Chapter 3 Test -- Form A **Name:** Practice

Use identities to simplify each expression.

1. $\cos\theta \cdot \csc\theta \cdot \tan\theta$

2. $\dfrac{\sin x + \cos x}{\sin x}$

1. _____

2. _____

3. $\dfrac{2\tan\left(\frac{\pi}{12}\right)}{1-\tan^2\left(\frac{\pi}{12}\right)}$

4. $\sin\left(x+\frac{\pi}{3}\right) - \cos\left(x+\frac{\pi}{6}\right)$

3. _____

4. _____

Prove that each of the following equations is an identity.

5. $\cot\theta \cdot \cos\theta = \csc\theta - \sin\theta$

6. $(\cot x + 1)^2 - \csc^2 x = \dfrac{2\cos x}{\sin x}$

7. $\dfrac{\csc\beta}{\tan\beta + \cot\beta} = \cos\beta$

8. $\tan\beta + \dfrac{\cos\beta}{1+\sin\beta} = \sec\beta$

Solve each problem.

9. If $\sec \alpha = \frac{2}{\sqrt{3}}$ and α is in Q IV, find the exact value of $\tan \alpha$.

10. Determine whether the function $f(x) = x^2 \cos x$ is odd, even, or neither.

9. _____

10. _____

11. Write $y = \sin x + \cos x$ in the form $y = A \sin(x + C)$ and graph one cycle of the function. Label axes appropriately. Determine the period, amplitude and phase shift.

$y =$ _____
amplitude: _____
phase shift: _____
period: _____

Omit (3.6 not covered)

12. Use an appropriate identity to find the exact value of $\tan 22.5°$.

12. _____

13. Prove that the equation $\sin 2\theta = 2 \sin \theta$ is not an identity.

14. Use a product-to-sum identity to find the exact value of $\cos(105°) \cdot \sin(75°)$.

Omit (3.6 not covered)

14. _____

Name: _____ Practice

Find the exact value of each expression.

1. $\arcsin\left(\frac{\sqrt{2}}{2}\right)$

1. _____

2. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

2. _____

3. $\cot^{-1}\left(-\sqrt{3}\right)$

3. _____

4. $\cos^{-1}(0)$

4. _____

5. $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$

5. _____

6. $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

6. _____

Find all real numbers that satisfy each equation.

7. $2\sin x = 1$

7. _____

8. $\tan 2x = 1$

8. _____

Find all values of α in $[0°, 360°)$ that satisfy each equation.

9. $\sec\alpha = 2$

10. $\csc(2\alpha) = 1$

Find all values of α in [0°, 360°) that satisfy each equation.

11. $\sin\alpha = \cos\alpha$

11. _____

12. $2\sin^2\alpha - \sin\alpha - 1 = 0$

12. _____

13. $\tan(\frac{1}{2}\alpha) = 1$

13. _____

14. $\cos(2\alpha) = \frac{1}{2}$

14. _____

Solve each problem.

15. Find all points at which the graph of $y = \cos x$ intersects the graph of $y = \cot x$.

15. _____

16. A utility pole is 25 ft tall. A guy wire is attached to the top of the pole and to the ground. The anchor for the wire on the ground is 10 ft from the base of the pole. What is the angle of depression formed by the wire?

Answers to Review 3-4

CHAPTER 3 (Answers to 5-8 on Forms A-D show one possible approach.)

Form A:

1. 1 **2.** $1 + \cot x$ **3.** $\frac{\sqrt{3}}{3}$ **4.** $\sin x$

5. $\cot\theta \cdot \cos\theta = \frac{\cos\theta}{\sin\theta} \cdot \cos\theta = \frac{\cos^2\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta} = \csc\theta - \sin\theta$

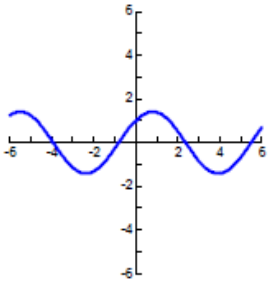
6. $(\cot x + 1)^2 - \csc^2 x = \cot^2 x + 2\cot x + 1 - \csc^2 x = 2\cot x = \frac{2\cos x}{\sin x}$

7. $\frac{\csc\beta}{\tan\beta + \cot\beta} = \frac{1/\sin\beta}{\sin\beta/\cos\beta + \cos\beta/\sin\beta} \cdot \frac{\sin\beta\cos\beta}{\sin\beta\cos\beta} = \frac{\cos\beta}{\sin^2\beta + \cos^2\beta} = \cos\beta$

8. $\tan\beta + \frac{\cos\beta}{1+\sin\beta} = \frac{\sin\beta}{\cos\beta} + \frac{\cos\beta}{1+\sin\beta} = \frac{\sin\beta + \sin^2\beta + \cos^2\beta}{\cos\beta(1+\sin\beta)} = \frac{\sin\beta + 1}{\cos\beta(1+\sin\beta)} = \frac{1}{\cos\beta} = \sec\beta$

9. $-\frac{\sqrt{3}}{3}$ **10.** Even: $f(-x) = (-x)^2\cos(-x) = x^2\cos(x) = f(x)$

11. $y = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$ period: 2π, amp: $\sqrt{2}$, phase shift: $\frac{\pi}{4}$ left **12.** $\sqrt{3-2\sqrt{2}}$ or $\sqrt{2}-1$



13. Show a counterexample, such as $\theta = 45°$: $\sin 2\theta = \sin[2(45°)] = \sin 90° = 1$, whereas $2\sin\theta = 2\sin 45° = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$ **14.** $-\frac{1}{4}$

CHAPTER 4

Form A:

1. $\frac{\pi}{4}$ **2.** $\frac{5\pi}{6}$ **3.** $-\frac{\pi}{6}$ **4.** $\frac{\pi}{2}$ **5.** $\frac{3}{5}$ **6.** $\frac{5\pi}{6}$

7. $\{x \mid x = \frac{\pi}{6} + 2\pi k$, or $\frac{5\pi}{6} + 2\pi k$, k an integer$\}$ **8.** $\{x \mid x = \frac{\pi}{8} + \frac{\pi}{2}k$, k an integer$\}$

9. 60°, 300° **10.** 45°, 225° **11.** 45°, 225° **12.** 90°, 210°, 330°

13. 90° **14.** 30°, 150°, 210°, 330° **15.** $\left\{\left(\frac{\pi}{2} + \pi k,\ 0\right)\right\}$

16. approximately 68.2°