Math 1050 Final Exam College Algebra

Fall Semester 2014

Form A

Instructor________________ Name key________________

Student ID________________ ID Verification________ Section Number ________

Time Limit: 120 minutes

All problems are weighted equally.

No graphing calculators, notes, books, cell phones, or any devices that can connect to the Internet are allowed.

Scientific calculators with no more than a basic numeric store and recall memory are allowed on the final exam. Reference formulas that are allowed are attached to the exam.

Each question is worth 8pts.

This exam has two parts:

- Part I - Ten multiple choice questions - answer all
- Part II - Fifteen open ended questions - answer all

INSTRUCTIONS PART I: Questions 1 - 10, Multiple Choice. Answer all TEN questions. Circle the correct answer. It is not necessary to show work. There will be no partial credit awarded on this part of the exam.

No partial credit for multiple choice questions!

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1) Determine whether the infinite geometric series converges or diverges. If it converges, find its sum.

$$18 + 6 + 2 + \frac{2}{3} + \ldots$$

A) Converges; 26 B) Converges; 27 C) Converges; - 9 D) Diverges

2) This matrix is nonsingular. Find the inverse of the matrix.

$$\begin{bmatrix} 2 & 5 \\ 1 & -5 \end{bmatrix}$$

A) $\begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{15} & \frac{2}{15} \end{bmatrix}$

B) $\begin{bmatrix} \frac{2}{15} & \frac{1}{3} \\ \frac{1}{15} & -\frac{1}{3} \end{bmatrix}$

C) $\begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{15} & \frac{2}{15} \end{bmatrix}$

D) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{15} & -\frac{2}{15} \end{bmatrix}$

3) List the potential rational zeros of the polynomial function.

$f(x) = -2x^3 + 3x^2 - 4x + 8$

A) $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4$

B) $\pm\frac{1}{8}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

C) $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

D) $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

4) Express as a single logarithm.

$2\log_4 x + 5\log_4 (x - 3)$

A) $10 \log_4 x(x - 3)$

B) $\log_4 x(x - 3)^{10}$

C) $\log_4 x(x - 3)$

D) $\log_4 x^2(x - 3)^5$

5) Determine whether the function is even, odd, or neither.

$f(x) = \frac{x}{x^2 + 2}$

A) even

B) odd

C) neither

6) Put the ellipse in standard form and identify the center.

$3x^2 + 5y^2 - 36x + 40y + 173 = 0$

A) $\frac{(x - 6)^2}{5} + \frac{(y + 4)^2}{3} = 1$

center: (6, -4)

B) $\frac{(x - 6)^2}{5} + \frac{(y + 4)^2}{3} = 1$

center: (-6, 4)

C) $\frac{(x + 6)^2}{5} + \frac{(y - 4)^2}{3} = 1$

center: (6, -4)

D) $\frac{(x + 6)^2}{5} + \frac{(y - 4)^2}{3} = 1$

center: (-6, 4)

7) Find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$. The graph is shifted down 5 units, reflected about the x-axis, and finally shifted left 9 units.

A) $y = -\sqrt{x + 9} - 5$ B) $y = -\sqrt{x - 9} + 5$ C) $y = \sqrt{-x - 9} - 5$ D) $y = -\sqrt{x + 9} + 5$

8) For the given functions f and g, find the requested composite function.

$f(x) = 3x + 7$, $g(x) = -2/x$; Find $(g \circ f)(3)$.

A) $\frac{46}{3}$ B) 5 C) $-\frac{1}{8}$ D) $-\frac{32}{3}$

9) The owner of a video store has determined that the cost C, in dollars, of operating the store is approximately given by $C(x) = 2x^2 - 30x + 770$, where x is the number of videos rented daily. Find the lowest cost to the nearest dollar.

A) \$658 B) \$545 C) \$320 D) \$883

10) Solve the inequality.

$x^3 - 4x^2 - 12x > 0$

A) $(-\infty,-2) \cup (0, 6)$ B) $(-2, 0) \cup (6, \infty)$ C) $(-6, 0) \cup (2, \infty)$ D) $(-2, \infty)$

Partial credit should be awarded as shown, but can be broken down further.

INSTRUCTIONS PART II: Questions 11 - 25. Answer all FIFTEEN questions carefully and completely showing all appropriate work and clearly indicating your answer.

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11) Find the amount that results from the investment.

$n=2$

\$1,000 invested at 8% compounded semiannually after a period of 12 years

2pts $\rightarrow$ $A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 1000\left(1+\frac{.08}{2}\right)^{2\cdot 12}$

$P=1000$
$r=.08$
$n=2$
$t=12$

1 point each

$\boxed{A = \$2563.30}$

2pts

12) Find the domain of the function.

$f(x) = \frac{x}{\sqrt{x-6}}$

5pts

$x-6 \neq 0$ denominator
$x-6 \geq 0$ square root

$x-6>0$

3pts

$x>6$ or
$\{x \mid x>6\}$ or
$(6,\infty)$

13) Graph the polynomial. Identify each x-intercept and whether the graph crosses or touches the x-axis at each x intercept.

$f(x) = x^3 - 4x^2 - 12x$

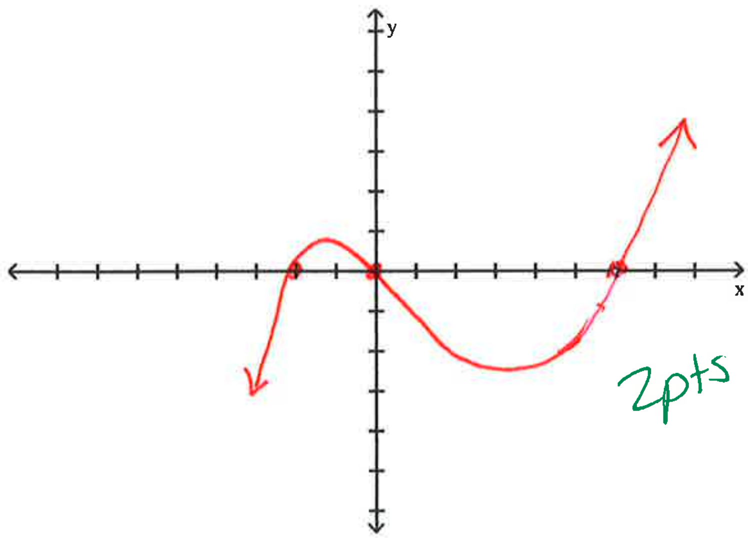
See question #10

$f(x) = x(x-6)(x+2)$

power function x^3 ↗ ↙

6pts (1pt each)

x-intercept	cross/touch
$x=0$	cross
$x=6$	cross
$x=-2$	cross



2pts

14) Find the real solutions of the equation.

$x^{1/2} - 8x^{1/4} + 12 = 0$

let $x^{1/4} = u$

2pts $(x^{1/4})^2 = u^2$

$x^{1/2} = u^2$

Substitute:

$u^2 - 8u + 12 = 0$ 2pts

$(u-6)(u-2) = 0$

$u = 6 \quad u = 2$ 2pts

Unsubstitute:

$(x^{1/4})^4 = (6)^4 \quad (x^{1/4})^4 = (2)^4$

$x = 1296 \quad$ 2pts $\quad x = 16$

15) Expand the expression.

$(4x + 3)^4$

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

3pts

3pts

$1(4x)^4(3)^0 + 4(4x)^3(3)^1 + 6(4x)^2(3)^2$
$+ 4(4x)^1(3)^3$
$+ 1(4x)^0(3)^4$

$= 256x^4 + 768x^3 + 864x^2 + 432x + 81$

2pts

Points may be given for using appropriate formula rather than using Pascal's Triangle for coefficients.

16) Find the twenty-second term of the arithmetic sequence -1, 4, 9, ...

2pts $a_n = a_1 + d(n-1)$

$a_1 = -1$
$d = 5$
$n = 22$

3pts

$a_{22} = -1 + 5(22-1)$
$= -1 + 5(21)$

2pts

$a_{22} = 104$ 1pt

17) Form a polynomial f(x) with real coefficients having the given degree and zeros.

Degree 3: zeros: 1 + i and -6

3pts $x = 1+i \quad x = 1-i \quad x = -6$

3pts $x-1-i=0 \quad x-1+i=0 \quad x+6=0$

$p(x) = (x-1-i)(x-1+i)(x+6)$

$p(x) = (x^2-2x+2)(x+6)$

or 2pts

$p(x) = x^3+4x^2-10x+12$

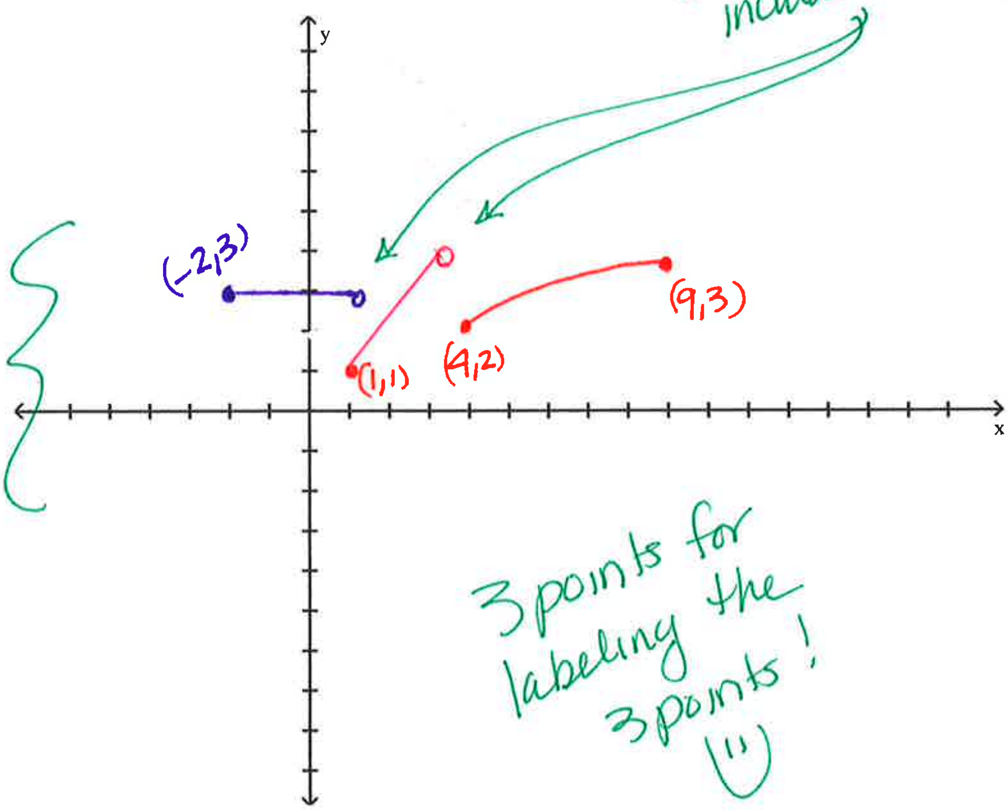
	x	-1	$-i$
x	x^2	$-x$	$\cancel{-ix}$
-1	$-x$	$+1$	$\cancel{+i}$
$+i$	$\cancel{+ix}$	$\cancel{-i}$	$-i^2$

$i^2 = (-1)$

$= -(-1) = 1$

18) Graph the function. Label or list at least three points on the graph.

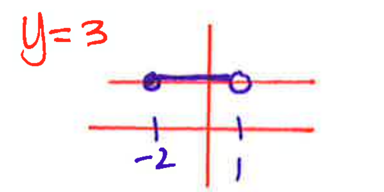
$$f(x) = \begin{cases} 3 & \text{if } -2 \le x < 1 \\ |x| & \text{if } 1 \le x < 4 \\ \sqrt{x} & \text{if } 4 \le x \le 9 \end{cases}$$

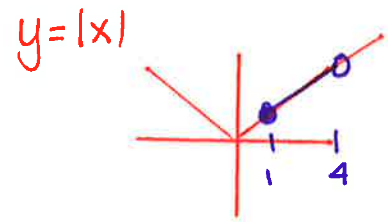


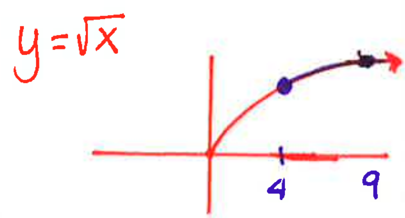
points may vary but they must not include

5 pts for correct graph

3 points for labeling the 3 points! (:







19) Solve the system of equations using Cramer's Rule.

$$\begin{cases} 5x + 6y = 8 \\ 4x + y = -5 \end{cases}$$

2pts $D = \begin{vmatrix} 5 & 6 \\ 4 & 1 \end{vmatrix} = (5)(1) - (4)(6) = 5 - 24 = -19$

2pts $D_x = \begin{vmatrix} 8 & 6 \\ -5 & 1 \end{vmatrix} = (8)(1) - (-5)(6) = 8 + 30 = 38$

2pts $D_y = \begin{vmatrix} 5 & 8 \\ 4 & -5 \end{vmatrix} = (5)(-5) - (4)(8) = -25 - 32 = -57$

1pt $x = \dfrac{D_x}{D} = \dfrac{38}{-19} = -2$

1pt $y = \dfrac{D_y}{D} = \dfrac{-57}{-19} = 3$

$(-2, 3)$

20) Write the partial fraction decomposition of the rational expression.

$$\frac{5x - 22}{(x + 4)(x - 3)}$$

$\dfrac{5x-22}{(x+4)(x-3)} = \dfrac{A}{(x+4)} + \dfrac{B}{(x-3)}$ 2pts

$5x-22 = A(x-3) + B(x+4)$ 2pts

let $x=3$: $5(3)-22 = \cancel{A(3-3)} + B(3+4)$

$-7 = 7B \qquad B=-1$ 2pts

let $x=-4$: $5(-4)-22 = A(-4-3) + \cancel{B(-4+4)}$

$-42 = -7A \qquad A=6$

2pts

$$\frac{5x-22}{(x+4)(x-3)} = \frac{6}{x+4} - \frac{1}{x-3}$$

21) Graph the function. Determine the domain, x and y intercepts, any vertical, horizontal, and/or oblique asymptotes. Label or list at least three points on the graph.

$f(x) = \frac{x^2 + 4x - 5}{(x-4)^2} = \frac{(x+5)(x-1)}{(x-4)^2}$

Domain: $x-4 \neq 0 \quad x \neq 4$ 1pt

$y=0 \rightarrow$ x - intercepts: $x = -5, \; x = 1 \rightarrow (-5,0)$ & $(1,0)$ 1pt

$x=0 \rightarrow$ y - intercept: $y = -5/16 \quad (0, -5/16)$ 1pt

Vertical Asymptote(s): $x = 4$

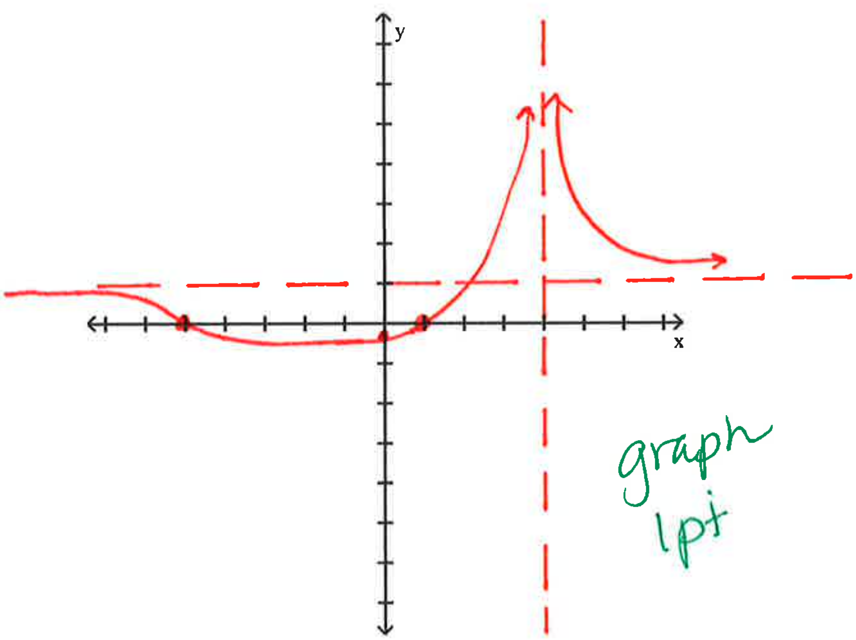
Horizontal Asymptote(s): $y = 1$ 1pt

Oblique Asymptote(s): none

Points may vary

3pts

x	y
−10	0.2806
−5	0
0	−0.3125
1	0
3	16
5	40
10	3.75



graph 1pt

22) Solve the equation.

$\log_3 (x + 5) + \log_3 (x - 1) = 3$

2pts $\rightarrow$ $\log_3 [(x+5)(x-1)] = 3$

3 $\leftarrow$ 2pts $\rightarrow$ 3

2pts $\begin{cases} (x+5)(x-1) = 27 \\ x^2 + 4x - 5 = 27 \\ \quad -27 \quad -27 \end{cases}$

$x^2 + 4x - 32 = 0$ $\leftarrow$ 2pts

$(x+8)(x-4) = 0$

$\cancel{x = -8}$ $\quad x = 4$

23) The size P(t) of a small herbivore population at time t (in years) obeys the function $P(t) = 700e^{0.12t}$ if they have enough food and the predator population stays constant. After how many years will the population reach 2800? Round your answer to the nearest year.

2pts $\longrightarrow$ $\dfrac{2800}{700} = \dfrac{700e^{0.12t}}{700}$

2pts $\quad 4 = e^{0.12t}$

$\ln 4 = \ln e^{0.12t}$

2pts $\quad \dfrac{\ln 4}{0.12} = \dfrac{\cancel{0.12} t \cancel{\ln e}^{1}}{\cancel{0.12}}$

$t \approx 11.5525$ years

$t = 12$ years

2pts

24) Graph the function. Determine the domain, range, and horizontal asymptote of the function. Label or list at least three points on the graph.

points may vary

x	y
-1	5
0	4
1	3.5

3pts

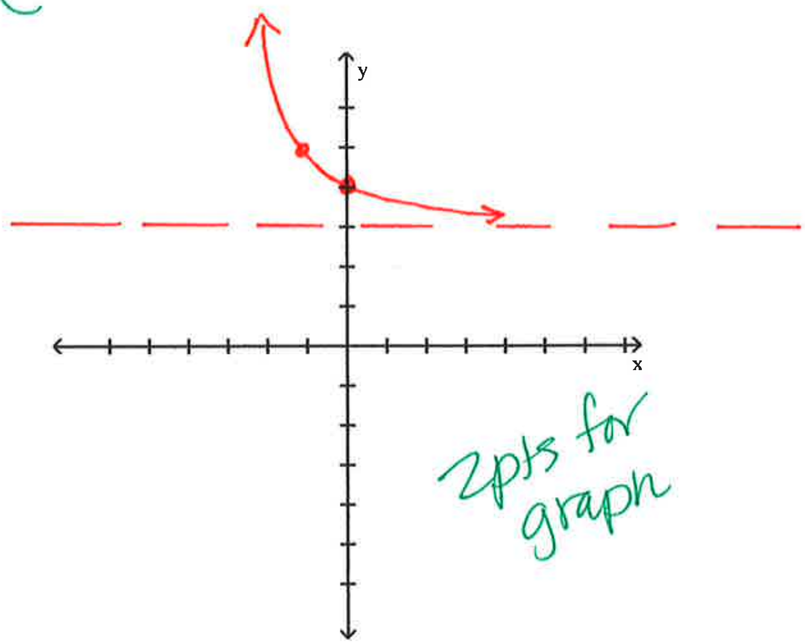
$f(x) = 2^{-x} + 3$

3pts

Domain: all real numbers

Range: $y > 3$

Horizontal Asymptote(s): $y = 3$



2pts for graph

25) A brick staircase has a total of 13 steps. The bottom step requires 105 bricks. Each successive step requires 4 less bricks than the prior one. How many bricks are required to build the staircase?

3pts

$n = 13$

$a_1 = 105$

$d = -4$

$a_n = a_1 + d(n-1)$ 1pt

$a_{13} = 105 + (-4)(13-1)$

$= 105 - 4(12)$

$= 57$ bricks in top step

1pt

1pt $S_n = \frac{n}{2}(a_1 + a_n)$

1pt $S_{13} = \frac{13}{2}(105 + 57) = \frac{13}{2}(162)$

$= 1053$ bricks

1pt