = 26$
$-3x \qquad -3x$

$-5y = -3x + 26$

$\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{26}{-5}$

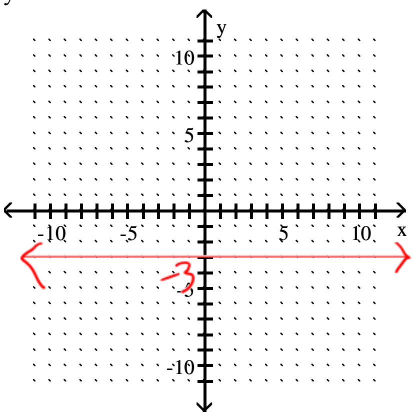
$y = \frac{-3}{-5}x + \frac{26}{-5}$

$y = \frac{3}{5}x - \frac{26}{5}$

$\boxed{m = \frac{3}{5}}$ (& y intercept is $(0, -\frac{26}{5})$)

Graph.

29) y + 3 = 0



When a linear function only has one variable, it is either a horizontal line or a vertical line. In this case we have y = -3, so we get a line that crosses the y axis at -3. This is horizontal. Whatever variable the equation has is what axis the line crosses. Solve for the variable to determine where the line crosses.

$y + 3 = 0$

$\quad -3 \quad -3$

$y = -3$

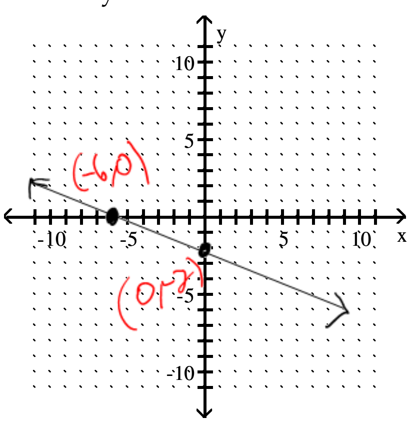
CROSSES Y AXIS

@ this location

(An x = 3 line would cross the x axis @ 3. (vertical))

Find the y- and x-intercepts for the equation. Then graph the equation.

30) −5x − 15y = 30



If a linear equation is in standard form, like this one, the fastest way to graph is to get the x intercept (y=0) and the y intercept (x=0) and plot

x intercept

$-5x - 15(0) = 30$

$\frac{-5x}{-5} = \frac{30}{-5}$

$x = -6$

$\boxed{(-6,0)}$

y intercept

$-5(0) - 15y = 30$

$\frac{-15y}{-15} = \frac{30}{-15}$

$y = -2$

$\boxed{(0,-2)}$

Determine whether the equation is linear.

31) $10x - 8y = 20$

If you can get the equation in either standard form $Ax + By = C$ or slope intercept form $y = mx + b$, then the equation is linear. If not, it is nonlinear.

$Ax + By = C$ ✓

LINEAR

$y = x^2$

$y = |x|$

$y = x^3$

$y = \sqrt{x}$

$y = \frac{1}{x}$

$y = \ln x$

$y = 2^x$

$y = \sqrt[3]{x}$

2.5 Find an equation in point–slope form of the line having the specified slope and containing the point indicated.

32) $m = \frac{-1}{2}$, $(-8, -5)$

POINT SLOPE FORM

$y - y_1 = m(x - x_1)$

$y - y_1 = m(x - x_1)$

$y - {}^-5 = \frac{-1}{2}(x - {}^-8)$

$\boxed{y + 5 = -\frac{1}{2}(x + 8)}$

Find an equation of the line containing the given pair of points. Write your final answer as a linear function in slope–intercept form.

33) $(8, -5)$ and $(1, -3)$

To use point slope form, we need to get the slope first, and choose a

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - {}^-5}{1 - 8} = \frac{-3 + 5}{-7} = \frac{-2}{7}$

USING $m = \frac{-2}{7}$ & $(8, -5)$ (you could also use $(1, -3)$)

$7 \cdot y - 7 \cdot {}^-5 = 7 \cdot \frac{-2}{7}(x - 8)$ Clear fractions & distribute parentheses

$7y + 35 = -2x + 16$ Solve for y

$\quad -35 \qquad\qquad -35$

$\frac{7y}{7} = \frac{-2x}{7} - \frac{19}{7}$

$y = \frac{-2}{7}x - \frac{19}{7}$ function!

$\boxed{f(x) = -\frac{2}{7}x - \frac{19}{7}}$

Solve the problem.

34) Persons taking a 30–hour review course to prepare for a standardized exam average a score of 620 on that exam. Persons taking a 70–hour review course average a score of 780. Find a linear function S(t), which fits this data, and which expresses score as a function of time.

$(x_1, y_1), (x_2, y_2)$

$(30, 620), (70, 780)$

$m = \frac{780-620}{70-30} = \frac{160}{40} = 4$

$y - 620 = 4(x - 30)$

$y - 620 = 4x - 120$

$+620 \quad +620$

$y = 4x + 500$

$y = S(t)$ & x is t so

$\boxed{S(t) = 4t + 500}$

If your story problem asks for a linear function, you will almost always be given information equivalent to 2 points. You job is to determine what are the two points, and in what order do they go. the order is (independent variable, dependent variable) In this case the variables are hours reviewed and exam score. The score depends on the review, so the score is the dependent variable, our usual y. So we have
(review time, score) for our point modelof (x,y). I usually use x and y for the main part

Tell whether the lines are "parallel", "perpendicular", or "neither."

35) $9x + 3y = 12$
$12x + 4y = 17$

Parallel lines have equal slope, so to determine if two lines are parallel, find their slopes

$9x + 3y = 12$

$-9x \quad -9x$

$\frac{3y}{3} = \frac{-9x}{3} + \frac{12}{3}$

$y = -3x - 4$

$m_1 = -3$

$12x + 4y = 17$

$-12x \quad -12x$

$\frac{4y}{4} = \frac{-12x}{4} + \frac{17}{4}$

$y = -3x + \frac{17}{4}$

$m_2 = -3$

Since $m_1 = -3 = m_2$

These slopes are equal, so the lines are

$\boxed{\text{PARALLEL}}$

Find an equation for the described linear function.

36) Through $\left(0, \frac{6}{7}\right)$ and parallel to $6x - 4y = 9$

$6x - 4y = 9$

$-6x \quad -6x$

$\frac{-4y}{-4} = \frac{-6x}{-4} + \frac{9}{-4}$

$y = \frac{3}{2}x - \frac{9}{4}$

Same Slope

$m = \frac{3}{2}$

so $\left(0, \frac{6}{7}\right), m = \frac{3}{2}$

$y = \frac{3}{2}x + \frac{6}{7}$

$\boxed{f(x) = \frac{3}{2}x + \frac{6}{7}}$

To get an equation for a line, we need to get the slope. In this case, we can find the slope of the reference line and use the same slope for our line, since our line is parallel to the reference line. Note the point we are given is the y-intercept, enabling us to jump to the slope intercept answer. Had the point been another point besides the y intercept, we would proceed to point slope form. Also notice they ask for the linear function, so therefore our f(x)

37) Through $\left(0, \frac{5}{8}\right)$ and perpendicular to $3x - 5y = 1$

Perpendicular lines have opposite and reciprocal slopes, for example -2/3 and 3/2 or 5 and -1/5. For this problem, we need to find the slope of th3e reference line and then create its opposite reciprocal to get the slope for our line. Again we are given the y intercept for the point. This time I will use point slope form so you can see that it also works, though is a

$3x - 5y = 1$

$-3x \quad -3x$

$-5y = -3x + 1$

$\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{1}{-5}$

$y = \frac{3}{5}x - \frac{1}{5}$

$m = -\frac{5}{3}, \ (0, \frac{5}{8})$

$y - \frac{5}{8} = -\frac{5}{3}(x - 0)$

$y - \frac{5}{8} = -\frac{5}{3}x$

$+\frac{5}{8} \quad +\frac{5}{8}$

$y = -\frac{5}{3}x + \frac{5}{8}$

$\boxed{f(x) = -\frac{5}{3}x + \frac{5}{8}}$

3.2 Solve using the substitution method. If the system has an infinite number of solutions, use set–builder notation to write the solution set. If the system has no solution, state this.

38) $4y + x = -3$
$x = 5y + 4$

To solve a system by substitution, you need to solve for one of the variables and substitute it into the other equation. Then back substitute into your solved equation and check your answer. I use substitution when, like in this problem, one of the equations already is either x = or y = . In this problem, I plugged the x from the second equation into the first equation, giving me y. Then I plugged the y into the x = equation to get x. It is appropriate to give the answer in an (x,y) point form. Also, it is important to check the answer in all of the initial equations.

$4y + (5y + 4) = -3$

$4y + 5y + 4 = -3$

$9y + 4 = -3$

$-4 \quad -4$

$9y = -7$

$\frac{9y}{9} = \frac{-7}{9}$

$y = -\frac{7}{9}$

Back Substitute

$x = 5\left(-\frac{7}{9}\right) + 4$

$x = \frac{-35}{9} + 4 \cdot \frac{9}{9}$

$x = \frac{1}{9}$

$\boxed{\left(\frac{1}{9}, \frac{-7}{9}\right)}$

Check:

$4y + x = -3$

$4\left(\frac{-7}{9}\right) + \left(\frac{1}{9}\right) = -3$

$\frac{-28}{9} + \frac{1}{9} = -3$

$\frac{-27}{9} = -3$ ✓

$x = 5y + 4$

$\frac{1}{9} = 5\left(\frac{-7}{9}\right) + 4$

$\frac{1}{9} = -\frac{35}{9} + 4$

$\frac{1}{9} = \frac{-35}{9} + \frac{36}{9} = \frac{1}{9}$ ✓

Solve using the elimination method. If the system has an infinite number of solutions, use set–builder notation to write the solution set. If the system has no solution, state this.

39) $x + 6y = 5$
$-3x + 5y = 31$

The Elimination Method consists of adding equations together and getting one of the variables to add to 0. In this case, I was able to eliminate x by multiplying the first equation by 3 and then adding the two equations together. I could have also multiplied the first by 5 and the second by -6 to add and eliminate the y, but that was more work. Then back substitute into either of the original equations to get the other variable and check. I use elimination for solving systems if all of the equations are in

$3 \cdot (x + 6y = 5)$

$-3x + 5y = 31$

$3x + 18y = 15$

$+ \ -3x + 5y = 31$

$23y = 46$

$\frac{23y}{23} = \frac{46}{23}$

$y = 2$

$x + 6(2) = 5$

$x + 12 = 5$

$-12 \quad -12$

$x = -7$

$\boxed{(-7, 2)}$

Check:

$x + 6y = 5$

$-7 + 6(2) = 5$

$-7 + 12 = 5$ ✓

$-3x + 5y = 31$

$-3(-7) + 5(2) = 31$

$21 + 10 = 31$ ✓

3.3 Solve the problem.

40) The sum of two numbers is 68. The second number is three times as large as the first number. What are the numbers?

This story problem is a translation story problem. Make sure you give a story answer. A statement like more than or less than is translated like this:

5 more than a number is	x + 5
3 less than a number is	x - 3

The sum of two numbers is 68.

$x + y = 68$

The second number is three times as large as the first number.

$y = 3x$

$x + y = 68$

$y = 3x$

substitution

$x + (3x) = 68$

$4x = 68$

$\frac{4x}{4} = \frac{68}{4}$

$x = 17$

BACK SUBSTITUTION $y = 3(17) = 51$

$(17, 51)$

The first # is 17 & the 2nd # is 51 story answer

Solve the problem.

41) The perimeter of a rectangle is 32 cm. The length is 12 cm longer than the width. Find the dimensions.

$P = 2L + 2W$

$L = W + 12$

$32 = 2L + 2W$

Common formulas you should know include the perimeter and area of a rectangle (same for square), the area and circumference of a circle, the area of a triangle, D=RT, and sum of angles in a triangle is 180 degrees.

$L = W + 12$

$32 = 2L + 2W$

$32 = 2(W + 12) + 2W$ SUBSTITUTION

$32 = 2W + 24 + 2W$

$32 = 4W + 24$

$-24 \qquad -24$

$8 = 4W$

$\frac{8}{4} = \frac{4W}{4}$

$W = 2$

$L = W + 12$ BACK SUBSTITUTION

$L = 2 + 12$

$L = 14$

The length is 14 cm & the width is 2 cm.

Story ANSWER includes words & units

42) The speed of a current is 6 mph. If a boat travels 56 miles downstream in the same time that it takes to travel 28 miles upstream, what is the speed of the boat in still water?

LET X = SPEED OF BOAT IN STILL WATER

This is a common D=RT problem. If you read the last statement of most story problems, you will find what is asked for in the problem. Let that be = x. I used substitution for this problem. Elimination could work, but is not as clear. The reason is that this system is not linear. Substitution is the main method for nonlinear systems. The reason this system is nonlinear is if you multiplied it all out, you would havex t imes t, which acts like

	D	= R	T
UP	28	$x-6$	T
DOWN	56	$x+6$	T

$28 = (x-6)T$

$56 = (x+6)T$

substitution

$T = \frac{28}{x-6}$ so

$(x-6) \cdot 56 = (x+6)\left(\frac{28}{x-6}\right) \cdot (x-6)$

$(x-6)56 = (x+6)28$

$56x - 336 = 28x + 168$

$-28x \quad +336 \qquad -28x \quad +336$

$\frac{28x}{28} = \frac{504}{28}$

$x = \boxed{18 \text{ mph}}$

43) Don runs a charity fruit sale, selling boxes of oranges for \$11 and boxes of grapefruit for \$10. If he sold a total of 762 boxes and took in \$8125 in all, then how many boxes of oranges did he sell?

Let x = # boxes of oranges
y = # boxes of grapefruit

Then $x+y = 762$ This is the total boxes

& $11x+10y = 8125$ This is the total cost equation. If you multiply the cost per box by the

$-10(x+y=762)$

$11x+10y=8125$

$-10x-10y=-7620$

$+ \quad 11x+10y=8125$

$x = 505$

$x+y = 762$

$505+y = 762$

$-505 \qquad -505$

$y = 257$

so $\boxed{\text{505 boxes of oranges \& 257 boxes of grapefruit}}$

44) A contractor mixes concrete from bags of pre−mix for small jobs. How many bags with 4% cement should he mix with 3 bags of 8% cement to produce a mix containing 5% cement?

Let x = # bags of 4% cement
3 = # bags of 8% cement

$$.04x + .08(3) = .05(x+3)$$

$$.04x + .24 = .05x + .15$$

$$-.05x \quad -.24 \qquad -.05x \quad -.24$$

$$\frac{-.01x}{-.01} = \frac{-.09}{-.01}$$

$$x = 9$$

$\boxed{\text{9 bags of 4\% cement}}$

45) Walt made an extra $9000 last year from a part−time job. He invested part of the money at 10% and the rest at 9%. He made a total of $860 in interest. How much was invested at 9%?

Let x = $ in 10%
y = $ in 9%

$$x + y = 9000$$
$$.10x + .09y = 860$$

$$-.10(x + y = 9000)$$
$$.10x + .09y = 860$$

$$-.10x - .10y = -900$$
$$+ \quad .10x + .09y = 860$$

$$\frac{-.01y}{-.01} = \frac{-40}{-.01}$$

$$y = 4000$$

$$x + y = 9000$$
$$x + 4000 = 9000$$
$$-4000 \qquad -4000$$
$$x = 5000$$

$5000 @ 10%
$4000 @ 9%

3.4 **Solve the system.**

46) $2x + 5y + z = -18$ R_1
$3x - 4y - z = 24$ R_2
$4x + y + 2z = 0$ R_3

$R_1 + R_2$

$$\begin{array}{rl} & 2x + 5y + z = -18 \\ + & 3x - 4y - z = 24 \\ \hline & 5x + y = 6 \quad R_4 \end{array}$$

$5x + y = 6$ R_4
$10x - 7y = 48$ R_5

$2R_2 + R_3$

$$\begin{array}{rl} & 6x - 8y - 2z = 48 \\ + & 4x + y + 2z = 0 \\ \hline & 10x - 7y = 48 \quad R_5 \end{array}$$

To solve a 3 by 3 system, work on eliminating one variable twice, and then eliminating another variable. Then back substitute and check. I this problem I eliminated z twice, and then y once to get x, then back to get y and then back to get z. The answer is given in a

$7R_4 + R_5$

$$\begin{array}{rl} & 35x + 7y = 42 \\ + & 10x - 7y = 48 \\ \hline & 45x = 90 \end{array}$$

$\frac{45x}{45} = \frac{90}{45}$

$x = 2$

Back substitute

$5x + y = 6$ R_4
$5(2) + y = 6$
$10 + y = 6$
$y = -4$

$2x + 5y + z = -18$ R_1
$2(2) + 5(-4) + z = -18$
$4 - 20 + z = -18$
$-16 + z = -18$
$z = -2$

$$\boxed{(2, -4, -2)}$$

& Check in all 3 equations, R_1, R_2 & R_3!

47) $x - y + 5z = 13$ R_1
$2x + z = 3$ R_2 & R_4
$x + 3y + z = 9$ R_3

$3R_1 + R_3$

$$\begin{array}{rl} & 3x - 3y + 15z = 39 \\ + & x + 3y + z = 9 \\ \hline & 4x + 16z = 48 \end{array}$$

$\frac{4x}{4} + \frac{16z}{4} = \frac{48}{4}$

$x + 4z = 12$ R_5

$2x + z = 3$ R_4
$x + 4z = 12$ R_5

In this problem, notice that the second equation already has no y variable. So we only need to eliminate y once more and y is eliminated out of 2 equations. Then I eliminated the z and found x...

$-4R_4 + R_5$

$$\begin{array}{rl} & -8x - 4z = -12 \\ + & x + 4z = 12 \\ \hline & -7x = 0 \end{array}$$

$\frac{-7x}{-7} = \frac{0}{-7}$

$x = 0$

Back Substitute

$2x + z = 3$ R_4
$2(0) + z = 3$
$z = 3$

$x - y + 5z = 13$ R_1
$0 - y + 5(3) = 13$
$-y + 15 = 13$
$\quad -15 \quad -15$
$\frac{-y}{-1} = \frac{-2}{-1}$
$y = 2$

$$\boxed{(0, 2, 3)}$$

Solve the system. If the system's equations are dependent or if there is no solution, state this.

48) $x - y + 5z = 17$ R_1
$-2x + 2y - 10z = 3$ R_2
$x + 5y + z = 13$ R_3

I eliminated x twice since it looked easy, though y or z would have been about the same work. In eliminating x the second time, I get a contradiction, leading to the answer of NO SOLUTION

$-R_1 + R_3$

$$-x + y - 5z = -17$$
$$+\ x + 5y + z = 13$$
$$6y - 4z = -4$$
$$\frac{6y}{2} - \frac{4z}{2} = \frac{-4}{2}$$
$$3y - 2z = -2 \quad R_4$$

$2R_1 + R_2$

$$2x - 2y + 10z = 34$$
$$+\ -2x + 2y - 10z = 3$$
$$0 = 37$$

CONTRADICTION EQUATION,

SO $\boxed{\emptyset}$ NO SOLUTION

49) $x + y + z = 9$ R_1
$2x - 3y + 4z = 7$ R_2
$x - 4y + 3z = -2$ R_3

For a change, I chose to eliminate z twice. Then I eliminated x and I found that all of the variables eliminated, leaving an identity equation. When this happens, the equations and therefore the system is DEPENDENT

$-4R_1 + R_2$

$$-4x - 4y - 4z = -36$$
$$+\ 2x - 3y + 4z = 7$$
$$-2x - 7y = -29$$
$$\frac{-2x}{-1} - \frac{7y}{-1} = \frac{-29}{-1}$$
$$2x + 7y = 29 \quad R_4$$

$2x + 7y = 29$ R_4
$2x + 7y = 29$ R_5

$-3R_1 + R_3$

$$-3x - 3y - 3z = -27$$
$$+\ x - 4y + 3z = -2$$
$$-2x - 7y = -29$$
$$\frac{-2x}{-1} - \frac{7y}{-1} = \frac{-29}{-1}$$
$$2x + 7y = 29 \quad R_5$$

$-R_4 + R_5$

$$-2x - 7y = -29$$
$$+\ 2x + 7y = 29$$
$$0 = 0$$

IDENTITY EQUATION

SO $\boxed{\text{DEPENDENT SYSTEM}}$