Salt Lake Community College Math 1060 Final Exam v2 - Fall Semester 2013

Name:_____ Instructor:_____

**This exam has two parts. Please read carefully the directions for each part. All problems are of equal point value. No notes, books, cell phones, or any devices that can connect to the internet are allowed.**

**PART ONE**

You must complete this portion of the test **without** using a calculator. For full credit you must show all appropriate work and clearly indicate your answers. After you have finished part one, your instructor will give you the remaining part of the exam.

When simplifying answers, it is **not** necessary to rationalize denominators.

1) Given that $\sin\alpha = \frac{1}{4}$ and $\alpha$ is an angle in quadrant II, find the exact values of all the remaining trigonometric functions.

$\cos\alpha$ _____ $\sec\alpha$ _____

$\tan\alpha$ _____ $\cot\alpha$ _____

$\csc\alpha$ _____

2) Find the exact value of each expression. If the expression is undefined, state so.

a) $\csc\left(\frac{5\pi}{3}\right)$

b) $\cot\left(-\frac{\pi}{6}\right)$

c) $\sec\left(450^\circ\right)$

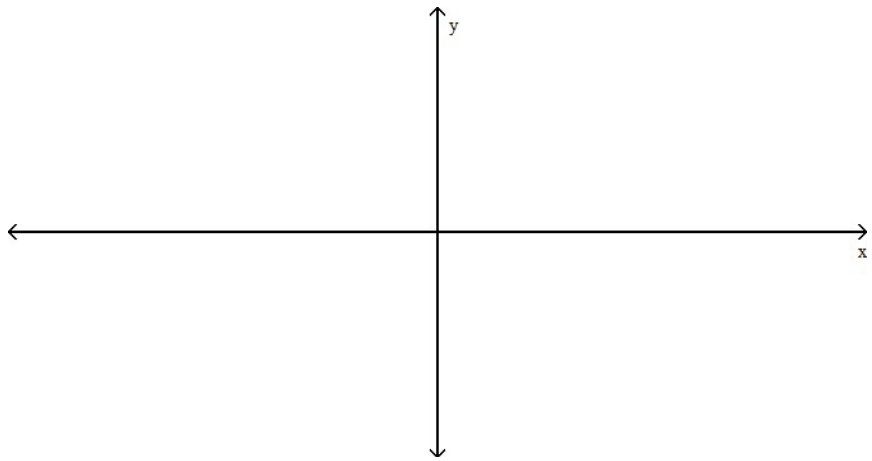
3) Find the exact value of each expression in radians.

a) $\arcsin\left(-\frac{1}{2}\right)$

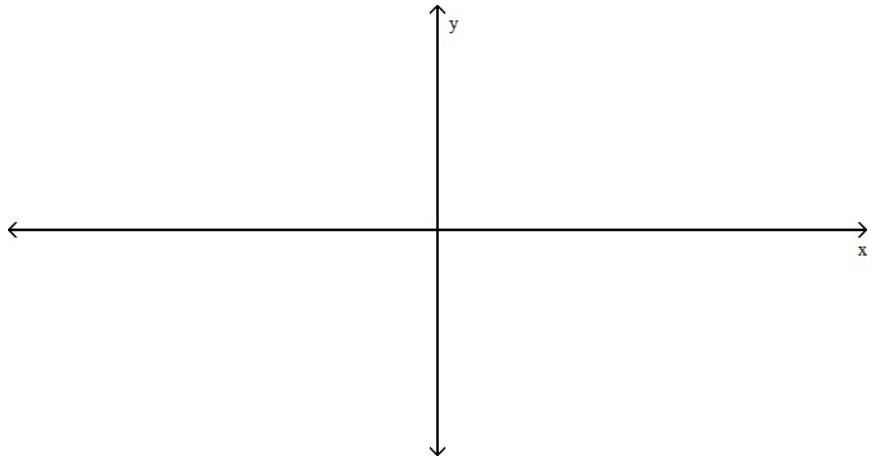
b) $\operatorname{arcsec}(2)$

c) $\arctan\left(-\frac{\sqrt{3}}{3}\right)$

4) Sketch the graph of $y = -3\sin[\pi(x-1)]$. Sketch at least one full cycle. Clearly label the "5 key points": the *x*-intercepts and the points corresponding to maxima or minima.



5) Sketch the graph of $y = \cot\left(x + \frac{\pi}{4}\right)$. Sketch at least one full cycle. Accurately label any asymptotes and *x*-intercepts. Give the equations of **all** the vertical asymptotes.




Equations of **all** the Vertical Asymptotes: _____

6) Prove that the following equation is an identity.

$$\frac{1+\cos x-\cos^2 x}{\sin x} = \cot x + \sin x$$

7) Find the exact value of the following expression. Simplify your answer.
Hint: You will need to use an identity.

$$2\sin^2\left(\frac{\pi}{8}\right)-1$$

8) Find the exact value of $\cos(\alpha-\frac{\pi}{3})$ if $\sin\alpha=\frac{1}{3}$ with $\alpha$ in quadrant II.

9) With $x$ in radians, find all real numbers (in radians) that satisfy the equation.

$2\sin(2x) - \sqrt{3} = 0$

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**PART TWO**

A NON-computer algebra system calculator is allowed. When directions specify an "exact value" a calculator should not be used.

Work all of the following problems. For full credit you must show all appropriate work and clearly indicate your answers.

10) A bicycle wheel with a 28 inch diameter is making 170 revolutions per minute.

a) What is the angular velocity in **radians per hour** for a point on the tire? Please show your work and give an exact answer (do not give a decimal approximation). Include units with your answer.

b) What is the linear velocity in **miles per hour** for a point on the tire? Please show your work and round the answer to the nearest tenth. Include units with your answer. [Please note that 1 mile = 5280 feet.]

11) Two sides and an angle are given. Solve any triangle(s) that results. Please round to the nearest tenth if necessary.

$a = 13,\ b = 9,\ \alpha = 65^\circ$

12) Find the following for the vectors $\mathbf{v} = \langle 3, -1 \rangle$ and $\mathbf{w} = \langle -2, 4 \rangle$. Please simplify your answers and give exact values.

a) $\mathbf{v} - 3\mathbf{w}$

b) $|\mathbf{w}|$

c) $\mathbf{v} \bullet \mathbf{w}$

13) The lengths of three sides of a triangle are ***a* = 4, *b* = 10 and *c* = 9**.
Find the measure of the angle $\beta$ in degrees. Please round to the nearest tenth.

14) Perform the indicated operation and **write the answer in the form *a* + *b*i**. Give **exact** values, not decimal approximations.

$$\frac{14\left(\cos 157^\circ + i\sin 157^\circ\right)}{7\left(\cos 127^\circ + i\sin 127^\circ\right)}$$

15) a) Convert the complex number to trigonometric form with $0^\circ \le \theta < 360^\circ$

$2-2i$

b) Simplify the following by using the trigonometric form from part (a) and DeMoivre's Theorem. **Leave the final answer in trigonometric form,** $0^\circ \le \theta < 360^\circ$ **.**

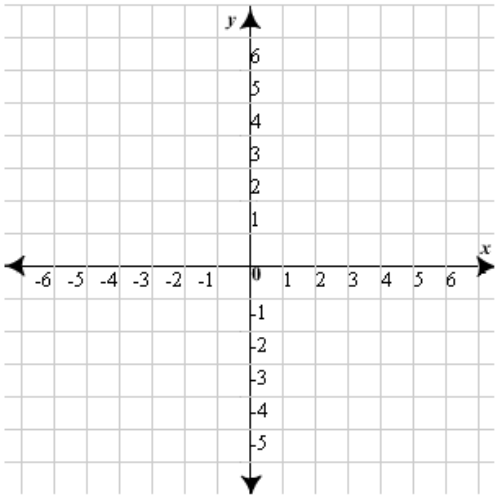
$(2-2i)^3$

16) Graph the curve whose parametric equations are given. List at least three exact points on the curve.

$x = t - 1$

$y = \sqrt{t}$

$0 \le t \le 4$

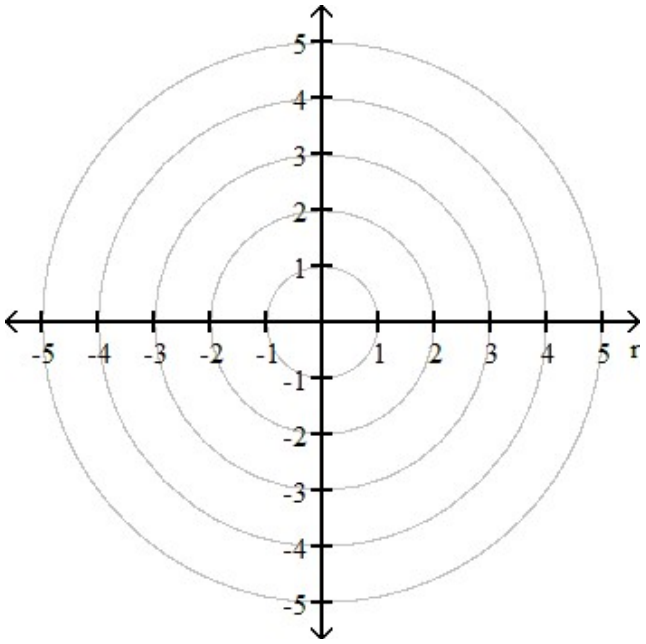


17) Forces of 3 lb and 11 lb act at an angle of $58^\circ$ to each other. Find the magnitude of the resultant force. Please round your answer to the nearest tenth and include appropriate units with your answer.
Your work must include a sketch showing the given situation.

18) A hot air balloon is between two spotters who are 1.3 miles apart. One spotter reports that the angle of elevation of the balloon is $65^\circ$, and the other reports that it is $72^\circ$. What is the altitude of the balloon? Please round your answer to the nearest tenth and include appropriate units with your answer. Your work must include a sketch showing the given situation.

19) Graph the polar equation. List at least three **exact** ($r$, θ) points that lie on the graph.

$r = 3\cos\theta$



20) A solid steel ball is placed on $12^{\circ}$ incline. If a force of 5.8 pounds in the direction of the incline is required to keep the ball in place, then what is the weight of the ball? Please round your answer to the nearest tenth and include appropriate units with your answer. Your work must include a sketch showing the given situation.

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Name: Key Instructor: _____

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**PART ONE**

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When simplifying answers, it is **not** necessary to rationalize denominators.

1) Given that $\sin\alpha = \frac{1}{4}$ and $\alpha$ is an angle in quadrant II, find the exact values of all the remaining trigonometric functions.

$\cos\alpha = -\frac{\sqrt{15}}{4}$ $\sec\alpha = -\frac{4}{\sqrt{15}}$ or $-\frac{4\sqrt{15}}{15}$

$\tan\alpha = -\frac{1}{\sqrt{15}}$ or $-\frac{\sqrt{15}}{15}$ $\cot\alpha = -\sqrt{15}$

$\csc\alpha = 4$

2) Find the exact value of each expression. If the expression is undefined, state so.

a) $\csc\left(\frac{5\pi}{3}\right) = -\frac{2}{\sqrt{3}}$ or $-\frac{2\sqrt{3}}{3}$

b) $\cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$

c) $\sec(450^\circ)$ undefined

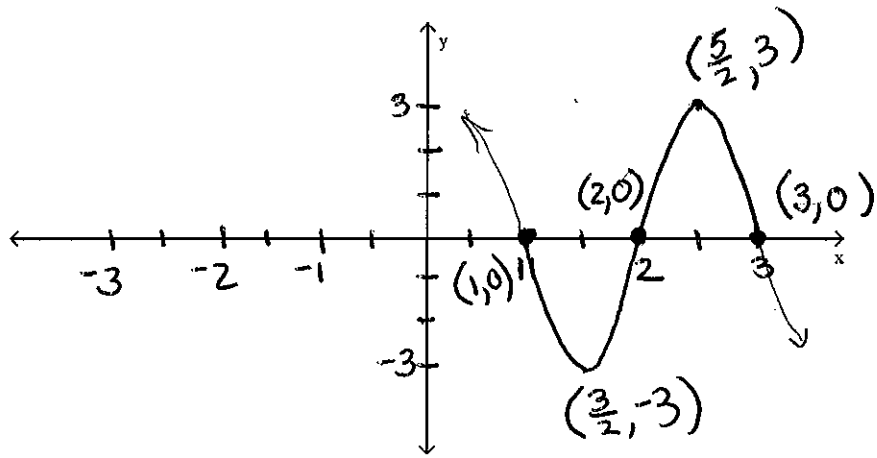
3) Find the exact value of each expression in radians.

a) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

b) $\operatorname{arcsec}(2) = \frac{\pi}{3}$

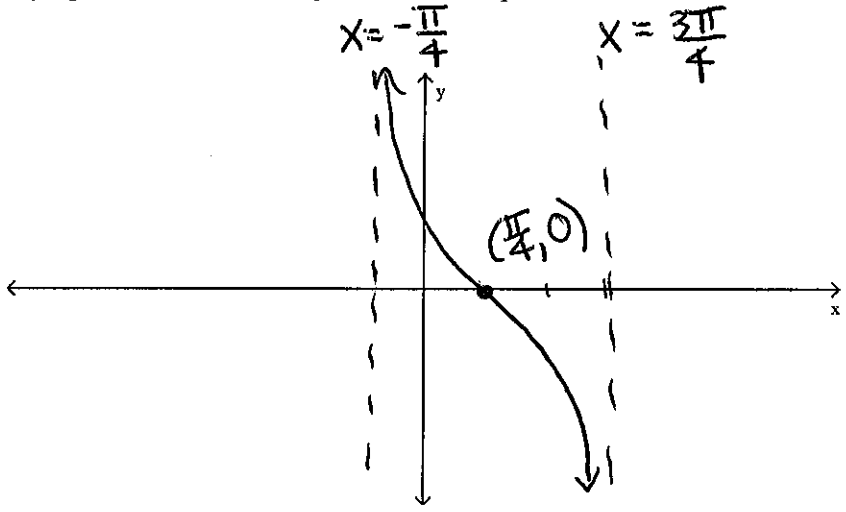
c) $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

4) Sketch the graph of $y = -3\sin[\pi(x-1)]$. Sketch at least one full cycle. Clearly label the "5 key points": the $x$-intercepts and the points corresponding to maxima or minima.




Period $= \frac{2\pi}{\pi} = 2$

5) Sketch the graph of $y = \cot\left(x + \frac{\pi}{4}\right)$. Sketch at least one full cycle. Accurately label any asymptotes and $x$-intercepts. Give the equations of **all** the vertical asymptotes.




Equations of **all** the Vertical Asymptotes: $X = \frac{3\pi}{4} + K\pi$ or $X = -\frac{\pi}{4} + K\pi$ where K is an integer

6) Prove that the following equation is an identity.

$$\frac{1 + \cos x - \cos^2 x}{\sin x} = \cot x + \sin x$$

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$$\frac{1 + \cos X - \cos^2 X}{\sin X} = \frac{\cos X + 1 - \cos^2 X}{\sin X}$$

$$= \frac{\cos X + \sin^2 X}{\sin X}$$

$$= \frac{\cos X}{\sin X} + \frac{\sin^2 X}{\sin X}$$

$$= \cot X + \sin X$$

7) Find the exact value of the following expression. Simplify your answer.
Hint: You will need to use an identity.

$$2\sin^2\left(\frac{\pi}{8}\right)-1$$

$$= -\cos\left(2\cdot\frac{\pi}{8}\right)$$

$$= -\cos\left(\frac{\pi}{4}\right)$$

$$= -\frac{\sqrt{2}}{2}$$

8) Find the exact value of $\cos(\alpha-\frac{\pi}{3})$ if $\sin\alpha=\frac{1}{3}$ with $\alpha$ in quadrant II.

$$\cos\left(\alpha-\frac{\pi}{3}\right)=\cos\alpha\cos\frac{\pi}{3}+\sin\alpha\sin\frac{\pi}{3}$$

$$=\left(-\frac{2\sqrt{2}}{3}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$=\frac{-2\sqrt{2}+\sqrt{3}}{6}$$

$\sin\alpha=\frac{1}{3}$:

$\cos^2\alpha+\frac{1}{9}=1$

$\cos^2\alpha=\frac{8}{9}$

$\cos\alpha=\ominus\frac{2\sqrt{2}}{3}$

$\alpha$ in QII

9) With $x$ in radians, find all real numbers (in radians) that satisfy the equation.

$2\sin(2x)-\sqrt{3}=0$

$\sin(2x)=\frac{\sqrt{3}}{2}$

$2x=\frac{\pi}{3}+2k\pi \Rightarrow x=\boxed{\frac{\pi}{6}+k\pi}$

OR

$2x=\frac{2\pi}{3}+2k\pi \Rightarrow x=\boxed{\frac{\pi}{3}+k\pi}$

for $k$ an integer

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Work all of the following problems. For full credit you must show all appropriate work and clearly indicate your answers.

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10) A bicycle wheel with a 28 inch diameter is making 170 revolutions per minute.

a) What is the angular velocity in **radians per hour** for a point on the tire? Please show your work and give an exact answer (do not give a decimal approximation). Include units with your answer.

$$\omega = \left(170 \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right) = 20{,}400\pi \text{ rad/hr}$$

b) What is the linear velocity in **miles per hour** for a point on the tire? Please show your work and round the answer to the nearest tenth. Include units with your answer. [Please note that 1 mile = 5280 feet.]

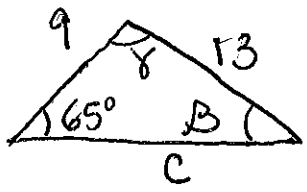
$$V = \omega r$$

$$V = \left(20{,}400\pi \frac{\text{rad}}{\text{hr}}\right)\left(14 \text{ in}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{1 \text{ mile}}{5280 \text{ ft}}\right)$$

$$V \approx 14.2 \text{ mile/hr}$$

11) Two sides and an angle are given. Solve any triangle(s) that results. Please round to the nearest tenth if necessary.

$a = 13, b = 9, \alpha = 65°$




$\beta \approx 38.9°$

$\gamma \approx 76.1°$

$C \approx 13.9$

$\frac{\sin 65°}{13} = \frac{\sin \beta}{9}$

$\sin \beta = \frac{9 \sin 65°}{13}$

$\beta = \sin^{-1}\left(\frac{9 \sin 65°}{13}\right) \approx 38.9°$

$\gamma \approx 180° - 65° - 38.9°$

$\gamma \approx 76.1°$

$\frac{c}{\sin 76.1°} = \frac{13}{\sin 65°}$

$C \approx \frac{13 \sin 76.1°}{\sin 65°}$

$C \approx 13.9$

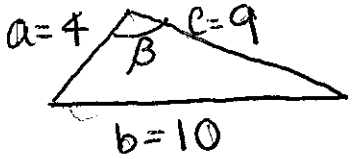
12) Find the following for the vectors $\mathbf{v} = \langle 3, -1 \rangle$ and $\mathbf{w} = \langle -2, 4 \rangle$. Please simplify your answers and give exact values.

a) $\mathbf{v} - 3\mathbf{w} = \langle 3, -1 \rangle - \langle -6, 12 \rangle = \langle 9, -13 \rangle$

b) $|\mathbf{w}| = \sqrt{(-2)^2 + (4)^2} = \sqrt{20} = 2\sqrt{5}$

c) $\mathbf{v} \bullet \mathbf{w} = (3)(-2) + (-1)(4) = -10$

13) The lengths of three sides of a triangle are $a = 4$, $b = 10$ and $c = 9$.
Find the measure of the angle $\beta$ in degrees. Please round to the nearest tenth.




$$10^2 = 9^2 + 4^2 - 2(9)(4)\cos\beta$$

$$\cos\beta = \frac{10^2 - 9^2 - 4^2}{-2(9)(4)}$$

$$\beta = \cos^{-1}\left(\frac{10^2 - 9^2 - 4^2}{-2(9)(4)}\right) \approx 92.4^\circ$$

14) Perform the indicated operation and **write the answer in the form** $a + bi$. Give **exact** values, not decimal approximations.

$$\frac{14(\cos 157^\circ + i\sin 157^\circ)}{7(\cos 127^\circ + i\sin 127^\circ)} = 2\left(\cos(157^\circ - 127^\circ) + i\sin(157^\circ - 127^\circ)\right)$$

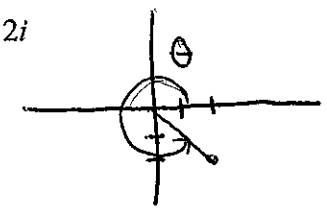
$$= 2\left(\cos(30^\circ) + i\sin(30^\circ)\right)$$

$$= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$= \sqrt{3} + i$$

15) a) Convert the complex number to trigonometric form with $0^\circ \le \theta < 360^\circ$

$2-2i$




$r = \sqrt{(2)^2+(-2)^2}$ $\quad \theta = 315^\circ$

$r = 2\sqrt{2}$

$2-2i = 2\sqrt{2}\left(\cos 315^\circ + i \sin 315^\circ\right)$

b) Simplify the following by using the trigonometric form from part (a) and DeMoivre's Theorem. **Leave the final answer in trigonometric form,** $0^\circ \le \theta < 360^\circ$ .

$$(2-2i)^3 = \left(2\sqrt{2}\right)^3 \left(\cos(3\cdot 315^\circ) + i \sin(3\cdot 315^\circ)\right)$$

$$= 16\sqrt{2}\left(\cos(945^\circ) + i \sin(945^\circ)\right)$$

$$= 16\sqrt{2}\left(\cos(225^\circ) + i \sin(225^\circ)\right)$$

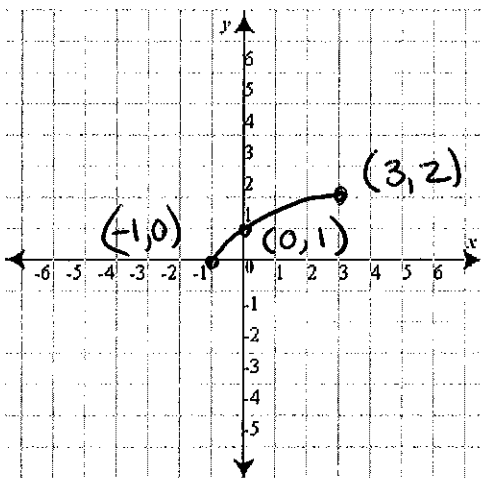
16) Graph the curve whose parametric equations are given. List at least three exact points on the curve.

$x = t-1$

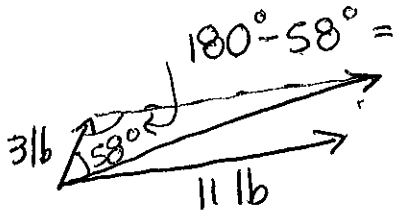
$y = \sqrt{t}$

$0 \le t \le 4$

| $t$ | $x$ | $y$ |
|---|---|---|
| 0 | -1 | 0 |
| 1 | 0 | 1 |
| 4 | 3 | 2 |



17) Forces of 3 lb and 11 lb act at an angle of 58° to each other. Find the magnitude of the resultant force. Please round your answer to the nearest tenth and include appropriate units with your answer.
Your work must include a sketch showing the given situation.

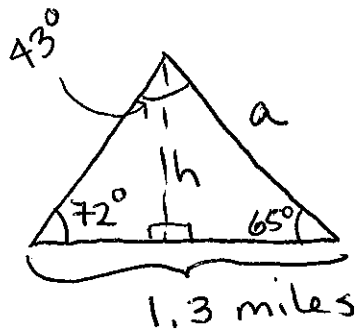



$\vec{F}$ is the resultant vector

$$|\vec{F}| = \sqrt{11^2 + 3^2 - 2(11)(3)\cos(122°)}$$

$$|\vec{F}| \approx 12.8 \text{ lb}$$

18) A hot air balloon is between two spotters who are 1.3 miles apart. One spotter reports that the angle of elevation of the balloon is 65°, and the other reports that it is 72°. What is the altitude of the balloon? Please round your answer to the nearest tenth and include appropriate units with your answer. Your work must include a sketch showing the given situation.




$$\frac{a}{\sin 72°} = \frac{1.3}{\sin 43°}$$

$$a = \frac{1.3 \sin 72°}{\sin 43°}$$

$$\sin 65° = \frac{h}{a}$$

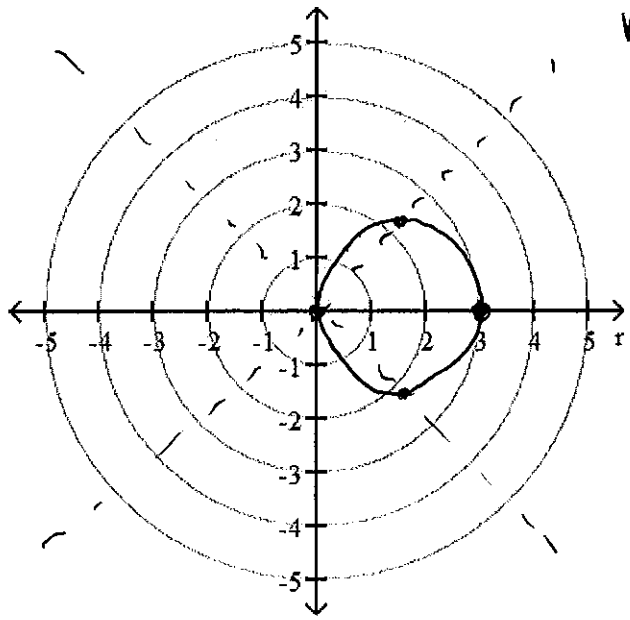
$$a \sin 65° = h$$

$$h \approx 1.6 \text{ miles}$$

( $a \approx 1.81$ miles )

19) Graph the polar equation. List at least three **exact** $(r, \theta)$ points that lie on the graph.

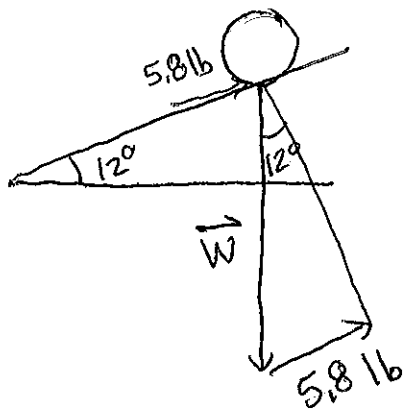
$r = 3\cos\theta$




| $\theta$ | $0$ | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ |
|---|---|---|---|---|---|
| $r$ | $3$ | $\frac{3\sqrt{2}}{2}$ | $0$ | $\frac{-3\sqrt{2}}{2}$ | $-3$ |

etc.

20) A solid steel ball is placed on 12° incline. If a force of 5.8 pounds in the direction of the incline is required to keep the ball in place, then what is the weight of the ball? Please round your answer to the nearest tenth and include appropriate units with your answer. Your work must include a sketch showing the given situation.




$$\sin 12^\circ = \frac{5.8}{|\vec{W}|}$$

$$|\vec{W}| = \frac{5.8}{\sin 12^\circ}$$

$$|\vec{W}| \approx 27.9 \text{ lb}$$