PRACTICE

Chapter 1 Test -- Form A Name: PRACTICE

Find the exact value of each expression.

1. $\sin(570°)$

$570 - 360 = 210$

210 is 30° more than 180°, so Reference angle = 30°, Quadrant is III.

$\sin\theta$ is $-$ in QIII.

$\sin 30° = \frac{1}{2}$

so $\sin 570° = -\frac{1}{2}$

2. $\cos\left(-\frac{\pi}{3}\right)$

$\frac{-\pi}{3}$: Ref Angle is $\frac{\pi}{3}$.

QUAD: IV

$\cos\theta$ is $+$ in QIV

$\cos\frac{\pi}{3} = \frac{1}{2}$

so $\cos -\frac{\pi}{3} = \frac{1}{2}$

3. $\tan\left(\frac{2\pi}{3}\right)$

$\frac{2\pi}{3}$ Ref is $\frac{\pi}{3}$

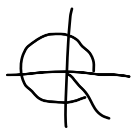
$\frac{2\pi}{3}$ in QII

$\tan\theta$ $-$ in QII

$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

4. $\csc(315°)$



45° Reference Angle

$\sin$ is $-$ in QIV

$\sin 45° = \frac{1}{\sqrt{2}}$

so $\csc 45° = \sqrt{2}$

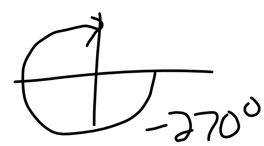
$\csc 315° = -\sqrt{2}$

5. $\sec\left(\frac{\pi}{4}\right)$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

so $\sec\frac{\pi}{4} = \sqrt{2}$

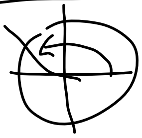
6. $\cot(-270°)$



Ref is 90°

$\cot 90° = \frac{\cos 90°}{\sin 90°} = \frac{0}{1}$

$= 0$

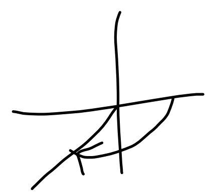


7. $\sin\left(\frac{5\pi}{6}\right)$

$\frac{5\pi}{6}$ Ref is $\frac{\pi}{6}$, $\sin$ $+$ in QII

$\sin\frac{\pi}{6} = \frac{1}{2}$, so $\sin\frac{5\pi}{6} = +\frac{1}{2}$

8. $\tan(-135°)$

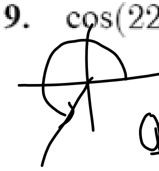


Ref $= 45°$, QIII

$\tan$ is $+$ in QIII

$\tan 45° = 1$, so $\tan(-135°) = +1$

9. $\cos(225°)$



QIII.

$\cos$ is $-$ in QIII

$\cos 45° = \frac{\sqrt{2}}{2}$

so $\cos(225°) = -\frac{\sqrt{2}}{2}$

10. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$,

so $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

11. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$0 < \cos^{-1} < \pi$

$\cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2}$

so $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

12. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

so $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

Solve each problem.

13. Find the exact value of the arc length intercepted by a central angle of 105° in a circle with a radius of 9 centimeters.

$S = r\theta$, r in RADIANS. $105° \cdot \frac{\pi}{180°} = \frac{7\pi}{12}$ RAD

$S = 9\left(\frac{7\pi}{12}\right) = \frac{21\pi}{4}$ cm

14. Find the degree measure (to the nearest hundredth of a degree) for an angle of 3 radians.

$3\text{RAD} \cdot \frac{180°}{\pi \text{ rad}} = \frac{540}{\pi}° \approx 171.89°$

15. Determine whether $-340°$ and $1060°$ are coterminal. Explain your answer.

ADD 360° TO GET A COTERMINAL ANGLE:

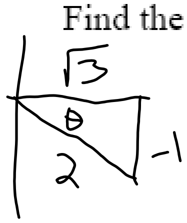
$-340° + 360° = \underline{\underline{20°}} + 360° = \underline{\underline{380°}} + 360° = \underline{\underline{740°}} + 360° = \underline{\underline{1100°}}$

1060° IS NOT COTERMINAL.

OR $1060° - (-340°) = 1400°$, $\frac{1400°}{360}$ IS NOT AN INTEGER,

SO NOT COTERMINAL

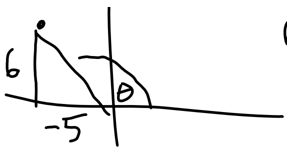
16. Find the exact value of $\tan\theta$ if $\cos\theta = \frac{\sqrt{3}}{2}$ and θ is in quadrant IV.



$\leftarrow -1$, down & pythagorean theorem

$\tan\theta = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

17. Find the exact value of cos θ for an angle θ in standard position whose terminal side contains the point $(-5, 6)$.

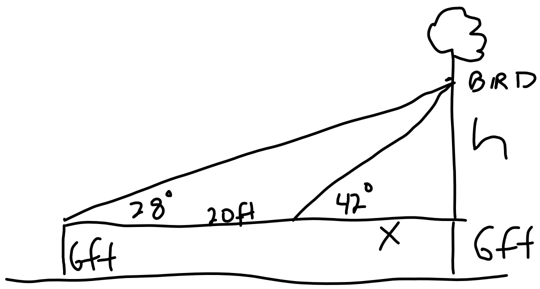


$r^2 = (-5)^2 + 6^2 = 25 + 36 = 61, \quad r = \sqrt{61}$

Ref ANGLE $= \alpha$, QII, cos is $-$ in QII

$$\cos\alpha = \frac{-5}{\sqrt{61}} = \frac{-5\sqrt{61}}{61}$$

18. Matt, a six-footer, spies a red-headed woodpecker on the branch of a tree in his yard. From the ground, the angle of elevation along his line of sight to the bird is 28°. He walks 20 feet towards the tree, and in the same plane as before, now sees the bird at an angle of elevation of 42°. How high in the tree is the bird? Round answer to the nearest foot.



$$\tan 28^\circ = \frac{h}{x+20}$$

$$\tan 42^\circ = \frac{h}{x} \Rightarrow h = x\tan 42$$

$$\tan 28 = \frac{x\tan 42}{x+20} \Rightarrow (x+20)\tan 28 = x\tan 42$$

$$\Rightarrow x\tan 28 + 20\tan 28 = x\tan 42$$

$$\Rightarrow x\tan 28 - x\tan 42 = -20\tan 28 \Rightarrow x(\tan 28 - \tan 42) = -20\tan 28$$

$$\Rightarrow x = \frac{-20\tan 28}{\tan 28 - \tan 42}; \quad h = x\tan 42 \approx 25.97$$

BIRD HEIGHT $= h + 6 = 25.97 + 6 \approx 32$ ft

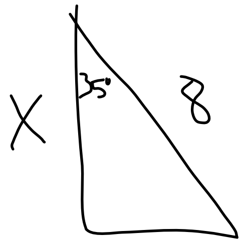
19. At what speed in miles per hour will a bicycle travel if the rider can cause the 26-inch diameter wheel to rotate 90 revolutions per minute? Round answer to the nearest tenth.

1 REV $= \pi D$ circumference
$= 26\pi$ in
60 min $=$ 1 hr
12 in $=$ 1 ft
5280 ft $=$ 1 mi

$$90\frac{\cancel{\text{rev}}}{\cancel{\text{min}}} \cdot \frac{26\pi\,\cancel{\text{in}}}{1\,\cancel{\text{rev}}} \cdot \frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}} \cdot \frac{1\text{ mi}}{5280\,\cancel{\text{ft}}} \cdot \frac{60\,\cancel{\text{min}}}{1\text{ hr}}$$

$$= 6.96 \approx 7.0 \text{ mi/hr}$$

20. One of the acute angles in a right triangle is 35° and the hypotenuse is 8 cm. Find the length of the leg of this right triangle which is adjacent to this 35° angle. Round answer to the nearest hundredth of a centimeter.



$\cos 35° = \frac{X}{8} \Rightarrow X = 8\cos 35°$

≈ 6.55 cm

Dugopolski's *Trigonometry*
Chapter 2 Test -- Form A 25

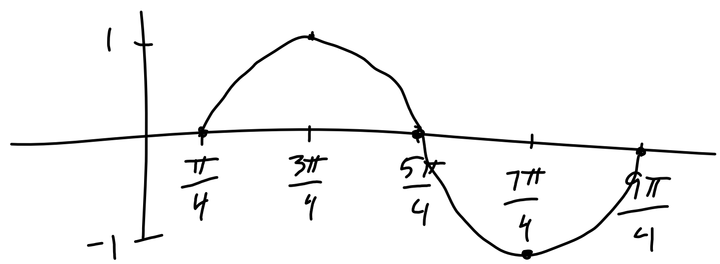
Name: PRACTICE

Sketch at least one cycle of the graph of each function. Draw and label the axes appropriately. Determine the period, range, and amplitude for each function.

1. $y = \sin(x - \frac{\pi}{4})$

period: 2π
range: $[-1, 1]$
amplitude: 1

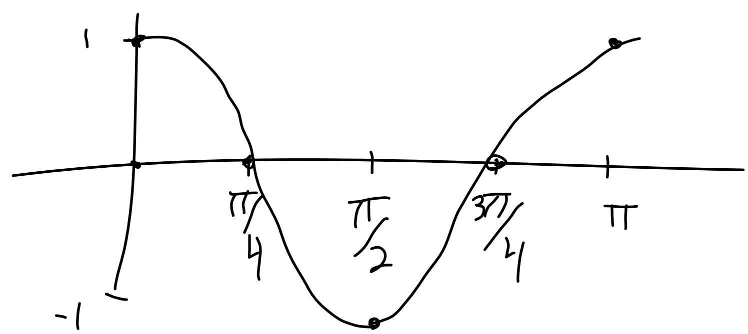
RIGHT $\pi/4$ phaseshift



2. $y = \cos(2x)$

period: $\frac{2\pi}{2} = \pi$
range: $[-1, 1]$
amplitude: 1

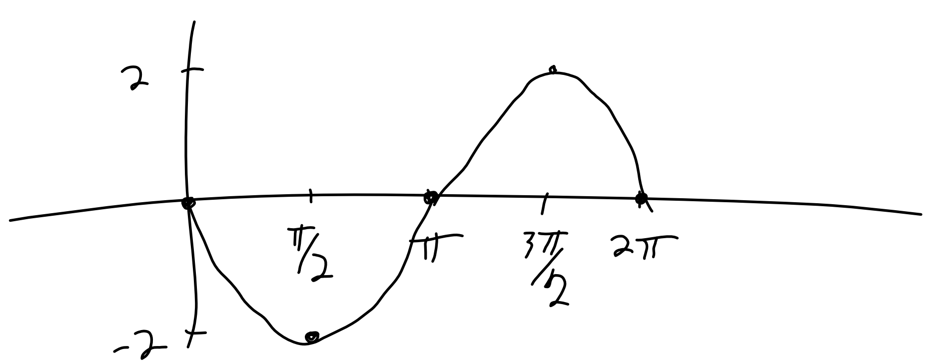
0 PHASE SHIFT



3. $y = -2\sin(x)$

period: 2π
range: $[-2, 2]$
amplitude: 2

0 PHASE SHIFT



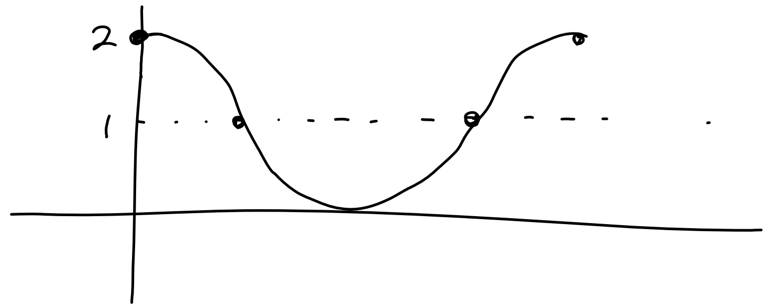
4. $y = \cos(x) + 1$

period: 2π

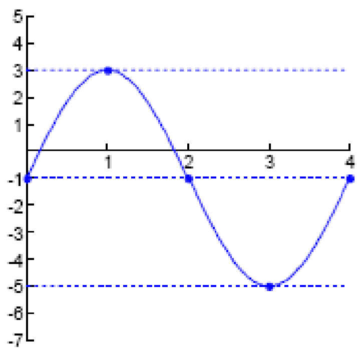
range: $[0, 2]$

amplitude: 1

0 Phase Shift



5. **Determine the amplitude and period for the sine curve in the accompanying graph. Write its equation in the form $y = A\sin(B[x - C]) + D$.**



"EQUILIBRIUM" is at $-1 \Rightarrow D = -1$

Period is 4, so $\frac{2\pi}{B} = 4 \Rightarrow B = \pi/2$

FROM -1 to 3 is 4, so $A = 4$

sin starts @ equilib, so no phase shift $C = 0$

period: 4 *amplitude:* 4

equation: $y = 4\sin\left(\frac{\pi}{2}x\right) - 1$

Sketch at least one cycle of the graph of each function. Draw and label the axes appropriately. Determine the period, asymptotes, and range for each function.

6. $y = \tan(x - \pi)$

period: π

$\frac{\pi}{2} + k\pi \rightarrow$ *asymptotes:* $\ldots \pi/2, 3\pi/2, 5\pi/2, \ldots$

range: $(-\infty, \infty)$

NORMAL TAN INFO:

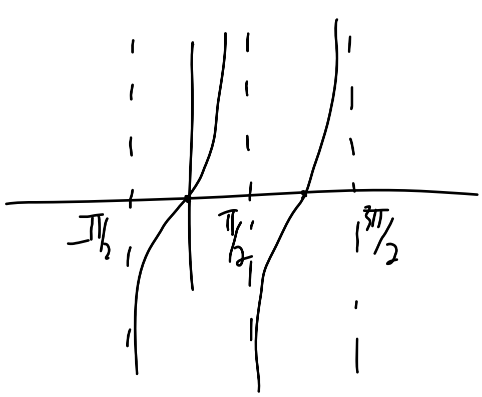
Asymptotes $-\pi/2$, $\pi/2$, crosses ORIGIN

Per π, RAN $(-\infty, \infty)$

$\rightarrow$ PHASE SHIFT RIGHT π.

SO Asymptotes: $-\frac{\pi}{2} + \pi = \pi/2$, $\pi/2 + \pi = 3\pi/2$

Crosses origin $+\pi = \pi$



SHIFTING A TRIG FUNCTION BY ITS PERIOD RESULTS IN NO CHANGE.

7. $y = -\sec(x) + \pi$

GRAPH COSINE TO GET SEC:

$-\cos x + \pi$: STARTS DOWN @ AMPLITUDE & UP π,

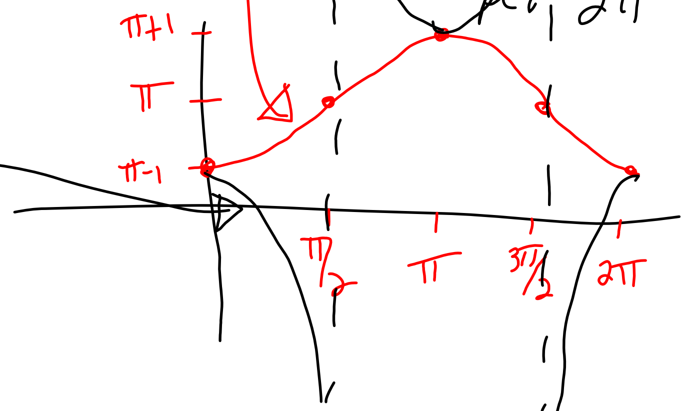
Per 2π Amp $= 1$

period: 2π

$\frac{\pi}{2} + k\pi$ asymptotes: $\pi/2, 3\pi/2, 5\pi/2 \ldots$

range: $(-\infty, \pi - 1] \cup [\pi + 1, \infty)$

sec has asymptote when cos is @ equilibrium & touches cos @ amplitudes



8. $y = \csc(3x)$

period: $2\pi/3$

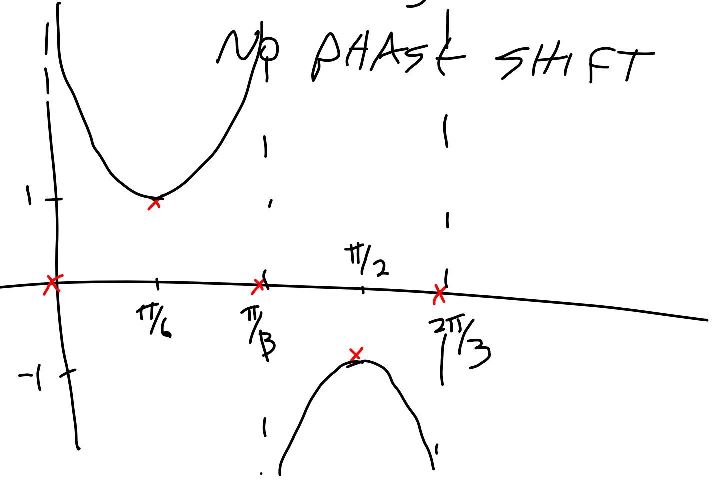
$\frac{k\pi}{3} \rightarrow$ asymptotes: $0, \pi/3, 2\pi/3, \ldots$

range: $(-\infty, -1] \cup [1, \infty)$

FOR CSC, THINK SIN.

$\sin 3x$: Per $\frac{2\pi}{3}$, Amp $= 1$, NO PHASE SHIFT

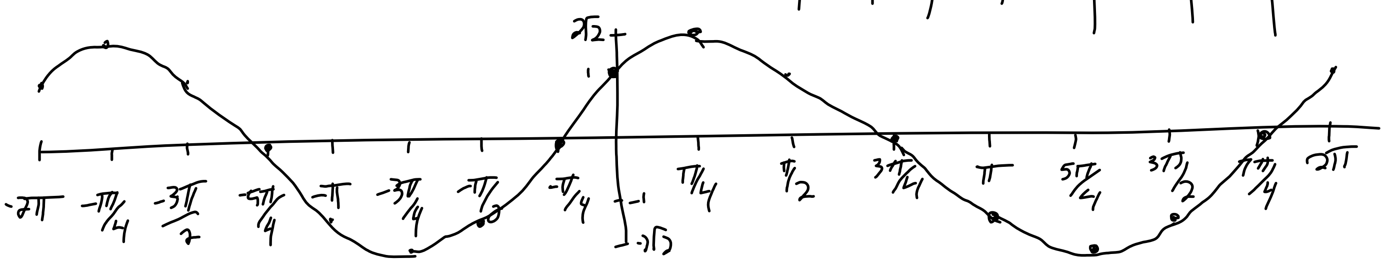
CSC HAS ASYMPTOTES WHEN SIN @ EQUILIBRIUM & TOUCHES SIN @ AMPLITUDE



Solve each problem.

9. Graph the function $y = \sin x - \cos x$ for x between -2π and 2π using the technique of adding the y-coordinates. Draw and label the axes appropriately.

x	-2π	$-7\pi/4$	$-3\pi/2$	$-5\pi/4$	$-\pi$	$-3\pi/4$	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin x$	0	$\sqrt{2}$	1	$\sqrt{2}$	0	$-\sqrt{2}$	-1	$-\sqrt{2}$	0	$\sqrt{2}$	1	$\sqrt{2}$	0	$-\sqrt{2}$	-1	$-\sqrt{2}$	0
$\cos x$	1	$\sqrt{2}$	0	$-\sqrt{2}$	-1	$-\sqrt{2}$	0	$\sqrt{2}$	1	$\sqrt{2}$	0	$-\sqrt{2}$	-1	$-\sqrt{2}$	0	$\sqrt{2}$	1
$\sin x - \cos x$	1	$2\sqrt{2}$	1	0	-1	$-2\sqrt{2}$	-1	0	1	$2\sqrt{2}$	1	0	-1	$-2\sqrt{2}$	-1	0	1



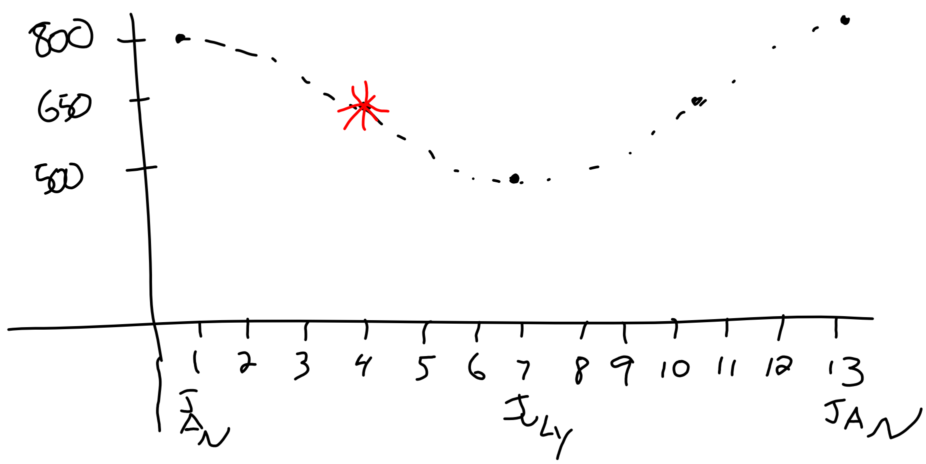
10. The population in a particular herd of antelope in South Africa oscillates between approximately 500 and 800. The maximum number can be found at the beginning of January, while the minimum number of can be found at the beginning of July. Express the population as a function of time in the form $y = A\sin[B(x - C)] + D$, where January is counted as month one ($x = 1$).

500 TO 800 : $\frac{500+800}{2} = 650$ = EQUILIBRIUM = D

FROM 500 TO 650 IS $\boxed{150 = A}$

FROM MAX TO MIN IS 6 MONTHS & IS ½ PERIOD, SO PERIOD IS $2(6) = \boxed{12} = \frac{2\pi}{B}$, SO $B = \frac{\pi}{6}$

HERE IS THE PICTURE:



* IF WE "START" OUR SINE HERE, IT IS RIGHT 4 UP 650 & goes down, so our final equation CAN BE

$$y = -150\sin\left[\frac{\pi}{6}(x-4)\right] + 650$$