

# Math 1010 Final Exam

Form A, Fall 2011

Name: KEY

Instructor: \_\_\_\_\_ S#: \_\_\_\_\_

Each problem is equally weighted. Scientific calculators are permitted.

Time Limit: Two hours

Not allowed: notes, books, graphing/programmable calculators, cell phones, handhelds, headphones.

1. Determine the system of equations that models the following situation:

"The sum of two numbers is 42. The first number is 6 less than the second number. Find the two numbers."

(a)  $\begin{cases} x + y = 42 \\ x = y - 6 \end{cases}$   $\rightarrow x + y = 42 \rightarrow x = y - 6$

(b)  $\begin{cases} x + y = 42 \\ x = 6 - y \end{cases}$

(c)  $\begin{cases} xy = 42 \\ x + y = 6 \end{cases}$

(d)  $\begin{cases} x + y = 42 \\ x - y = 6 \end{cases}$

(e) Impossible to determine from the information given

2. Solve  $9m - 15 - 2m = 6m - 1 - m$

(a)  $m = 7$

(b)  $m = 0$

(c)  $m = 8$

(d)  $m = 14$

(e) No solution

$$9m - 15 - 2m = 6m - 1 - m$$

$$7m - 15 = 5m - 1$$

$$2m = 14$$

$$m = 7$$

3. Solve the following system of linear equations. What is the x-coordinate of the solution?

$$\begin{array}{r} 2x + y = 6 \\ x - y = 3 \\ \hline 3x = 9 \\ \hline x = 3 \end{array}$$

(a)  $x = 0$

(b)  $x = 6$

(c)  $x = -6$

(d)  $x = 3$

(e) No solution.

4. Solve for  $F$  in the following equation:  $C = \frac{5}{9}(F - 32)$

(a)  $F = \frac{9}{5}C - 32$

(b)  $F = \frac{5}{9}(C - 32)$

(c)  $F = \frac{9}{5}(C + 32)$

(d)  $F = \frac{9}{5}C + 32$

(e)  $F = \frac{5}{9}C + 32$

$$\frac{9}{5} \cdot C = \frac{5}{9} (F - 32) \cdot \frac{9}{5}$$

$$\frac{9}{5}C = F - 32$$

$$\frac{9}{5}C + 32 = F$$

5. How much 5% acid solution and how much 10% acid solution must be mixed to make 2.5 liters of a 7% acid solution?

(a) 1 liter of the 5% solution

1.5 liters of the 10% solution

(b) 1.5 liters of the 5% solution

1 liter of the 10% solution

(c) 2 liters of the 5% solution

0.5 liters of the 10% solution

(d) 0.5 liter of the 5% solution

2 liters of the 10% solution

(e) Impossible to determine from the information given

$x = \text{Liters of } 5\%$ ,  $y = \text{Liters of } 10\%$

$$\begin{cases} x + y = 2.5 \Rightarrow y = 2.5 - x \\ -0.05x + 0.10y = 0.07(2.5) \end{cases}$$

$$-0.05x + 0.10(2.5 - x) = 0.175$$

$$-0.05x + 0.25 - 0.10x = 0.175$$

$$-0.05x = -0.075$$

$$x = 1.5 \text{ Liters of } 5\%$$

$$y = 2.5 - x = 2.5 - 1.5 = 1.0 \text{ Liters of } 10\%$$

6. Let  $f(x) = x^2 + x - 5$ . Find  $f(-3)$ .

(a) 1

(b) -17

(c) 7

(d) 25

(e) 26

$$f(\quad) = (\quad)^2 + (\quad) - 5$$

$$f(-3) = (-3)^2 + (-3) - 5$$

$$f(-3) = 9 - 3 - 5 = \boxed{1}$$

7. Find the distance between the points  $(\frac{1}{3}, \frac{1}{2})$  and  $(\frac{4}{3}, -\frac{1}{2})$ .

(a)  $3\sqrt{2}$

(b) 0

(c) 2

(d)  $\sqrt{2}$

(e)  $\frac{5}{3}$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(\frac{4}{3} - \frac{1}{3})^2 + (-\frac{1}{2} - \frac{1}{2})^2} \\ &= \sqrt{(\frac{3}{3})^2 + (-1)^2} \\ &= \sqrt{1 + 1} \\ &= \boxed{\sqrt{2}} \end{aligned}$$

8. Solve for  $x$ :  $\log_4(2x + 6) = 2$ .

(a)  $x = 3$

(b)  $x = 1$

(c)  $x = -2$

(d)  $x = 2$

(e)  $x = 5$

$$\log_4(2x+6) = 2 \Rightarrow 2x+6 = 4^2 \Rightarrow 2x+6 = 16 \Rightarrow 2x = 10 \Rightarrow x = 5$$

9. Find the equation of the line perpendicular to  $3x - 2y = 6$  passing through the point  $(-5, 1)$ .

Write your answer in slope-intercept form.

(a)  $y = -\frac{2}{3}x - \frac{7}{3}$

(b)  $y = -\frac{2}{3}x + \frac{13}{3}$

(c)  $y = \frac{3}{2}x + \frac{17}{2}$

(d)  $y = -\frac{2}{3}x - \frac{13}{3}$

(e)  $y = \frac{3}{2}x - \frac{13}{2}$

$$3x - 2y = 6 \Rightarrow -2y = -3x + 6 \Rightarrow y = \frac{3}{2}x - 3 \Rightarrow m = \frac{3}{2} \Rightarrow$$

$$m_{\perp} = -\frac{2}{3}$$

$$(-5, 1), m = -\frac{2}{3}$$

$$y - y_1 = m(x - x_1) \Rightarrow y - 1 = -\frac{2}{3}(x - (-5)) \Rightarrow$$

$$\Rightarrow y - 1 = -\frac{2}{3}(x + 5) \Rightarrow y - 1 = -\frac{2}{3}x - \frac{10}{3} \Rightarrow$$

$$y = -\frac{2}{3}x + \frac{3}{3} - \frac{10}{3} \Rightarrow \boxed{y = -\frac{2}{3}x - \frac{7}{3}}$$

10. Simplify to the form  $a+bi$ :  $\frac{1}{1+i} + i^2$

(a)  $i$

(b)  $-i$

(c)  $\frac{1}{2} - \frac{1}{2}i$

(d)  $-\frac{1}{2} - \frac{1}{2}i$

(e)  $-1 + i$

$i^2 = -1$

$$\frac{1}{1+i} + i^2 \Rightarrow \frac{1}{1+i} \cdot \frac{(1-i)}{(1-i)} - 1 \Rightarrow$$

FOIL

$$\Rightarrow \frac{1-i}{1-i+i-i^2} - 1 \Rightarrow \frac{1-i}{1+i} \Rightarrow \frac{1-i}{2}$$

$-i^2 = 1$

$$\Rightarrow \frac{1}{2} - \frac{1}{2}i$$

11. Divide

(a)  $x + 4 + \frac{27}{x-5}$

(b)  $x^2 + 5x + 24 + \frac{127}{x-5}$

(c)  $x^2 - x + 2$

(d)  $-x^2 - 5x - 24 + \frac{127}{x-5}$

(e)  $x^2 - 2x + 12$

$$\begin{array}{r} x^3 - x + 7 \\ x - 5 \\ \hline x^2 + 5x + 24 + \frac{127}{x-5} \\ x-5 \overline{) x^3 + 0x^2 - x + 7} \\ \underline{-x^3 + 5x^2} \phantom{+ 7} \downarrow \\ 5x^2 - x \phantom{+ 7} \\ \underline{-5x^2 + 25x} \phantom{+ 7} \downarrow \\ 24x + 7 \\ \underline{-24x + 120} \\ +127 \end{array}$$

12. Solve the following equation for  $x$

(a)  $x = -5$

(b)  $x = -2$

(c)  $x = -5, x = -2$

(d)  $x = 0$

(e) No solution

$$(\sqrt{x+6})^2 - (x+4)^2 = (x+4)(x+4) \text{ FOIL!}$$

$$\Rightarrow x+6 = x^2 + 8x + 16$$

$$\Rightarrow 0 = x^2 + 7x + 10$$

$$\Rightarrow 0 = (x+5)(x+2)$$

$$\Rightarrow x = -5, x = -2$$

Check  $x = -5$ ;  $\sqrt{-5+6} = \sqrt{1} = 1$  No!

$x = -2$ ;  $\sqrt{-2+6} = \sqrt{4} = 2$  YES  $\Rightarrow x = -2$

13. Huck can paint a fence in 3 hours. Tom takes 5 hours to paint a fence. How long will it take them together to paint the fence?

(a) 2 hours

(b) 4 hours

(c)  $1\frac{7}{8}$  hours

(d) 8 hours

(e)  $1\frac{1}{8}$  hours

$$\frac{1}{3} + \frac{1}{5} = \frac{1}{T}$$

*Handwritten notes:  $3 \cdot 5 \cdot T$  and  $3 \cdot 5 \cdot T$  are written above the denominators. An arrow points from the circled question to the  $T$  in the denominator.*

$$5T + 3T = 15$$

$$8T = 15$$

$$T = \frac{15}{8} = 1\frac{7}{8} \text{ hours}$$

14. Simplify and write the answer using positive exponents only. Assume the variables are positive.

(a)  $\frac{x^{10}}{4y^6}$

(b)  $\frac{x^2}{4y^{10}}$

(c)  $-\frac{x^2}{4y^{10}}$

(d)  $\frac{2x^2}{y^{10}}$

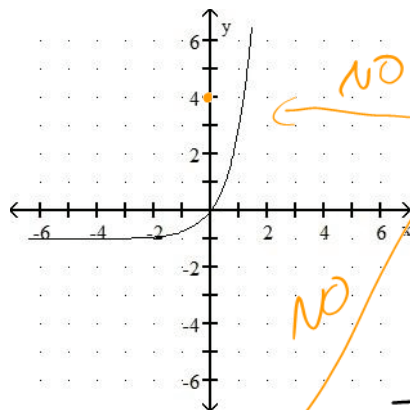
(e)  $\frac{x^{10}}{4y^6}$

$$\left(\frac{8x^{-3}y^4}{4x^{-2}y^{-1}}\right)^{-2} \Rightarrow \left(\frac{2x^{-3}y^4}{x^{-2}y^{-1}}\right)^{-2} \Rightarrow \frac{2^{-2}x^6y^{-8}}{x^4y^2} \Rightarrow \frac{x^2y^8}{4y^{10}}$$

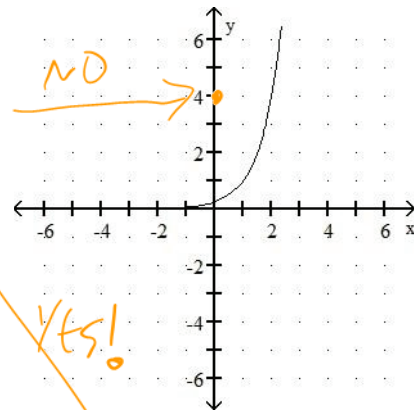
*Handwritten notes: The final result is  $\frac{x^2y^8}{4y^{10}}$ . There are additional scribbles and a circled  $2^2$  in the denominator.*

15. Graph the following function:  $f(x) = 4^{x+1}$

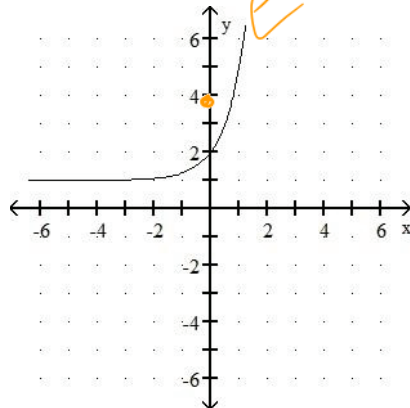
(a)



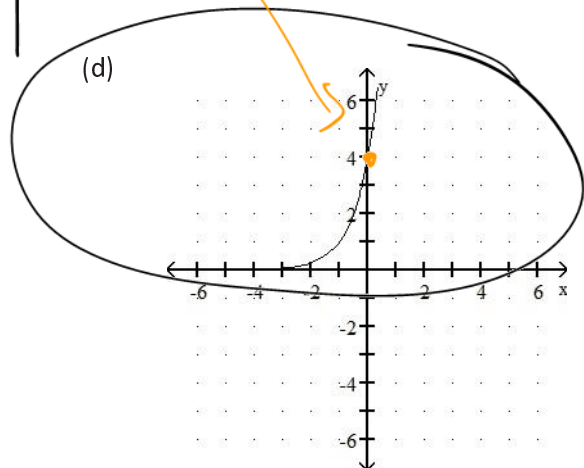
x	y
-3	$4^{-2} = \frac{1}{16}$
-2	$4^{-1} = \frac{1}{4}$
-1	$4^0 = 1$
0	$4^1 = 4$
1	$4^2 = 16$



(b)



(d)



*Handwritten notes: 'NO' is written next to graphs (a), (b), and (c). 'YES!' is written next to graph (d). Arrows point from the 'NO' and 'YES!' labels to their respective graphs.*

16. Solve the following equation for y

(a)  $y = 3$

(b)  $y = -1, y = 3$

(c)  $y = -1$

(d)  $y = \frac{12}{5}$

(e)  $y = \frac{12}{5}, y = -\frac{2}{5}$

$3 + 2|y - 1| = 7$

$\Rightarrow \frac{2|y-1|}{2} = \frac{4}{2}$

$\Rightarrow |y-1| = 2$

$\Rightarrow \begin{matrix} \downarrow & & \downarrow \\ y-1 = 2 & , & y-1 = -2 \\ +1 & +1 & +1 & +1 \end{matrix}$

$\Rightarrow y = 3, y = -1$

17. Find the domain D of the following relation.

$\{(2,3), (8,11), (7,9), (2,14)\}$

(a)  $D = \{2, 8, 7\}$

(b)  $D = \{3, 11, 9, 14\}$

(c)  $D = \{x | 2 \leq x \leq 8\}$

(d)  $D = \{x | 3 \leq x \leq 14\}$

(e)  $D = \{all \text{ real numbers}\}$

$D = \{2, 8, 7\}$

18. Simplify

(a)  $-9xy^2\sqrt[3]{x^2y}$

(b)  $-9x^2y^3\sqrt[3]{x^2y^4}$

(c)  $3x^2y^3\sqrt[3]{x^2y^4}$

(d)  $-3xy^3\sqrt[3]{xy}$

(e)  $-3xy^2\sqrt[3]{x^2y}$

Look for groups of 3!

$= \sqrt[3]{(-3)(-3)(-3) \cdot xxx \cdot yyy \cdot yyy \cdot y}$

$= -3 \cdot xy^2 \cdot \sqrt[3]{x^2y}$

19. Solve the following equation for  $x$

(a)  $x = 13$

(b)  $x = 9$

(c)  $x = -3$

(d)  $x = -1, x = 3$

(e)  $x = 1$

$$\frac{2}{x+4} - \frac{1}{x^2-16} = \frac{1}{x-4}$$

$$\Rightarrow \frac{2}{\cancel{x+4}} - \frac{1}{(\cancel{x+4})(x-4)} = \frac{1}{\cancel{x-4}}$$

$$\Rightarrow 2(x-4) - 1 = 1(x+4)$$

$$\Rightarrow 2x - 8 - 1 = x + 4 \Rightarrow 2x - 9 = x + 4 \Rightarrow$$

$$-x + 9 = -x + 4$$

$$\Rightarrow \boxed{x = 13}$$

ANSWER DOESN'T CAUSE  $\div 0$  IN EQUATION  
SO IT IS NOT EXTRANEOUS.

20. Simplify

(a)  $\frac{x}{x-7}$

(b)  $\frac{x-7}{x^2+8x}$

(c)  $x - 7$

(d)  $\frac{x}{x^2+11x+24}$

(e)  $\frac{x-7}{x}$

$$\frac{x^2+8x+15}{x^2+11x+24} \div \frac{x^2+5x}{x^2+x-56}$$

$$\Rightarrow \frac{x^2+8x+15}{x^2+11x+24} \cdot \frac{x^2+x-56}{x^2+5x}$$

$$\Rightarrow \frac{(x+5)(x+3)}{(x+8)(x+3)} \cdot \frac{(x+8)(x-7)}{x(x+5)}$$

$$\Rightarrow \boxed{\frac{x-7}{x}}$$

21. Solve the following equation for  $x$

(a)  $\left\{ \frac{7-\sqrt{97}}{8}, \frac{7+\sqrt{97}}{8} \right\}$

(b)  $\left\{ \frac{3}{4}, 1 \right\}$

(c)  $\left\{ \frac{7}{8} + \sqrt{97}, \frac{7}{8} - \sqrt{97} \right\}$

(d)  $\left\{ \frac{7}{4}, \frac{7}{3} \right\}$

(e)  $\left\{ \frac{8-\sqrt{97}}{7}, \frac{8+\sqrt{97}}{7} \right\}$

$$4x^2 - 7x = 3$$

$$\Rightarrow 4x^2 - 7x - 3 = 0$$

DISCRIMINANT

$$b^2 - 4ac = 49 - 4(-12) = 97$$

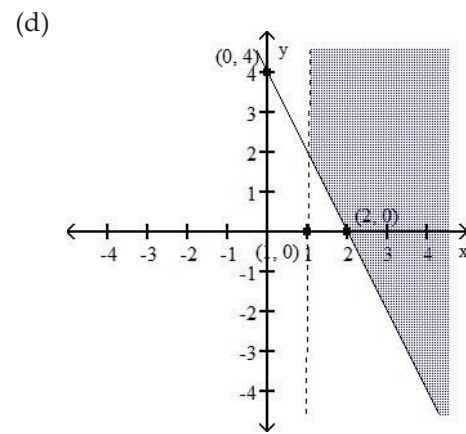
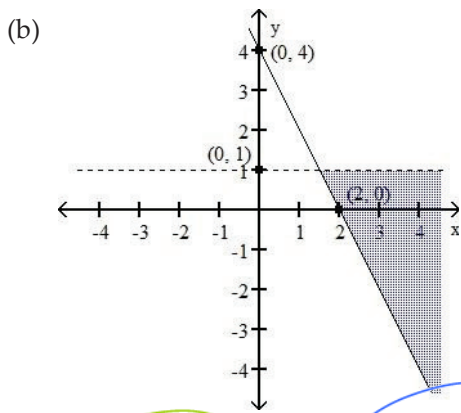
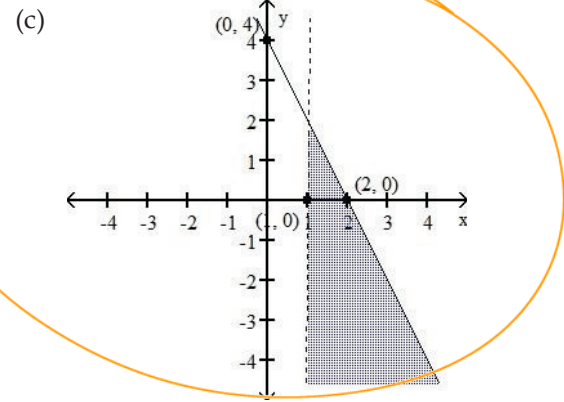
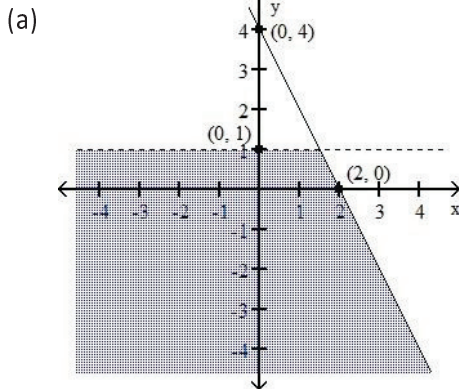
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \boxed{\frac{7 \pm \sqrt{97}}{8}}$$

22. A ball is launched off an 84-foot-tall cliff. The height of the ball is given by the function:  
 $h(t) = -16t^2 + 32t + 84$ . How long after the ball is launched will it hit the ground?

- (a) 0 seconds
- (b) 1.5 seconds
- (c) 2.5 seconds
- (d) 3 seconds
- (e) 3.5 seconds**

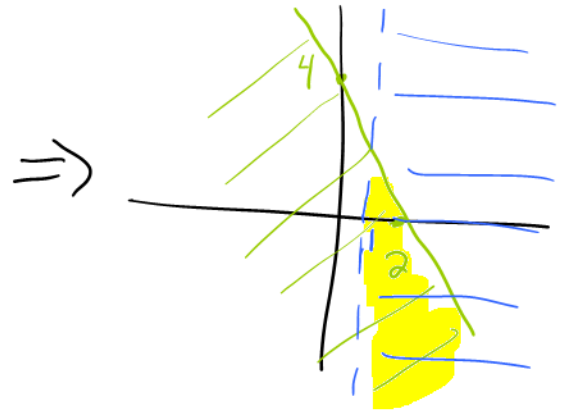
GROUND  $\Rightarrow h = 0$   
 $0 = -16t^2 + 32t + 84$   
 $0 = -4(4t^2 - 8t - 21)$   
 $0 = -4(2t + 3)(2t - 7)$   
 $2t + 3 = 0 \quad 2t - 7 = 0$   
 $t = -\frac{3}{2} \quad t = \frac{7}{2} = 3.5 \text{ hrs}$

23. Graph the system of inequalities  
 $2x + y \leq 4$  and  $x - 1 > 0$



$2x + y \leq 4$   
 $\begin{array}{r|l} 2 & 4 \\ 0 & 4 \\ 2 & 0 \end{array}$   
 SOLID,  
 SHADE (0,0)

$x - 1 > 0$   
 $x > 1$   
 VERTICAL THROUGH 1,  
 DASHED,  
 DON'T SHADE (0,0)





24. Find the center and radius of the circle defined by the equation:

$$x^2 + y^2 - 10x + 4y + 20 = 0$$

(a) Center = (5, 2) Radius = 9

(b) Center = (-2, 5) Radius = 3

(c) Center = (-5, 2) Radius = 9

(d) Center = (5, -2) Radius = 3

(e) Center = (2, -5) Radius = 1

$$x^2 - 10x + 25 + y^2 + 4y + 4 = -20 + 25 + 4$$

$$(x-5)^2 + (y+2)^2 = 9$$

Center (5, -2); r = 3

25. Meriwether and William can row their boat 39 miles downstream in the same time it takes them to row 15 miles upstream. The speed of their boat in still water is 9 miles per hour. Find the speed of the river's current.

(a) 1 mph

(b) 2 mph

(c) 4 mph

(d) 6 mph

(e) 9 mph

26. Solve the following equation:

(a)  $x = 0, x = \frac{2}{3}, x = -4$

(b)  $x = \frac{2}{3}, x = -4$

(c)  $x = \pm\sqrt[3]{40}$

(d)  $x = -\frac{2}{3}, x = 4$

(e)  $x = 0, x = -\frac{4}{3}, x = 2$

$$3x^3 - 8x = -10x^2$$

$$3x^3 + 10x^2 - 8x = 0$$

$$x(3x^2 + 10x - 8) = 0$$

$$x(3x - 2)(x + 4) = 0$$

$$x = 0, x = \frac{2}{3}, x = -4$$

RATE of current = R

UPSTREAM:  $9 - R$

DOWNSTREAM:  $9 + R$

	D	R	T
UP	15	$9 - R$	$\frac{15}{9 - R}$
Down	39	$9 + R$	$\frac{39}{9 + R}$

$$\frac{15}{9 - R} = \frac{39}{9 + R} \Rightarrow 15(9 + R) = 39(9 - R)$$

$$\Rightarrow 135 + 15R = 351 - 39R$$

$$\Rightarrow 54R = 216$$

$$\Rightarrow R = 4 \text{ mph}$$

27. Let  $f(x) = x + 1$  and  $g(x) = x^2 - 2$ . Find  $(f \cdot g)(x)$ .

(a)  $x^2 + x - 1$

(b)  $x^2 + 2x - 1$

(c)  $x^2 - 1$

(d)  $x^3 + x^2 - 2x - 2$

(e)  $x^2 + x + 2$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$= (x + 1) \cdot (x^2 - 2) \text{ FOIL}$$

$$= x^3 - 2x + x^2 - 2$$

$$= x^3 + x^2 - 2x - 2$$

28. Solve for x. Write the answer in interval notation.

$$|2x + 3| \leq 4$$

→ OR  $2x + 3 \leq 4$  AND

$$2x + 3 \geq -4$$

(a)  $[\frac{1}{2}, \infty)$

(b)  $(-\infty, \frac{1}{2}]$

(c)  $(-\infty, -\frac{7}{2}] \cup [\frac{1}{2}, \infty)$

(d)  $[-\frac{7}{2}, \frac{1}{2}]$

(e)  $(-\infty, -\frac{1}{2}] \cup [\frac{7}{2}, \infty)$

$$\begin{matrix} -4 & \leq & 2x+3 & \leq & 4 \\ -3 & & -3 & & -3 \end{matrix}$$

$$\begin{matrix} -7 & \leq & 2x & \leq & 1 \\ \frac{-7}{2} & & \frac{2}{2} & & \frac{1}{2} \end{matrix}$$

$$\begin{matrix} -\frac{7}{2} & \leq & x & \leq & \frac{1}{2} \end{matrix}$$

$$\boxed{[-\frac{7}{2}, \frac{1}{2}]}$$

29. Perform the indicated operation and simplify if possible

$$\frac{x+1}{x+3} - \frac{x+17}{x^2-x-12}$$

(a)  $-\frac{16}{(x+3)(x-4)}$

(b)  $\frac{x+12}{(x+3)(x-4)}$

(c)  $\frac{x-7}{x-4}$

(d)  $\frac{x-16}{x-4}$

(e)  $\frac{(x+5)}{x+3}$

$$\Rightarrow \frac{(x+1)}{(x+3)} - \frac{(x+17)}{(x-4)(x+3)} \Rightarrow \frac{(x+1)(x-4)}{(x+3)(x-4)} - \frac{(x+17)}{(x-4)(x+3)}$$

$$\Rightarrow \frac{x^2 - 3x - 4 - x - 17}{(x+3)(x-4)} \Rightarrow \frac{x^2 - 4x - 21}{(x+3)(x-4)}$$

$$\Rightarrow \frac{(x-7)(x+3)}{(x+3)(x-4)} \Rightarrow \boxed{\frac{x-7}{x-4}}$$

30. Simplify  $(2x + 5)^2 - (2x + 5)(2x - 5)$

(a) 8

(b)  $20x \Rightarrow (2x+5)(2x+5) - (4x^2 - 10x + 10x - 25)$

(c)  $20x + 50 \Rightarrow 4x^2 + 10x + 10x + 25 - 4x^2 + 25$

(d)  $20x^2 + 8x$

(e)  $8x + 6$

$$\boxed{20x + 50}$$