Math 1050 Final Exam Form A **College Algebra** **Fall Semester 2010**

Instructor_____ **Name**_____

Student ID_____ **ID Verification**_____ **Section Number** _____

Time Limit: 120 minutes

Graphing calculators without CAS are allowed. All problems are weighted equally.

Computers, cell phones, CAS calculators and other hand-held devices are not allowed. Students may not bring notes or books into the exam.

Allowed reference formulas are attached to the back of the exam.

This exam has three parts

Part I - Ten multiple choice questions - answer all
Part II - Ten open ended questions - answer all
Part III - Ten open ended questions - choose FIVE

INSTRUCTIONS PART I: Questions 1 - 10, Multiple Choice. Answer all TEN questions. Circle the correct answer. It is not necessary to show work. There will be no partial credit awarded on this part of the exam.

1) The function $f(x) = \frac{2}{5x + 3}$ is one-to-one. Find its inverse.

A) $f^{-1}(x) = \frac{2}{5y} - \frac{3}{5}$

B) $f^{-1}(x) = \frac{5x + 3}{2}$

C) $f^{-1}(x) = \frac{2}{5x} - \frac{3}{5}$

D) $f^{-1}(x) = \frac{3}{5} - \frac{2}{5x}$

2) Form a polynomial f(x) with real coefficients having degree 3 and zeros −2 and 3 + i.

A) $f(x) = x^3 - 4x^2 - 10x + 20$
B) $f(x) = x^3 - 8x^2 + 2x + 20$
C) $f(x) = x^3 - 6x^2 - 10x + 20$
D) $f(x) = x^3 - 4x^2 - 2x + 20$

3) Solve the system using the inverse matrix method.

$$\begin{cases} x + 2y + 3z = 6 \\ x + y + z = 2 \\ x - 2z = -10 \end{cases}$$

NOTE: The inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$ is $\begin{bmatrix} -2 & 4 & -1 \\ 3 & -5 & 2 \\ -1 & 2 & -1 \end{bmatrix}$.

A) x = 4, y = −6, z = 8; (4, −6, 8)
B) x = 6, y = 0, z = 0; (6, 0, 0)
C) x = 10, y = 8, z = 0; (10, 8, 0)
D) x = 6, y = −12, z = 8; (6, −12, 8)

4) The number of mosquitoes M(x), in millions, in a certain area depends on the June rainfall x, in inches: $M(x) = 8x - x^2$. What rainfall produces the maximum number of mosquitoes?

A) 64 in.
B) 4 in.
C) 0 in.
D) 8 in.

5) Determine whether the infinite geometric series $1 + \frac{1}{3} + \frac{1}{9} + \cdots$ converges or diverges. If it converges, find its sum.

A) Converges; $\frac{3}{2}$

B) Converges; $\frac{1}{3}$

C) Converges; 1

D) Diverges

6) Perform the indicated row operations on the augmented matrix that is shown below. The matrix obtained from step (a) should be use in step (b). Then, the matrix obtained in step (b) should be used in step (c). After completing the row operation in step (c), circle the resulting matrix.

$$\left[\begin{array}{ccc|c} 1 & -3 & -5 & 2 \\ -2 & -5 & 2 & 5 \\ -2 & -5 & 4 & 6 \end{array}\right]$$

(a) $R_2 = 2r_1 + r_2$

(b) $R_3 = 2r_1 + r_3$

(c) $R_3 = 3r_2 + r_3$

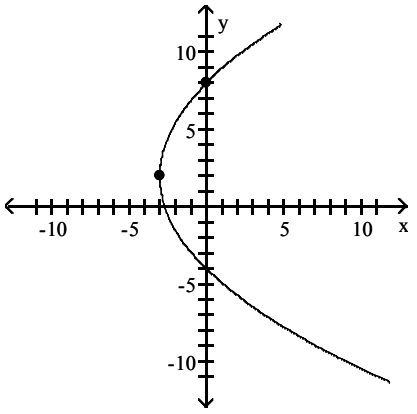
A) $\left[\begin{array}{ccc|c} 1 & -3 & -5 & 2 \\ 0 & -11 & -8 & 9 \\ 0 & -8 & 30 & 37 \end{array}\right]$

B) $\left[\begin{array}{ccc|c} 1 & -3 & -5 & 2 \\ 0 & -11 & -8 & 9 \\ 0 & -32 & -10 & 37 \end{array}\right]$

C) $\left[\begin{array}{ccc|c} 1 & -3 & -5 & 2 \\ 0 & -11 & -8 & 9 \\ 0 & -44 & -30 & 37 \end{array}\right]$

D) $\left[\begin{array}{ccc|c} 1 & -3 & -5 & 2 \\ 0 & -1 & -8 & 9 \\ 0 & -12 & -14 & 19 \end{array}\right]$

7) Write an equation for the parabola that is graphed below. The parabola has vertex (−3, 2) and contains the point (0, 8).



A) $(x + 2)^2 = 12(y - 3)$
B) $(x - 2)^2 = 12(y + 3)$
C) $(y - 3)^2 = 12(x + 2)$
D) $(y - 2)^2 = 12(x + 3)$

8) Find the function that is finally graphed after the following transformations are applied to the graph of $y = |x|$. The graph is shifted right 3 units, then stretched by a factor of 3, then shifted vertically down 2 units, and finally reflected across the x−axis.
Hint: Draw graph and then choose equation.

A) $y = -(3|x - 3| - 2)$
B) $y = 3|-x - 3| - 2$
C) $y = -3|x - 3| - 2$
D) $y = -(3|x + 3| - 2)$

9) Determine whether the function $f(x) = \frac{x}{x^2 - 3}$ is even, odd, neither, or both.

A) even
B) odd
C) neither
D) both

10) Solve the inequality $x^4 < 49x^2$. Express the solution using interval notation.

A) $(-\infty, -7)$ or $(7, \infty)$
B) $(-7, 0)$ or $(7, \infty)$
C) $(-7, 0)$ or $(0, 7)$
D) $(-\infty, -7)$ or $(0, 7)$

INSTRUCTIONS PART II: Questions 11 - 20, Short Response. Answer all TEN questions carefully and completely, showing your work and clearly indicating your answer.

11) Find the composite function $f \circ g$ and the domain of $f \circ g$ for

$f(x) = \frac{2}{x + 9}$ and $g(x) = x + 4$.

composite function $f \circ g$ _____ (simplified)

domain of $f \circ g$ _____ (in set notation)

12) Solve the system of equations using Cramer's Rule. Show your work!

$$\begin{cases} 4x - 7y = 5 \\ 2x + 5y = -3 \end{cases}$$

solution _____

13) The sequence is defined recursively. Write the first four terms.

$a_1 = -9;\ a_n = n - a_{n-1}$

a_1 = _____

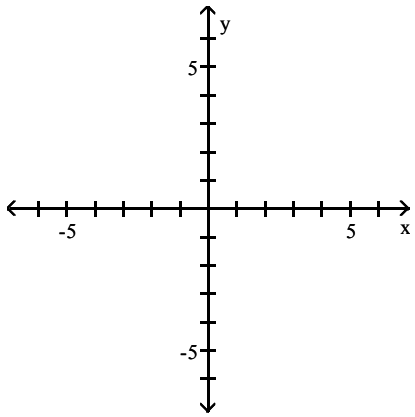
a_2 = _____

a_3 = _____

a_4 = _____

14) Graph the function

$$f(x) = \begin{cases} x - 2 & \text{if } x < 1 \\ 5 & \text{if } x \geq 1 \end{cases}$$



15) A brick staircase has a total of 16 steps The bottom step requires 111 bricks. Each successive step requires 4 less bricks than the prior one. How many bricks are required to build the staircase? *Hint: This is an arithmetic series.*

number of bricks required to build the staircase _____

16) Solve the equation $\log_3 x + \log_3(x - 24) = 4$.

solution(s) _____

17) A hall 130 feet in length was designed as a whispering gallery. If the ceiling is 25 feet high at the center, how far from the center are the foci located? *Recall: A whispering gallery is shaped like an ellipse.*

Along with your work in solving this problem, show your drawing of the whispering gallery below, labelling as necessary.

distance that the foci are from the center _____

18) Find the real solution(s) of the equation $x^{1/2} - 7x^{1/4} + 10 = 0$.

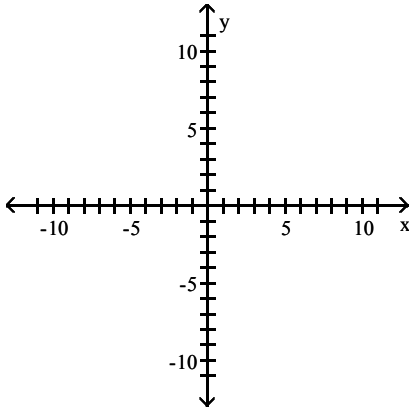
solution(s) _____

19) Find the center and radius of the circle with the equation $x^2 + y^2 - 6x - 4y = -9$. Use the center and radius to graph the circle.

center _____

radius _____

graph

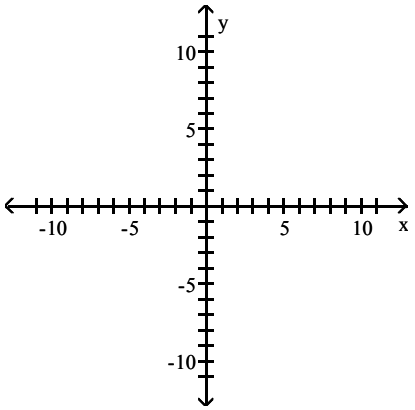


20) Find the domain, asymptotes, and intercepts. Use this information to graph the function $f(x) = \frac{x}{x^2 - 9}$.

domain _____

asymptotes _____

intercepts _____



INSTRUCTIONS PART III: Questions 21 - 30, Short Response. Answer FIVE questions only. Put an X through the 5 problems you do not want graded. If you do not cross out any problems,the first 5 problems that show any work will be the ones that are graded.

21) Randy invested his inheritance in an account that paid 6.6% interest, compounded continuously. After 5 years, he found that he now had $44,821.17. What was the original amount of his inheritance? (Round your answer to the nearest dollar.)

original amount of inheritance _____

22) Write the partial fraction decomposition of the rational expression $\frac{x - 8}{(x - 2)(x - 4)}$.

partial fraction decomposition _____

23) Find the sum of the sequence $\sum_{k=2}^{5} 8k$.

sum of the sequence _____

24) Find all complex zeros of the function $f(x) = x^3 + 7x^2 + 16x + 10$.

complex zeros _____

25) Solve the equation $2^{(7+3x)} = \frac{1}{4}$.

Solution(s) _____

26) Find an equation for the hyperbola described.

Vertices at $(0, \pm 6)$; asymptote the line $y = \frac{3}{2}x$

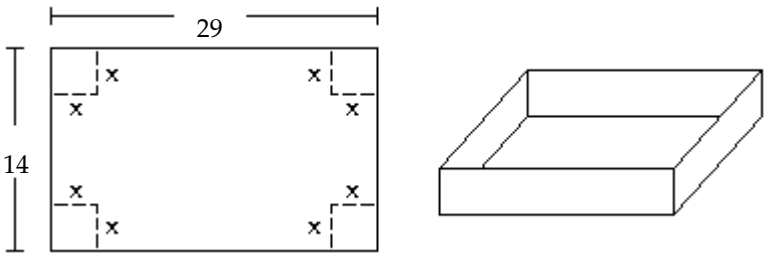
equation _____

27) Find the value of the determinant by hand. Do not use a calculator. Show your work!

$$\begin{vmatrix} 4 & 2 & 4 \\ 2 & 3 & 0 \\ 3 & 2 & -5 \end{vmatrix}$$

determinant _____

28) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 14 inches by 29 inches by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x and state the domain.



volume as a function of x _____

Leave the volume function in factored form.

real world domain _____

29) Analyze the graph of the function $f(x) = -x^2(x - 4)(x + 3)$ as follows:

(a) Determine the end behavior: find the power function that the graph of f resembles for large values of $|x|$.

power function _____

(b) Find the x- and y-intercepts of the graph.

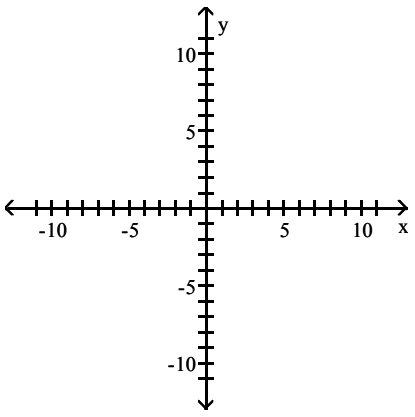
x-intercepts _____

y-intercept _____

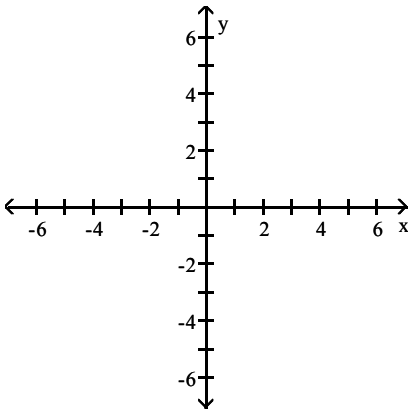
(c) Determine whether the graph crosses or touches the x-axis at each x-intercept.

list x-intercepts below state whether graph crosses or touchees at each

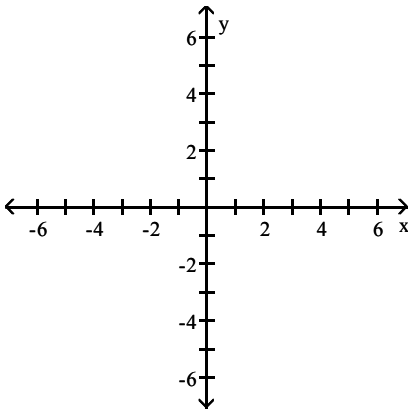
(d) Use the information obtained in (a) – (c) to sketch a graph of f by hand. Label all intercepts .



30) Graph the function $y = 2^x$. Then use transformations to graph $f(x) = 2^{(x+2)} - 1$. Label each graph.



$y = 2^x$



$f(x) = 2^{(x+2)} - 1$

3 points used in graphing $y = 2^x$ _____, _____, _____
(give exact values – do not use a calculator)

domain of $f(x) = 2^{(x+2)} - 1$ _____ *(use interval notation)*

range of $f(x) = 2^{(x+2)} - 1$ _____ *(use interval notation)*

asymptote(s) of $f(x) = 2^{(x+2)} - 1$ _____

The following reference formulas may be used during the Math 1050 final exam. Students are NOT allowed to use any additional books or notes.

$$(x-h)^2+(y-k)^2=r^2$$

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \qquad \frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \qquad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

$$(y-k)^2=4a(x-h) \qquad (y-k)^2=-4a(x-h)$$

$$(x-h)^2=4a(y-k) \qquad (x-h)^2=-4a(y-k)$$

$$A=Pe^{rt} \qquad A=P\left(1+\tfrac{r}{n}\right)^{nt} \qquad A(t)=A_0e^{kt}$$

$$S_n=\frac{n}{2}(a_1+a_n) \qquad S_n=a_1\frac{1-r^n}{1-r}\ ,r\neq 0,1$$