Math 1050 Final Exam College Algebra Fall Semester 2011 Form A KEY

Find the domain of the function.

1) $\frac{x}{\sqrt{x-9}}$

A) $\{x \mid x \neq 0, 9\}$ B) $\{x \mid x \neq 9\}$ C) $\{x \mid x \geq 9\}$ [✓ D) $\{x \mid x > 9\}$]   E) $\{x \mid x \neq 0\}$

$$\left.\begin{array}{l} \text{DENOMINATOR; } \sqrt{x-9} \neq 0 \Rightarrow x-9 \neq 0 \Rightarrow x \neq 9 \\ \text{EVEN ROOT; } x-9 \geq 0 \Rightarrow x \geq 9 \end{array}\right\} x > 9$$

Find the center (h, k) and radius r of the circle with the given equation.

2) $x^2 + y^2 - 4x + 14y = 28$

A) $(h, k) = (7, -2);\ r = 81$ [✓ B) $(h, k) = (2, -7);\ r = 9$]

C) $(h, k) = (-7, 2);\ r = 9$ D) $(h, k) = (-2, 7);\ r = 81$

$$x^2 - 4x + 4 + y^2 + 14y + 49 = 28 + 4 + 49$$

½   ½

$$(x-2)^2 + (y+7)^2 = 81$$

$x-2=0$ $y+7=0$ $\sqrt{81}$

$$(h,k) = (2,-7),\ r = 9$$

Write the equation of a function that has the given characteristics.

3) The graph of $f(x) = x^3$, that is first shifted 6 units upward, shifted left 3 units, then reflected about the x axis.

A) $y = -(x-3)^3+6$

B) $y = -(x+6)^3-3$

C) $y = -(x+6)^3+3$

D) $y = -(x+3)^3-6$

[✓ E) $y = -(x+3)^3+6$]

$$x^3 \xrightarrow{\text{up } 6} x^3 + 6 \xrightarrow{\text{Left } 3} (x+3)^3 - 6$$

$$\xrightarrow{\text{X REF}} -\left[(x+3)^3 - 6\right] \Rightarrow$$

$$y = -(x+3)^3 + 6$$

Solve the equation.

4) $\log_2 (x + 4) + \log_2 (x - 2) = 4$

[✓ A) {4}]   B) {-6}   C) {4, -6}   D) {5}

$$2^{\log_2 (x+4)(x-2)} = 2^{4}$$

$$\Rightarrow (x+4)(x-2) = 16$$

FOIL

$$\Rightarrow x^2+2x-8 = 16$$

$$-16 \qquad -16$$

$$\Rightarrow x^2+2x-24=0$$

$$(x+6)(x-4)=0$$

$$x=-6 \quad , \boxed{x=4}$$

check

$\log_2(-6+4)$

$\downarrow$

$\log_2(-2)$ $\emptyset$

check

$\log_2(4+4)+\log_2(4-2)=4$

$\log_2 8 + \log_2 2 = 4$

$3 + 1 = 4$ ✓

Solve the inequality. Express the solution using interval notation.

5) $(x - 3)^2(x + 4) > 0$

A) $(-4, \infty)$

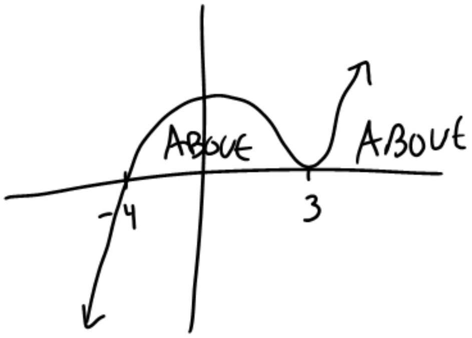
B) $(-\infty, -4)$

C) $(-\infty, -4]$

[✓ D) $(-4, 3) \cup (3, \infty)$]

E) $(-\infty, 3]$

>0 IS ABOVE X-AXIS :



$(-4, 3) \cup (3, \infty)$

For the given functions f and g, find the requested composite function value.

6) $f(x) = \sqrt{x + 3}$, $g(x) = 3x$; Find $(f \circ g)(2)$.

A) $3\sqrt{5}$ B) $\sqrt{15}$ [✓ C) 3]   D) $3\sqrt{15}$

$$(f \circ g)(2) = f(g(2)) = f(6) = \sqrt{6+3} = \sqrt{9}$$

$$g(2) = 3(2) = 6$$

$$= 3$$

Write as the sum and/or difference of logarithms. Express powers as factors.

7) $\ln\left(\frac{w^3(x-5))}{\sqrt{y}}\right)$, $y > 0$

A) $\ln w^3 + \ln(x-5) - \ln\sqrt{y}$

[✓ B) $3\ln w + \ln(x-5) - \frac{1}{2}\ln y$]

C) $\left(\frac{\ln w^3 + \ln(x-5))}{\ln\sqrt{y}}\right)$

D) $3\ln w + \ln x - \ln 5 - \frac{1}{2}\ln y$

E) $3\ln w + \ln(x-5) - 2\ln y$

$\Rightarrow \ln w^3 + \ln(x-5) - \ln\sqrt{y}$

$\Rightarrow \ln w^3 + \ln(x-5) - \ln y^{1/2}$

$\Rightarrow 3\ln w + \ln(x-5) - \frac{1}{2}\ln y$

Use the given matrices to compute the given expression, where I_2 is the 2x2 identity matrix.

8) If $A = \begin{bmatrix} 2 & -1 \\ 7 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -3 \\ 4 & 7 \end{bmatrix}$, find $-2A + 4B + 2I_2$

[✓ A) $\begin{bmatrix} 18 & -10 \\ 2 & 12 \end{bmatrix}$]

B) $\begin{bmatrix} 19 & -8 \\ 4 & 13 \end{bmatrix}$

C) $\begin{bmatrix} 7 & 4 \\ 11 & 13 \end{bmatrix}$

D) $\begin{bmatrix} -3 & -6 \\ 3 & 5 \end{bmatrix}$

$-2A + 4B + 2I_2 \Rightarrow -2\begin{bmatrix} 2 & -1 \\ 7 & 9 \end{bmatrix} + 4\begin{bmatrix} 5 & -3 \\ 4 & 7 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -4 & 2 \\ -14 & -18 \end{bmatrix} + \begin{bmatrix} 20 & -12 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} -4+20+2 & 2-12+0 \\ -14+16+0 & -18+28+2 \end{bmatrix} \Rightarrow \begin{bmatrix} 18 & -10 \\ 2 & 12 \end{bmatrix}$

List the potential rational zeros of the polynomial function. Do not find the zeros.

9) $f(x) = 6x^4 + 3x^3 - 2x^2 + 2$

A) $\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm 1, \pm 2$

[✓ B) $\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{2}{3}, \pm 1, \pm 2$]

C) $\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{2}{3}, \pm 1, \pm 2, \pm 3$

D) $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm 1, \pm 2, \pm 3, \pm 6$

$\frac{\text{FACTORS OF } 2}{\text{FACTORS OF } 6} \Rightarrow \frac{1, 2}{1, 2, 3, 6} \Rightarrow \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{2}{1}, \cancel{\frac{2}{2}}, \frac{2}{3}, \cancel{\frac{2}{6}}$ ALL $\pm$

EXTRA COPIES

Find the real solutions of the equation.

10) $x^{2/3} - 7x^{1/3} + 10 = 0$

A) {-125, -8}

B) {-5, -2}

C) {2, 5}

[✓ D) {8, 125}]

Let $u = x^{1/3}$

$u^2 - 7u + 10 = 0$

$(u-5)(u-2) = 0$

$u = 5,\ u = 2$

$(x^{1/3})^3 = 5^3 \Rightarrow x = 125$

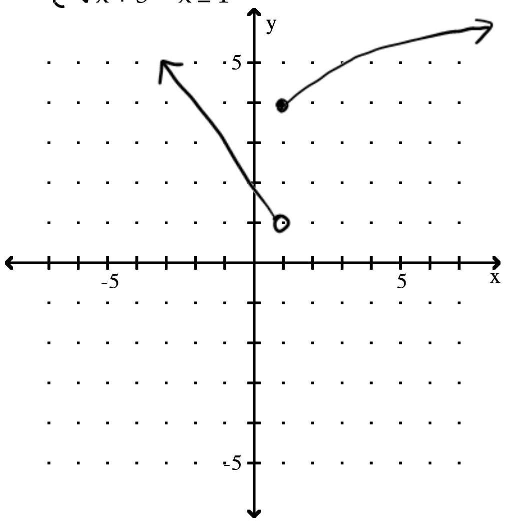
$(x^{1/3})^3 = 2^3 \Rightarrow x = 8$

INSTRUCTIONS PART II: Questions 11 – 20, Short Response. Answer all TEN questions carefully and completely, showing your work and clearly indicating your answer.

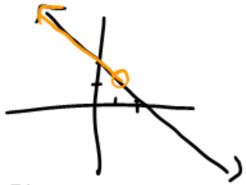
Graph the function.

11)

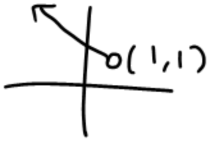
$$f(x) = \begin{cases} -x + 2 & x < 1 \\ \sqrt{x} + 3 & x \geq 1 \end{cases}$$



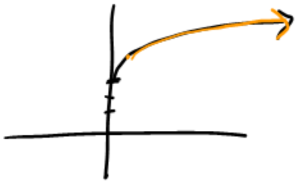
$y = -x+2 \Rightarrow$



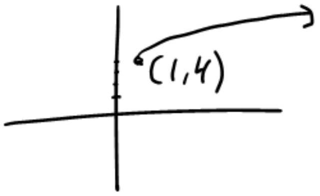
$x<1$;



$y = \sqrt{x} + 3 \Rightarrow$



$x \geq 1$



For the given functions f and g,

12) $f(x) = \frac{x+2}{x-1}$; $g(x) = 3 - x$

A) Find $\left(\frac{f}{g}\right)(x)$ $= f \cdot \frac{1}{g} = \frac{x+2}{x-1} \cdot \frac{1}{3-x} = \frac{x+2}{(x-1)(3-x)}$

B) State the domain of $\left(\frac{f}{g}\right)(x)$ $\{x \mid x \neq 1, 3\}$

$\left(\frac{f}{g}\right)$**(x)** $= \frac{x+2}{(x-1)(3-x)}$

Domain: $\{x \mid x \neq 1, 3\}$

Solve the problem.

13) The price p (in dollars) and the quantity x sold of a certain product obey the demand equation:

$$p = -\frac{1}{7}x + 200, \{x \mid 0 \le x \le 2000\}$$

What is the maximum revenue to the nearest dollar, where the revenue is the number of items sold times the price.

$R = xp = x(-\frac{1}{7}x + 200) \Rightarrow R(x) = -\frac{1}{7}x^2 + 200x.$

MAX @ vertex so $x = \frac{-b}{2a} = \frac{-200}{2(-1/7)} = 700$ items.

MAX REVENUE is $R(700) = -\frac{1}{7}(700)^2 + 200(700) =$

$= -70000 + 140000$

$= \boxed{\$\,70,000}$

Find the inverse function of f. State the domain and range of f.

14) $f(x) = \frac{2x - 5}{x + 3}$

$y = \frac{2x-5}{x+3} \Rightarrow x(y+3) = \frac{2y-5}{\cancel{y+3}}\cancel{(y+3)}$

$f^{-1}(x) = \frac{-3x-5}{x-2}$

Domain of f: $x \neq -3$

Range of f: $y \neq 2$

Domain of $f^{-1}(x)$: $x \neq 2$

Range of $f^{-1}(x)$: $y \neq -3$

$\Rightarrow x(y+3) = 2y - 5$

$\Rightarrow xy + 3x = 2y - 5$

$\quad -2y - 3x \quad -2y - 3x$

$\Rightarrow xy - 2y = -3x - 5$

$\Rightarrow \frac{y(x-2)}{x-2} = \frac{-3x-5}{x-2}$

$\Rightarrow y = \frac{-3x-5}{x-2}$ INVERSE

Determine algebraically whether the function is even, odd, or neither. You <u>must</u> show your work algebraically to receive full credit.

15) $f(x) = \frac{x}{x^2 - 5}$

$f(-x) = \frac{-x}{(-x)^2-5} = \frac{-x}{x^2-5}$

$-f(x) = -\left(\frac{x}{x^2-5}\right) = \frac{-x}{x^2-5}$

so $\boxed{\begin{array}{l} f(-x) = -f(x) \\ \text{\& } f \text{ is ODD} \end{array}}$

Graph the function. Identify the domain, label any intercepts and equations of any asymptotes.

16) $f(x) = \frac{3x}{(x - 4)(x + 2)}$

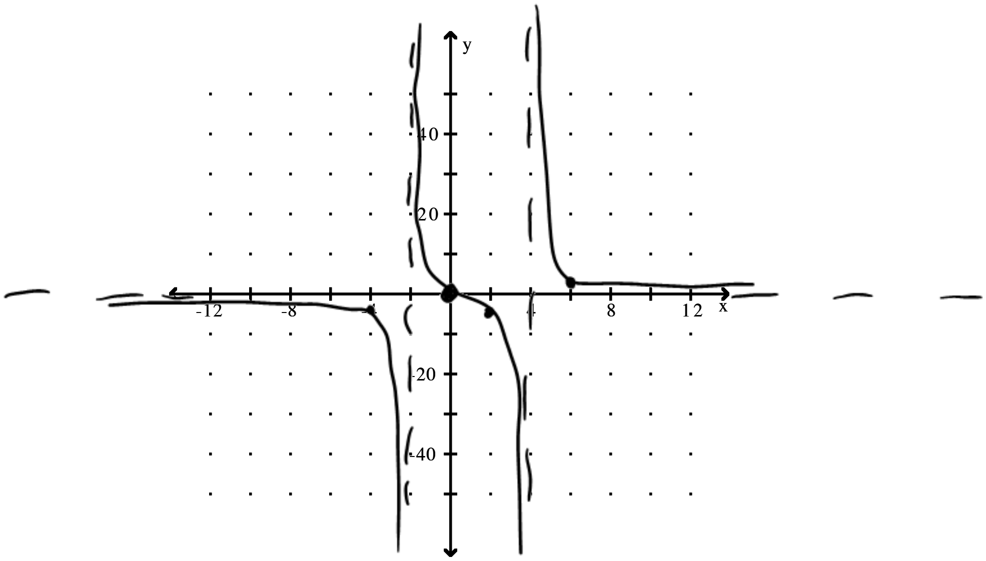
Domain: $\{x \mid x \neq -2, 4\}$

Equation(s) of Asymptote(s):

$\underset{VA}{x=-2}, \underset{VA}{x=4}, \underset{HA}{y=0}$

Intercepts: $(0,0)$

X	Y
-4	-3/4
2	-3/4
6	9/8



Find the present value, that is, the principal needed. Round to the nearest cent.

17) To get $10,000 after 3 years at 6% compounded monthly

$$A=\overset{\downarrow}{P}\left(1+\frac{r}{12}\right)^{(12t)}$$

$$10000=P\left(1+\frac{.06}{12}\right)^{(12\cdot 3)}$$

$$10000=P(1.005)^{36}$$

$$\frac{10000}{1.196681}=\frac{P(1.196681)}{1.196681}$$

$$\boxed{\$\,8356.45=P}$$

This matrix is nonsingular. Find the inverse of the matrix. You <u>must</u> show your work for full credit.

18)

$$\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$$

$$\begin{array}{r} 3 \\ -4 \end{array}\left[\begin{array}{cc|cc} 4 & 3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right] \qquad \begin{array}{rrrrr} & 12 & 9 & 3 & 0 \\ + & -12 & -8 & 0 & -4 \\ \hline & 0 & 1 & 3 & -4 \end{array}$$

$$\begin{array}{r} \\ -3 \end{array}\left[\begin{array}{cc|cc} 4 & 3 & 1 & 0 \\ 0 & 1 & 3 & -4 \end{array}\right] \leftarrow \qquad \begin{array}{rrrrr} & 0 & -3 & -9 & 12 \\ + & 4 & 3 & 1 & 0 \\ \hline & \frac{4}{4} & \frac{0}{4} & \frac{-8}{4} & \frac{12}{4} \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & -4 \end{array}\right] \leftarrow \; 1 \quad 0 \quad -2 \quad 3$$

$$\boxed{A^{-1}=\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}}$$

Graph the function. Label any intercepts and equations of any asymptotes. State the domain and the range of the function.

19) $f(x) = 3^{(x - 1)} + 1$

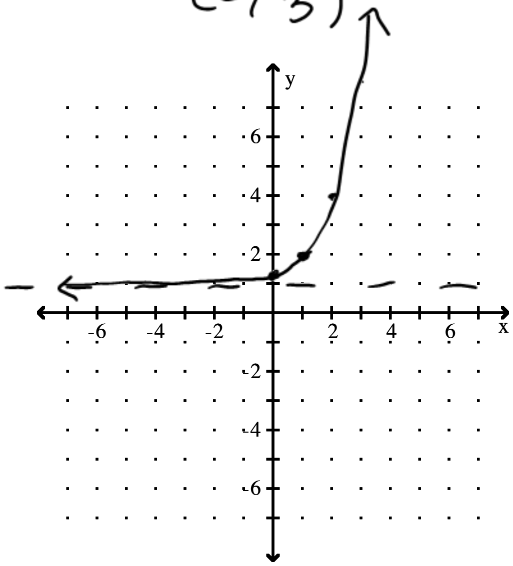
Domain: $(-\infty, \infty)$

Range: $(1, \infty)$

Equation(s) of Asymptote(s): $y = 1$ constant term

Intercept(s): $(0, \frac{4}{3})$

x	y
1	2



Find the first term, the common difference, and give a recursive formula for the arithmetic sequence.

20) 6th term is 31; 14th term is –49

$a_6 = a_1 + 5d \Rightarrow 31 = a_1 + 5d$

$a_{14} = a_1 + 13d \Rightarrow -49 = a_1 + 13d$

$$\Rightarrow \begin{cases} a_1 + 5d = 31 \\ a_1 + 13d = -49 \end{cases} \Rightarrow$$

$-a_1 - 5d = -31$

$+a_1 + 13d = -49$

$\frac{8d}{8} = \frac{-80}{8}$

$\boxed{d = -10}$

$a_1 + 5d = 31 \Rightarrow a_1 - 50 = 31$

$\boxed{a_1 = 81}$

$\boxed{a_1 = 81, \; a_n = a_{n-1} - 10}$

INSTRUCTIONS PART III: Questions 21 – 30, Short Response. Answer FIVE questions only. Put an X through the 5 problems you do not want graded. If you do not cross out any problems, the first 5 problems that show any work will be the ones that are graded.

Determine whether the infinite geometric series converges or diverges. If it converges, find its sum.

21) $\sum_{k=1}^{\infty} 3\left(\frac{2}{5}\right)^{k-1}$

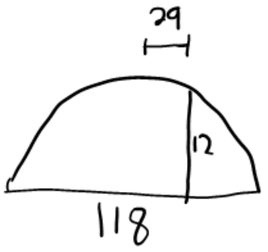
$r = \frac{2}{5}$, so $|r| < 1$ & $\boxed{\text{series converges.}}$

$a_1 = 3\left(\frac{2}{5}\right)^{1-1} = 3$

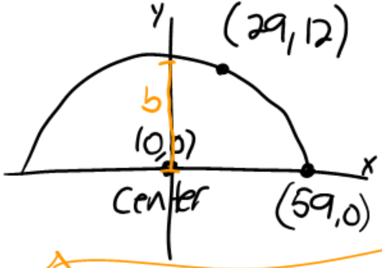
& $S_\infty = \frac{a_1}{1-r} = \frac{3}{1-\frac{2}{5}} = \frac{3}{\frac{3}{5}} = 3 \cdot \frac{5}{3} = \boxed{5}$

Solve the problem.

22) A bridge is built in the shape of a semielliptical arch. It has a span of 118 feet. The height of the arch 29 feet from the center is to be 12 feet. Find the height of the arch at its center. Round to the nearest tenth.



$\Rightarrow$



$\Rightarrow$ a = 59, h = 0, k = 0
x = 29, y = 12

Δ

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \Rightarrow \frac{(29-0)^2}{59^2} + \frac{(12-0)^2}{b^2} = 1 \Rightarrow \overset{3481b^2}{\frac{841}{\cancel{3481}}} + \overset{3481b^2}{\frac{144}{\cancel{b^2}}} = 1^{\,3481b^2}$$

$$\Rightarrow \underset{-841b^2}{841b^2} + 501264 = \underset{-841b^2}{3481b^2} \Rightarrow \frac{501264}{2640} = \frac{\cancel{2640}\,b^2}{\cancel{2640}}$$

$$\Rightarrow \sqrt{b^2} = \sqrt{\frac{501264}{2640}} \Rightarrow b \approx \boxed{13.78 \text{ ft high @ center}}$$

Given that a real zero to the function below is x = 1, find all the real zeros of the polynomial function.

23) $f(x) = x^3 - 5x^2 + 5x - 1$

$$\begin{array}{r|rrrr} & 1 & -5 & 5 & -1 \\ & & 1 & -4 & 1 \\ \hline 1 & \underbrace{1 \quad -4 \quad 1} & & & 0 \end{array}$$

$$x^2 - 4x + 1 = 0 \Rightarrow b^2 - 4ac = 16 - 4 = 12 = 4 \cdot 3$$

$$\boxed{x = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}, \ x = 1}$$

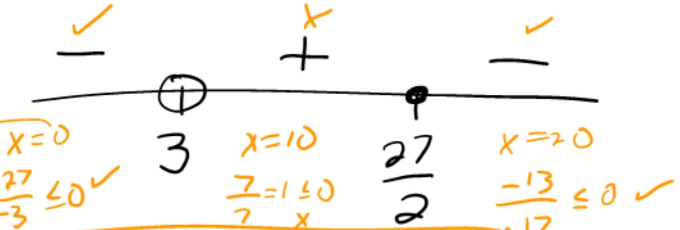
Solve the inequality. Express the solution using interval notation.

24) $\frac{x + 18}{x + 3} \leq 3$

$$\frac{x+18}{x+3} - 3 \leq 0 \Rightarrow \frac{x+18}{x-3} - 3\frac{(x-3)}{(x-3)} \leq 0 \Rightarrow \frac{x+18-3(x-3)}{x-3} \leq 0$$

LCD

$$\Rightarrow \frac{-2x+27}{x-3} \leq 0 \Rightarrow$$



$$\leq 0 \Rightarrow \boxed{(-\infty, 3) \cup [\frac{27}{2}, \infty)}$$

Solve the problem.

25) The half–life of plutonium–234 is 9 hours. If 90 milligrams is present now, how much will be present in 15 hours? (Round your answer to four decimal places.)

$$\tfrac{1}{2}\text{ life } 9\text{hrs} \Rightarrow A = A_0 e^{kt} \Rightarrow \frac{\frac{1}{2}A_0}{A_0} = \frac{A_0 e^{9k}}{A_0}$$

$$\Rightarrow \ln\left(\tfrac{1}{2}\right) = \ln e^{9k} \Rightarrow \frac{\ln(\frac{1}{2})}{9} = \frac{9k}{9} \Rightarrow k = \frac{\ln(\frac{1}{2})}{9} \approx -.0770$$

$$A_0 = 90 \Rightarrow A = 90e^{(-.0770\cdot 15)} \Rightarrow A \approx 28.3482 \text{ mg}$$

Solve. Leave your answer exact.

26) $2^{(x+1)}=5^x$

$$\Rightarrow \ln 2^{(x+1)} = \ln 5^x \Rightarrow (x+1)\ln 2 = x\ln 5$$

$$\Rightarrow \cancel{x\ln 2} + \ln 2 = x\ln 5 \quad \underset{-x\ln 2}{} \qquad \underset{-x\ln 2}{} \Rightarrow \ln 2 = x\ln 5 - x\ln 2$$

$$\Rightarrow \frac{\ln 2}{\ln 5 - \ln 2} = \frac{x(\ln 5 - \ln 2)}{\ln 5 - \ln 2} \Rightarrow \boxed{x = \frac{\ln 2}{\ln 5 - \ln 2}}$$

Find the center, transverse axis, vertices, foci, and any equations of asymptotes of the hyperbola.

27) $x^2 - 25y^2 - 8x - 50y - 34 = 0$

Center: $(4,-1)$

Vertices: $(9,-1),(-1,-1)$

Foci: $(4\pm\sqrt{26},-1)$

Equation(s) of Asymptote(s):

$$\boxed{y+1=\pm\frac{1}{5}(x-4)}$$

$x^2-8x \qquad -25y^2-50y \qquad =34$

$x^2-8x+16 \qquad -25(y^2+2y+1 \qquad)=34+16-25(1)$

$\frac{1}{2} \qquad \frac{1}{2}$

$\frac{(x-4)^2}{25} \qquad \frac{-25(y+1)^2}{25} \qquad =\frac{25}{25}$

$$\boxed{\frac{(x-4)^2}{25}-\frac{(y+1)^2}{1}=1}$$

$\uparrow \qquad \uparrow$

LR 5 $\qquad$ UD 1 , cen $(4,-1)$

$a=5 \qquad b=1$

$c^2=a^2+b^2=25+1$

$c=\sqrt{26}$ LR

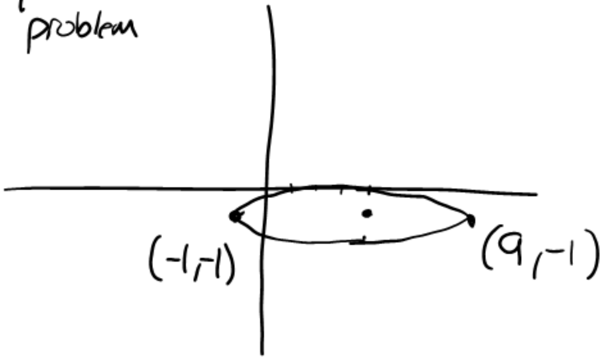
$\downarrow$

$(4\pm\sqrt{26},-1)$

FOCI

Asymptotes ← if b under y like this problem

$y-k=\pm\frac{b}{a}(x-h)$



Solve the system of equations using Cramer's Rule if it is applicable. If Cramer's Rule is not applicable, say so. You **<u>must</u> show your work to recieve full credit.**

28)

$$\begin{cases}4x - 7y = 5\\ 2x + 5y = -3\end{cases}$$

$$D=\begin{vmatrix}4 & -7\\ 2 & 5\end{vmatrix}=4\cdot5-2\cdot(-7)=20+14=34$$

$$D_x=\begin{vmatrix}5 & -7\\ -3 & 5\end{vmatrix}=5\cdot5-(-3)(-7)=25-21=4$$

$$D_y=\begin{vmatrix}4 & 5\\ 2 & -3\end{vmatrix}=4(-3)-2\cdot5=-12-10=-22$$

$$\boxed{x=\frac{D_x}{D}=\frac{4}{34}=\frac{2}{17}}\ ,\ \boxed{y=\frac{D_y}{D}=\frac{-22}{34}=\frac{-11}{17}}$$

Write the partial fraction decomposition of the rational expression.

29) $\frac{45 - 9x}{x^3 - 6x^2 + 9x} = \frac{45-9x}{x(x^2-6x+9)} = \frac{45-9x}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$$\Rightarrow \quad 45 - 9x = A(x-3)^2 + Bx(x-3) + Cx$$

Let $x = 0$; $\frac{45}{9} = \frac{9A}{9} \Rightarrow \boxed{A = 5}$

$x = 3$ $\quad \frac{45-27}{3} = \frac{3C}{3} \Rightarrow \boxed{C = 6}$

compare x^2 coefficents : $0 = A + B \Rightarrow B = -A \Rightarrow \boxed{B = -5}$

$$\boxed{\frac{5}{x} + \frac{-5}{x-3} + \frac{6}{(x-3)^2}}$$

Form a polynomial f(x) with real coefficients having the given degree and zeros.

30) Degree: 3; zeros: –3 and 4– 2i $, 4+2i$

$x = -3 \Rightarrow x + 3 = 0$

$x = 4 - 2i \Rightarrow x - 4 + 2i = 0 \quad \Rightarrow (x+3)(x-4+2i)(x-4-2i) = 0$

$x = 4 + 2i \Rightarrow x - 4 - 2i = 0$

$$\Rightarrow (x+3)(x^2 - 8x + 20) = 0$$

$$\Rightarrow x^3 - 5x^2 - 4x + 60 = 0$$

$$\Rightarrow \boxed{f(x) = x^3 - 5x^2 - 4x + 60}$$