PRACTICE

Chapter 3 Test -- Form A Name: ~~Practice~~

Use identities to simplify each expression.

1. $\cos\theta \cdot \csc\theta \cdot \tan\theta$

$\cancel{\cos\theta} \cdot \frac{1}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{\cancel{\cos\theta}}$

$= 1$

2. $\frac{\sin x + \cos x}{\sin x}$

$= \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}$

$= 1 + \cot x$

3. $\frac{2\tan\left(\frac{\pi}{12}\right)}{1-\tan^2\left(\frac{\pi}{12}\right)}$

Tan Double angle formula!

$= \tan 2\left(\frac{\pi}{12}\right) = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

4. $\sin\left(x+\frac{\pi}{3}\right) - \cos\left(x+\frac{\pi}{6}\right)$

$\sin x\cos\frac{\pi}{3} + \cos x\sin\frac{\pi}{3} - \left(\cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6}\right)$

$\Rightarrow \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x - \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$

$\Rightarrow \frac{1}{2}\sin x + \frac{1}{2}\sin x = \sin x$

Prove that each of the following equations is an identity.

5. $\cot\theta \cdot \cos\theta = \csc\theta - \sin\theta$

$\frac{\cos\theta}{\sin\theta} \cdot \cos\theta$

$\frac{\cos^2\theta}{\sin\theta}$

$\frac{1-\sin^2\theta}{\sin\theta}$

$\frac{1}{\sin\theta} - \sin\theta = \csc\theta - \sin\theta$

6. $(\cot x + 1)^2 - \csc^2 x = \frac{2\cos x}{\sin x}$

$\cot^2 x + 2\cot x + 1 - \csc^2 x$

$\underbrace{\cot^2 x - \csc^2 x}_{-1} + 2\cot x + 1$

$-1 \quad + 2\cot x + 1$

$2\cot x = 2\frac{\cos x}{\sin x}$

7. $\dfrac{\csc\beta}{\tan\beta+\cot\beta}=\cos\beta$

$$\frac{\frac{1}{\sin\beta}}{\frac{\sin\beta}{\cos\beta}+\frac{\cos\beta}{\sin\beta}} \Rightarrow \frac{\frac{1}{\sin\beta}}{\frac{\sin^2\beta+\cos^2\beta}{\sin\beta\cos\beta}} \Rightarrow \frac{\frac{1}{\sin\beta}}{\frac{1}{\sin\beta\cos\beta}}$$

$$\Rightarrow \frac{1}{\sin\beta}\cdot\frac{\sin\beta\cos\beta}{1} = \cos\beta$$

8. $\tan\beta+\dfrac{\cos\beta}{1+\sin\beta} = \sec\beta$

$$\frac{\sin\beta}{\cos\beta}+\frac{\cos\beta}{1+\sin\beta}$$

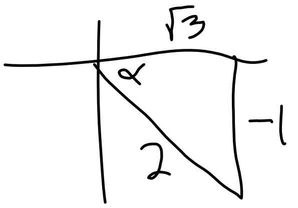
$$\Rightarrow \frac{\sin\beta(1+\sin\beta)+\cos\beta\cos\beta}{\cos\beta(1+\sin\beta)}$$

$$\Rightarrow \frac{\sin\beta+\sin^2\beta+\cos^2\beta}{\cos\beta(1+\sin\beta)}$$

$$\Rightarrow \frac{\cancel{\sin\beta+1}}{\cos\beta(\cancel{1+\sin\beta})} = \frac{1}{\cos\beta} = \sec\beta$$

Solve each problem.

9. If $\sec\alpha=\frac{2}{\sqrt{3}}$ and α is in Q IV, find the exact value of $\tan\alpha$.



$\cos\alpha=\dfrac{\sqrt{3}}{2}$

← by Pythagorean Theorem, & distance is down, so -1

$$\boxed{\tan\alpha=\frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}}$$

10. Determine whether the function $f(x)=x^2\cos x$ is odd, even, or neither.

$f(-x)=(-x)^2\cos(-x)$ (cos x is even)

$=x^2\cos x$

$=f(x)$,

so $f(x)$ is even

11. Write $y=\sin x+\cos x$ in the form $y=A\sin(x+C)$ and graph one cycle of the function. Label axes appropriately. Determine the period, amplitude and phase shift.

$y =$ $\sqrt{2}\sin(x+\frac{\pi}{4})$

amplitude: $\sqrt{2}$

phase shift: $-\pi/4$ or $\frac{\pi}{4}$ to LEFT

period: 2π

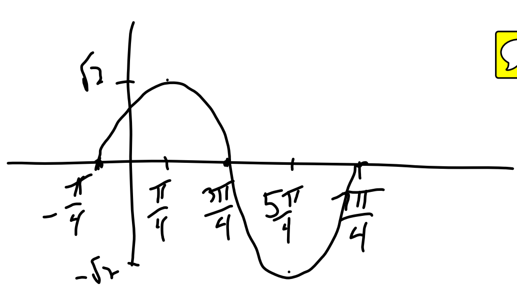
$a\sin x+b\cos x = A\sin(x+C)$

where $A=\sqrt{a^2+b^2}$

& $\tan C=\dfrac{b}{a}$

$A=\sqrt{1+1}=\sqrt{2}$

& $\tan C=\frac{1}{1}=1 \Rightarrow C=\dfrac{\pi}{4}$



12. Use an appropriate identity to find the exact value of tan 22.5°.

22.5 is ½ of 45, so ½ angle:

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$ $\qquad \tan 22.5^\circ = \tan\frac{45^\circ}{2}$, so $x = 45^\circ$

$$\frac{\sin 45}{1+\cos 45} = \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{2+\sqrt{2}}{2}} = \frac{\sqrt{2}}{\cancel{2}} \cdot \frac{\cancel{2}}{2+\sqrt{2}} = \frac{\sqrt{2}}{2+\sqrt{2}} \cdot \frac{2-\sqrt{2}}{2-\sqrt{2}}$$

$$= \frac{2\sqrt{2}-2}{4-2} = \frac{2\sqrt{2}-2}{2} = \sqrt{2}-1$$

13. Prove that the equation $\sin 2\theta = 2\sin\theta$ is not an identity.

$\sin 2\theta = 2\sin\theta\cos\theta$

& $2\sin\theta\cos\theta = 2\sin\theta$ ONLY IF $\cos\theta = 1$,

SO THIS IS A CONDITIONAL EQUATION NOT AN IDENTITY.

OR: LET $\theta = \frac{\pi}{2}$: $\sin 2\left(\frac{\pi}{2}\right) = \sin\pi = 0 \neq 2\sin\frac{\pi}{2} = 2$.

14. Use a product-to-sum identity to find the exact value of $\cos(105^\circ)\cdot\sin(75^\circ)$.

$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$\cos(105^\circ)\sin(75^\circ) = \frac{1}{2}\left[\sin 180^\circ - \sin 30^\circ\right] = \frac{1}{2}\left[0 - \frac{1}{2}\right] = -\frac{1}{4}$

Dugopolski's *Trigonometry*
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Find the exact value of each expression.

1. $\arcsin\left(\frac{\sqrt{2}}{2}\right)$ where is $\sin\theta = \frac{\sqrt{2}}{2}$ between $-\pi/2$ & $\pi/2$?

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, so

1. $\boxed{\frac{\pi}{4}}$

2. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ where is $\cos\theta = -\frac{\sqrt{3}}{2}$ between 0 & π? $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

so

2. $\boxed{\frac{5\pi}{6}}$

3. $\cot^{-1}\left(-\sqrt{3}\right)$ WHERE is $\cot\theta = -\sqrt{3}$ between 0 & π? $\cot\left(\frac{5\pi}{6}\right) = -\sqrt{3}$, so

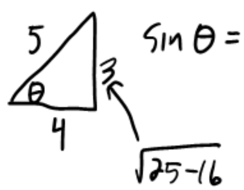
3. $\boxed{\frac{5\pi}{6}}$. Some books have the range of $\cot^{-1}(x)$ as $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$, & then the answer would be $-\frac{\pi}{6}$.

4. $\cos^{-1}(0)$ where does $\cos\theta = 0$ between 0 & π? $\cos\left(\frac{\pi}{2}\right) = 0$ so

4. $\boxed{\frac{\pi}{2}}$

5. $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$

Let $\theta = \cos^{-1}\left(\frac{4}{5}\right)$. THEN WE CAN EXPRESS θ WITH A TRIANGLE:



Now find $\sin\theta$.

$\sin\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$

5. $\boxed{\frac{3}{5}}$

6. $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

Lives between 0 & π!

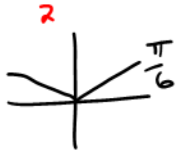
6. $\boxed{5\pi/6}$

Find all real numbers that satisfy each equation.

7. $2\sin x = 1$

$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$:

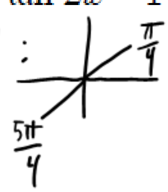


so $\boxed{x = \frac{\pi}{6} + 2k\pi \quad \& \quad x = \frac{5\pi}{6} + 2k\pi}$

7. ________________

8. $\tan 2x = 1$

$\tan\theta = 1$:



so $2x = \frac{\pi}{4} + k\pi$

& $\boxed{x = \frac{\pi}{8} + \frac{k\pi}{2}}$

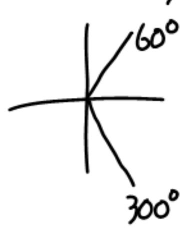
8. ________________

Find all values of α in $[0°, 360°)$ that satisfy each equation.
→ DEGREE ANSWERS

THE 2 Has us look 2 times around the circle.

9. $\sec\alpha = 2$

$\Rightarrow \cos\alpha = \frac{1}{2}$

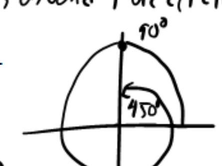


$\boxed{\alpha = 60°, 300°}$

10. $\csc(2\alpha) = 1$

$\Rightarrow \sin(2\alpha) = 1$

$2\alpha = 90°$ & $450°$

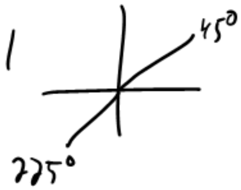


so $\boxed{\alpha = 45° \text{ \& } 225°}$

Find all values of α in $[0°, 360°)$ that satisfy each equation.

11. $\sin\alpha = \cos\alpha$ $\Rightarrow \tan\alpha = 1$

$\frac{\sin\alpha}{\cos\alpha} = \frac{\cos\alpha}{\cos\alpha}$



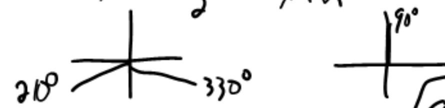
$\boxed{\alpha = 45°, 225°}$

11. ____________________

12. $2\sin^2\alpha - \sin\alpha - 1 = 0$

$(2\sin\alpha + 1)(\sin\alpha - 1) = 0$

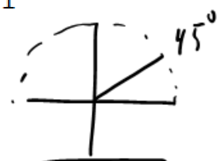
$\sin\alpha = -\frac{1}{2}$, $\sin\alpha = 1$



12. $\boxed{90°, 210°, 330°}$

13. $\tan(\frac{1}{2}\alpha) = 1$

Go ½ way around.

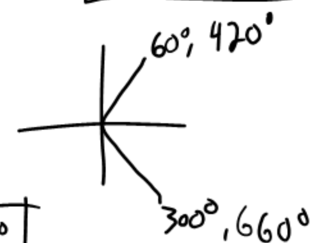


$\frac{1}{2}\alpha = 45° \Rightarrow \boxed{\alpha = 90°}$

13. ____________________

14. $\cos(2\alpha) = \frac{1}{2}$

2 times around



$2\alpha = 60°, 300°, 420°, 660°$

$\Rightarrow \boxed{\alpha = 30°, 150°, 210°, 330°}$

14. ____________________

Solve each problem.

15. Find all points at which the graph of $y = \cos x$ intersects the graph of $y = \cot x$.

$\cos x = \cot x \Rightarrow \cos x = \frac{\cos x}{\sin x} \Rightarrow \cos x - \frac{\cos x}{\sin x} = 0 \Rightarrow \cos x\left(1 - \frac{1}{\sin x}\right) = 0 \Rightarrow \cos x = 0, 1 - \frac{1}{\sin x} = 0$

$\cos x = 0$ @ $\frac{\pi}{2} + k\pi$. $1 - \frac{1}{\sin x} = 0 \Rightarrow 1 = \frac{1}{\sin x} \Rightarrow \sin x = 1$ @ $\frac{\pi}{2} + 2k\pi$

So when $x = \frac{\pi}{2} + k\pi$, $y = \cos(\frac{\pi}{2} + k\pi) = 0$

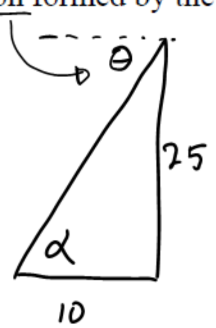
& The points of intersection are $\left\{\left(\frac{\pi}{2} + k\pi, 0\right)\right\}$

15. ____________________

16. A utility pole is 25 ft tall. A guy wire is attached to the top of the pole and to the ground. The anchor for the wire on the ground is 10 ft from the base of the pole. What is the angle of depression formed by the wire? FIND θ.

ANGLE OF ELEVATION

ANGLE OF DEPRESSION



$\theta = \alpha$. $\tan\alpha = \frac{25}{10}$

$\Rightarrow \alpha = \boxed{68.2°}$