Math 1050 Final Exam form A – Fall Semester 2009

Name KEY ______________________ Instructor ______________________

Student ID ______________________, ID Verification __________ Section Number ___________

This exam has three parts

Part I – Ten multiple choice questions – choose the best answer
Part II – Ten open ended questions – you MUST show all your work
Part III – Choose FIVE out of ten open ended questions – you MUST show your work and indicate which five problems are to be graded

Each problem is equally weighted. Graphing calculators without CAS systems are allowed.
Time limit: 2 hours.
Not allowed: notes, books, CAS calculators, cell phones, other hand–held devices.

PART I – Ten multiple choice questions – choose the best answer

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Solve.

1) A local civic theater has 22 seats in the first row and 21 rows in all. Each successive row contains 3 additional seats. How many seats are in the civic theater?

A) 790 seats B) 1092 seats C) 1070 seats D) 1010 seats

Solve the problem.

2) What is the domain of the function $f(x) = \sqrt{x^4 - 81}$?

A) $(-\infty, -3]$ or $[3, \infty)$ B) $(-\infty, 3)$
C) $(-\infty, 3)$ or $(3, \infty)$ D) $(-\infty, -3)$ or $(3, \infty)$

Find the real solutions of the equation.

3) $3x^{-2} - 2x^{-1} - 8 = 0$

A) $\{\frac{3}{4}, \frac{1}{2}\}$ B) $\{-\frac{4}{3}, 2\}$ C) $\{-\frac{4}{3}, -2\}$ D) $\{-\frac{3}{4}, \frac{1}{2}\}$

1. $a_1 = 22,\ a_2 = 25,\ \cdots \quad d = 3,\ n = 21$

$\boxed{a_n = a_1 + (n-1)d} \Rightarrow \quad a_{21} = a_1 + 20d = 22 + 20(3) = 82$

$\boxed{S_n = \frac{n}{2}(a_1 + a_n)} \Rightarrow S_{21} = \frac{21}{2}(22 + 82) = \boxed{1092}$

$\boxed{B}$

2. DOMAIN of $f(x) = \sqrt{x^4 - 81}$

$x^4 - 81 \geq 0 \Rightarrow (x^2+9)(x^2-9) \geq 0 \Rightarrow (x^2+9)(x+3)(x-3) \geq 0$

$\Rightarrow x = -3, 3$ are zeros $\Rightarrow$

$(-5)^4 - 81 = 544 \geq 0$ ✓

$0^4 - 81 \not\geq 0$ X

$5^4 - 81 = 544 \geq 0$ ✓

$\boxed{(-\infty, -3] \cup [3, \infty)}\ \boxed{A}$

3. $3x^{-2} - 2x^{-1} - 8 = 0$

Let $u = x^{-1}$ & $3u^2 - 2u - 8 = 0$

so $u^2 = x^{-2}$

$(3u+4)(u-2) = 0$

$\Rightarrow u = -\frac{4}{3},\ u = 2$

$\Rightarrow$

$x^{-1} = \frac{-4}{3} \Rightarrow \boxed{x = -\frac{3}{4}}$

$x^{-1} = 2 \Rightarrow \boxed{x = \frac{1}{2}}$

$\boxed{D}$

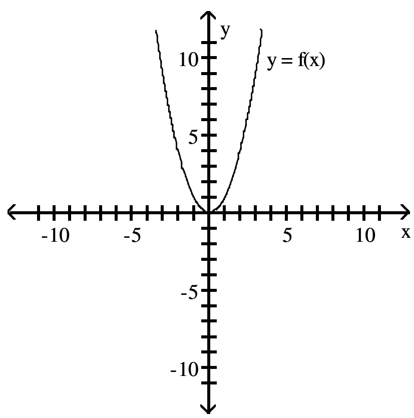
Solve the problem.

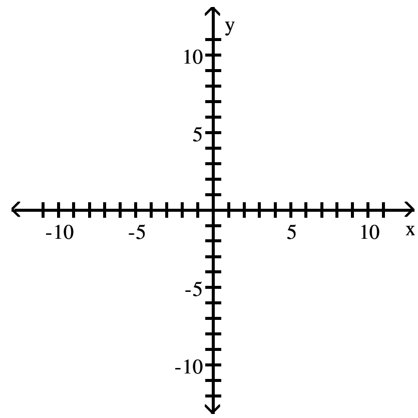
4) Consider the quadratic model $h(t) = -16t^2 + 40t + 50$ for the height (in feet), h, of an object t seconds after the object has been projected straight up into the air. Find the maximum height attained by the object. How much time does it take to fall back to the ground? Assume that it takes the same time for going up and coming down.

A) maximum height = 75 ft; time to reach ground = 1.25 seconds
B) maximum height = 75 ft; time to reach ground = 2.5 seconds
C) maximum height = 50 ft; time to reach ground = 1.25 seconds
D) maximum height = 50 ft; time to reach ground = 2.5 seconds

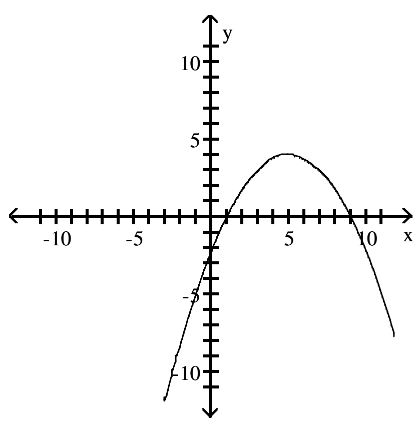
Use the accompanying graph of y = f(x) to sketch the graph of the indicated equation.

5) $y = -\frac{1}{4}f(x + 5) + 4$





A)



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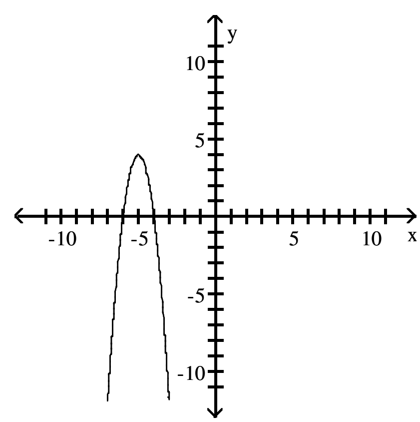

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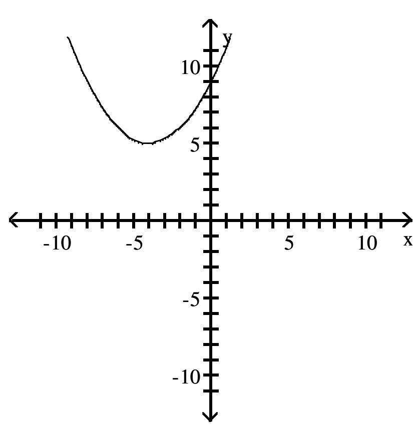

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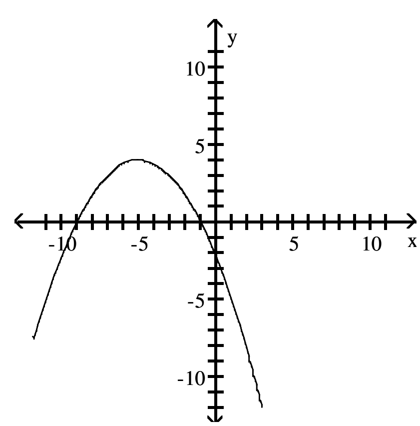


4. $h(t) = -16t^2 + 40t + 30$

MAX HEIGHT IS Y VALUE OF VERTEX

VERTEX: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$x = t = \frac{-b}{2a} = \frac{-40}{2(-16)} = \frac{-40}{-32} = \frac{5}{4}$ sec to go up to MAX

$y = h\left(\frac{5}{4}\right) = -16\left(\frac{5}{4}\right)^2 + 40\left(\frac{5}{4}\right) + 50 = -25 + 50 + 50$

$= \boxed{75 \text{ ft}}$

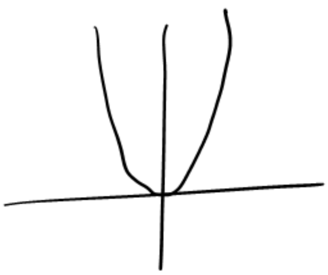
TIME TO GET BACK TO GROUND

= TIME UP + TIME DOWN

$= \frac{5}{4} + \frac{5}{4} = \boxed{5/2 \text{ sec}}$

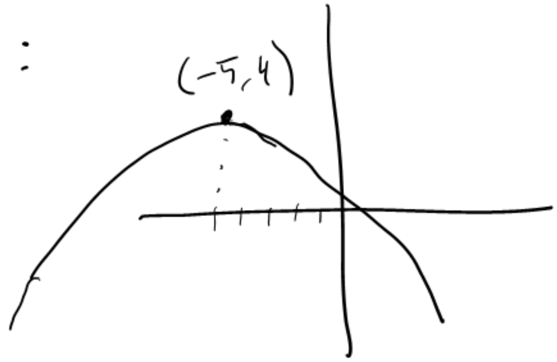
$\boxed{B}$

5. f(x):



$-\frac{1}{4} f(x+5) + 4 \Rightarrow$ Left 5,
UP 4,
VERTICAL COMPRESSION by $\cdot \frac{1}{4}$
& X-reflection

so $-\frac{1}{4} f(x+5) + 4$:



$\boxed{D}$

Write as the sum and/or difference of logarithms. Express powers as factors.

6) $\log_5\left(\dfrac{x-4}{z^2}\right)$

A) $\log_5(x-4) - \log_5 z$
B) $\log_5(x-4) - 2\log_5 z$
C) $\log_5 x - \log_5 4 - 2\log_5 z$
D) $\log_5(x-4) + 2\log_5 z$

Find the center (h, k) and radius r of the circle with the given equation.

7) $x^2 + y^2 + 4x - 5y + 2 = 0$

A) $(h, k) = (2, -\frac{5}{2});\ r = \frac{\sqrt{33}}{2}$
B) $(h, k) = (-4, 5);\ r = \sqrt{2}$
C) $(h, k) = (-2, \frac{5}{2});\ r = \frac{\sqrt{33}}{2}$
D) $(h, k) = (4, -5);\ r = 3\sqrt{3}$

Perform the indicated operations. Recall that I_2 is the 2 by 2 identity matrix.

8)
Let $A = \begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 3 & 5 \end{bmatrix}$. Find $BA - 3I_2$

A) $\begin{bmatrix} 15 & 1 \\ 11 & 2 \end{bmatrix}$
B) $\begin{bmatrix} -1 & -8 \\ -1 & 18 \end{bmatrix}$
C) $\begin{bmatrix} 15 & 4 \\ 14 & 2 \end{bmatrix}$
D) $\begin{bmatrix} -1 & -5 \\ -4 & 18 \end{bmatrix}$

Solve the inequality. Express the solution using interval notation.

9) $\dfrac{x+19}{x+2} < 7$

A) $\left(-2, \frac{5}{6}\right)$
B) $\left(-\infty, -2\right) \cup \left(\frac{5}{6}, \infty\right)$
C) $\left(-\infty, \frac{5}{6}\right) \cup \left(2, \infty\right)$
D) $\left(-\infty, -2\right) \cup \left(2, \infty\right)$

6. $\log_5\left(\frac{x-4}{z^2}\right) = \log_5(x-4) - \log_5 z^2 = \boxed{\log_5(x-4) - 2\log_5 z}$

$\boxed{\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n \quad \& \quad \log_b m^r = r\log_b m}$

$\boxed{B}$

7. $x^2+y^2+4x-5y+2=0$

$x^2+4x+4+y^2-5y+\frac{25}{4} = -2+4+\frac{25}{4}$

$(x+2)^2 + (y-\frac{5}{2})^2 = \frac{33}{4}$

$\boxed{\text{center } \left(-2, \frac{5}{2}\right), \text{ radius } \sqrt{\frac{33}{4}} = \frac{\sqrt{33}}{2}}$

$\boxed{C}$

8.

$$BA - 3I_2 = \begin{bmatrix} -1 & 4 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1\cdot -2 + 4\cdot 4 & -1\cdot 0 + 4\cdot 1 \\ 3\cdot -2 + 5\cdot 4 & 3\cdot 0 + 5\cdot 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 4 \\ 14 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} 15 & 4 \\ 14 & 2 \end{bmatrix}}$$

$\boxed{C}$

9. $\frac{x+19}{x+2} < 7 \Rightarrow \frac{x+19}{x+2} - 7\cdot\frac{(x+2)}{x+2} < 0 \Rightarrow \frac{x+19}{x-2} + \frac{-7x-14}{x-2} < 0$

$\Rightarrow \frac{x+19-7x-14}{x-2} < 0 \Rightarrow \frac{-6x+5}{x-2} < 0 \Rightarrow$ $-6x+5=0 \Rightarrow x=\frac{5}{6}$

$\& \; x+2=0 \Rightarrow x=-2$

TEST: -5, 0, 10

-2, $5/6$

$\frac{-5+19}{-5+2} < 7$ ✓ $\quad$ $\frac{0+19}{0+2} < 7$ X $\quad$ $\frac{10+19}{10+2} < 7$ ✓

$\Rightarrow \boxed{(-\infty, -2) \cup \left(\frac{5}{6}, \infty\right)}$

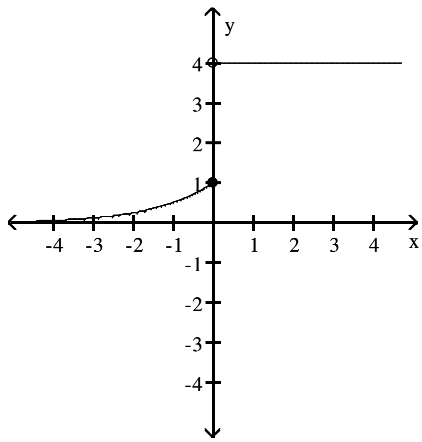
$\boxed{B}$

Graph the function.

10)

$$f(x) = \begin{cases} 4 & \text{if } x < 0 \\ 2^x & \text{if } x \ge 0 \end{cases}$$

A)



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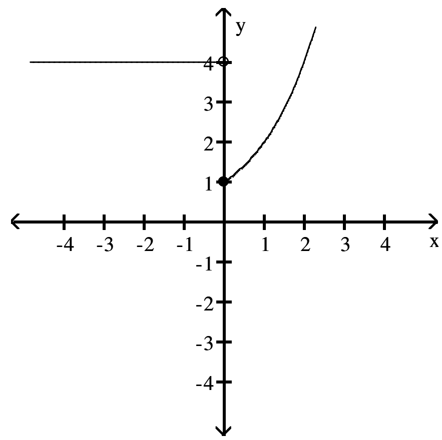

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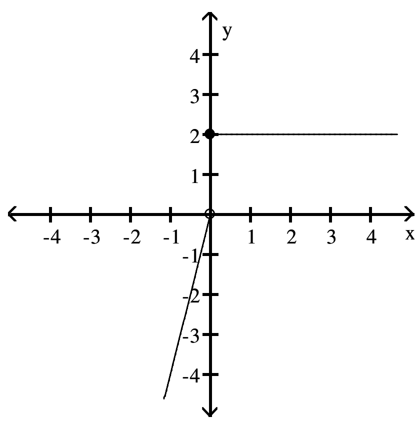

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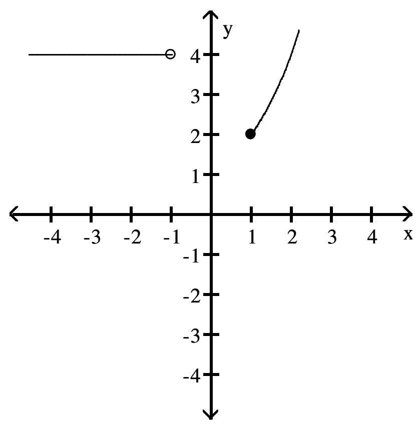
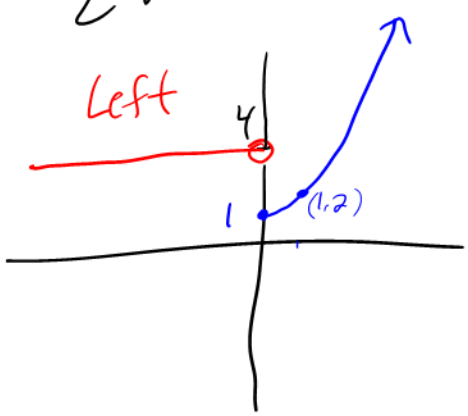


$f(x) = \begin{cases} 4 & x < 0 \\ 2^x & x \ge 0 \end{cases}$

$x < 0$ → LEFT IS A CONSTANT (HORIZONTAL @ 4)

$x \ge 0$ → RIGHT IS EXPONENTIAL

x	y
0	$2^0 = 1$
1	$2^1 = 2$



$\boxed{B}$

Answer Key
Testname: AB MC F09 V3

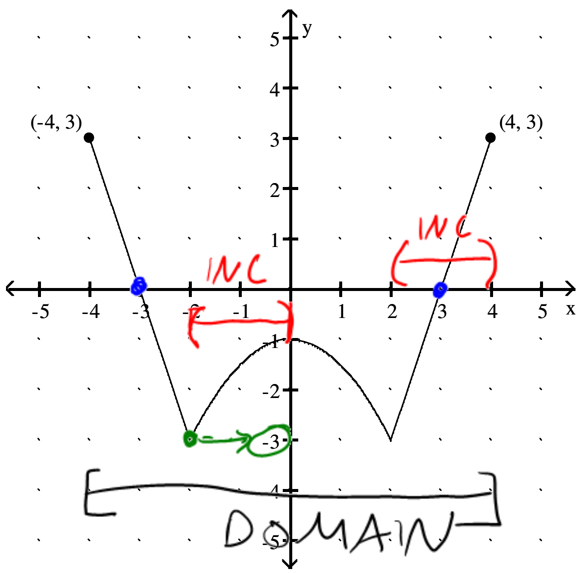
FORM A

1) B
2) A
3) D
4) B
5) D
6) B
7) C
8) C
9) B
10) B

PART II: Questions 11 – 20, Open ended
Answer all TEN questions. You must show all your work in a clear and logical progression and clearly indicate your answer to receive full credit.

Answer the questions using the given graph of the function f(x).

11)



a) State the domain of f. $[-4, 4]$

b) On what interval(s) is f increasing? $(-2, 0) \cup (2, 4)$

c) Find all values of x for which f(x) = 0

$x = -3, 3$

d) Find f(–2) $= -3$

e) Is the function even, odd, or neither?

Y-AXIS SYMMETRY $\Rightarrow$ EVEN

Find all complex zeros of the function.

12) $f(x) = x^3 + 9x^2 + 16x - 26$

RATIONAL zerotest: $\frac{26}{1} \Rightarrow \pm 1, 2, 13, 26$

$$\begin{array}{c|cccc} & 1 & 9 & 16 & -26 \\ \hline \boxed{1} & 1 & 10 & 26 & 0 \end{array} \Rightarrow x = 1$$

$x^2 + 10x + 26$

$$x = \frac{-10 \pm \sqrt{100 - 4(26)}}{2} = \frac{-10 \pm \sqrt{-4}}{2} = \frac{-10 \pm 2i}{2} = -5 \pm i$$

Zeros: $\boxed{1, -5+i, -5-i}$

Solve the problem.

13) Conservationists tagged 120 black–nosed rabbits in a national forest in 2004. In 2007 they tagged 240 black–nosed rabbits in the same range. If the rabbit population follows the exponential law, how many rabbits will be in the range 10 years from 2004? Round your answer to the nearest whole number.

$N = N_0 e^{kt}$ $N_0 = 120$. WHEN $t = 3$ (2007−2004), $N = 240$

$$\Rightarrow \frac{240}{120} = \frac{120}{120} e^{(k \cdot 3)} \Rightarrow \ln 2 = \ln e^{3k} \Rightarrow \frac{\ln 2}{3} = \frac{3k}{3}$$

$$\Rightarrow k = \frac{1}{3} \ln 2 \approx 0.23104906$$

so $N = 120 e^{0.23104906t}$. 10 yrs after 2004 is $t = 10$

so $N = 120 e^{(0.23104906 \cdot 10)} = 1209.5$, rounded

$= \boxed{1210 \text{ rabbits}}$

Analyze and graph the rational function.

14) $R(x) = \dfrac{2x^2 + 6x - 8}{x^2 - x - 6}$

a) State the domain.

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0 \Rightarrow x = 3, -2$ so $\boxed{\text{DOMAIN} = \{x \mid x \neq 3, -2\}}$

b) List all intercepts as ordered pairs.

$x = 0 \Rightarrow R = \frac{-8}{-6} = \frac{4}{3}$

$(0, 4/3)$

$y = 0 = 2x^2 + 6x - 8$

$2(x^2 + 3x - 4)$

$0 = 2(x+4)(x-1) \Rightarrow x = -4, 1$

so $\boxed{(0, 4/3) \quad (-4, 0) \quad (1, 0)}$

c) List the equations of any asymptotes and indicate if they are vertical, horizontal, or oblique.

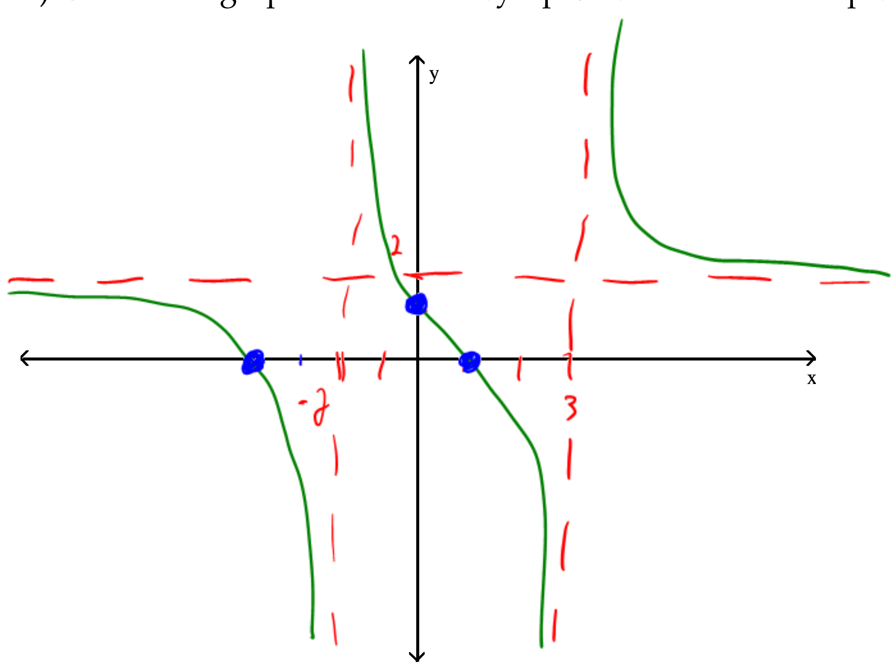
Power on top is $2x^2$

on bottom is x^2

$\frac{2x^3}{x^2} = 2$ so $\boxed{y = 2 \text{ is horizontal Asymptote}}$

$\boxed{X = 3 \text{ \& } X = -2 \text{ ARE VERTICAL ASYMPTOTES}}$

d) Sketch the graph. Label the asymptotes and the intercepts.



Find and simplify the difference quotient of f, $\frac{f(x + h) - f(x)}{h}$, h≠ 0, for the function.

15) $f(x) = x^2 + 9x - 6$

$$f(x+h) = (x+h)^2 + 9(x+h) - 6 = x^2 + 2xh + h^2 + 9x + 9h - 6$$

$$-f(x) = -(x^2 + 9x - 6) = -x^2 - 9x + 6$$

$$\text{so } \frac{f(x+h) - f(x)}{h} = \frac{\cancel{x^2} + 2xh + h^2 \cancel{+9x} + 9h \cancel{-6} \cancel{-x^2} \cancel{-9x} \cancel{+6}}{h}$$

$$= \frac{2x\cancel{h} + h^{\cancel{2}} + 9\cancel{h}}{\cancel{h}} = \boxed{2x + h + 9}$$

Find the inverse function of f. State the domain and range of f and its inverse.

16) $f(x) = \frac{3x - 2}{x + 5}$, DOMAIN $x \neq -5$

$$y = \frac{3x-2}{x+5} \Rightarrow (y+5) \cdot x = \frac{3y-2}{(y+5)} \cdot (y+5) \Rightarrow xy + 5x = 3y - 2$$

$-3y \quad -5x \qquad -3y \quad -5x$

$$\Rightarrow xy - 3y = -5x - 2 \Rightarrow y\cancel{(x-3)} = -5x - 2$$

$x-3 \qquad x-3$

$$\Rightarrow y = \frac{-5x-2}{x-3} = f^{-1}(x), \text{ DOMAIN } x \neq 3$$

domain of f: $\{x \mid x \neq -5\}$

range of f: $\{y \mid y \neq 3\}$

domain of f^{-1}: $\{x \mid x \neq 3\}$

range of f^{-1}: $\{y \mid y \neq -5\}$

OR $y = \frac{-(5x+2)}{-(3-x)} = \frac{5x+2}{3-x}$. BOTH ARE CORRECT...

Find the domain of the composite function f ∘ g.

17) $f(x) = \frac{x}{x+6}$; $g(x) = \frac{24}{x+4}$

$f \circ g = f(g(x))$

x is in g, so x satisfies g's domain $\Rightarrow \boxed{x \neq -4}$

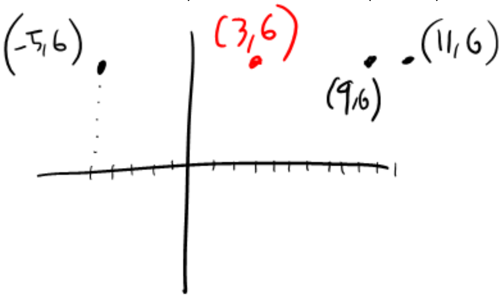
g is in f, so g satisfies f's domain $\Rightarrow (x+4)\frac{24}{x+4} \neq -6(x+4)$ so $\{x \mid x \neq -4, -8\}$

$\Rightarrow 24 \neq -6x - 24 \Rightarrow 48 \neq -6x \Rightarrow \boxed{x \neq -8}$

$+24 \quad +24 \quad \frac{48}{-6} \neq \frac{-6x}{-6}$

Find an equation for the ellipse described.

18) Vertices at (−5, 6) and (11, 6); focus at (9, 6)



½ way between vertices is center = (3, 6)

$\frac{-5+11}{2}$

Distance from center to focus is 6 = c

& Distance from center to vertex is 8 = a

so $c^2 = a^2 - b^2 \Rightarrow 6^2 = 8^2 - b^2 \Rightarrow b^2 = 64 - 36$

$b^2 = 28$

& $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \Rightarrow \boxed{\frac{(x-3)^2}{64} + \frac{(y-6)^2}{28} = 1}$

Solve the system using the inverse matrix method. Your work must demonstrate how the inverse matrix is used.

19)
$$\begin{cases} x + 2y + 3z = -9 \\ x + y + z = -11 \\ x - 2z = 3 \end{cases}$$

$$X = A^{-1} \cdot b = \begin{bmatrix} -2 & 4 & -1 \\ 3 & -5 & 2 \\ -1 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -9 \\ -11 \\ 3 \end{bmatrix}$$

The inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$ is $\begin{bmatrix} -2 & 4 & -1 \\ 3 & -5 & 2 \\ -1 & 2 & -1 \end{bmatrix}$.

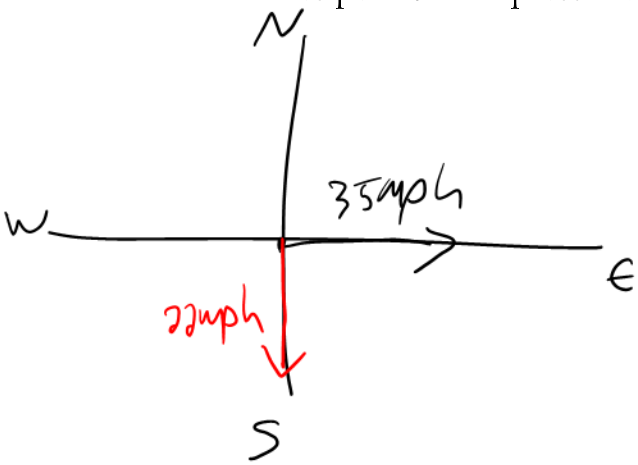
$$= \begin{bmatrix} -2 \cdot -9 + 4 \cdot -11 + -1 \cdot 3 \\ 3 \cdot -9 + -5 \cdot -11 + 2 \cdot 3 \\ -1 \cdot -9 + 2 \cdot -11 + -1 \cdot 3 \end{bmatrix} =$$

$$= \begin{bmatrix} 18 - 44 - 3 \\ -27 + 55 + 6 \\ 9 - 22 - 3 \end{bmatrix} = \begin{bmatrix} -29 \\ 34 \\ -16 \end{bmatrix}$$

so $\boxed{x = -29,\ y = 34,\ z = -16}$

Solve the problem.

20) Two boats leave a dock at the same time. One boat is headed directly east at a constant speed of 35 miles per hour, and the other is headed directly south at a constant speed of 22 miles per hour. Express the distance d between the boats as a function of the time t.



$D = rt$, so coordinates of EAST BOAT is $(35t, 0)$

& coordinates of south boat is $(0, -22t)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 35t)^2 + (-22t - 0)^2}$$

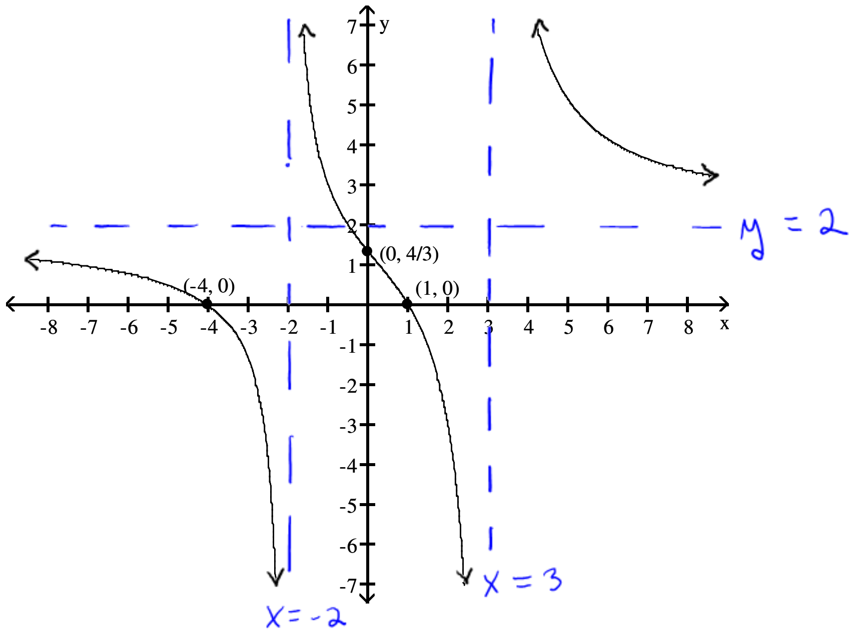
$$= \sqrt{(-35t)^2 + (-22t)^2} = \sqrt{1225t^2 + 484t^2} = \sqrt{1709t^2}$$

$$\boxed{d(t) = \sqrt{1709}\; t}$$

Answer Key
Testname: AB OE F09 V2

FORM A , open ended questions

11) a) $[-4, 4]$
b) $(-2, 0) \cup (2, 4)$
c) $x = 3$, $x = -3$
d) -3
e) even
12) f(x) = zeros: 1, $-5 + i$, $-5 - i$
13) 1210
14) a) $\{x \mid x \neq 3\ x \neq -2$
b) $(0, \frac{4}{3})$, $(1, 0)$, $(-4, 0)$
c) vertical: $x = 3$, $x = -2$; horizontal: $y = 2$
d)



15) $2x + h + 9$
16) $f^{-1}(x) = \frac{5x + 2}{3 - x}$; domain of f: $\{x \mid x \neq -5\}$; range of f: $\{y \mid y \neq 3\}$; domain of f^{-1}: $\{x \mid x \neq 3\}$; range of f^{-1}: $\{y \mid y \neq -5\}$
17) $\{x \mid x \neq -4, x \neq -8\}$
18) $\frac{(x - 3)^2}{64} + \frac{(y - 6)^2}{28} = 1$
19) $x = -29$, $y = 34$, $z = -16$; $(-29, 34, -16)$
20) $d(t) = \sqrt{1709}t$

PART III: Questions 21 – 30, Self select

Choose FIVE out of the next TEN questions to complete. You must show all your work and clearly indicate your answer for full credit. CROSS OUT the problems that you do not want graded. If no problems are crossed out, the first five problems showing work will be graded.

Write the partial fraction decomposition of the rational expression.

21) $\dfrac{12x + 3}{(x - 1)(x^2 + x + 1)}$

$$\frac{(12x+3)(x-1)(x^2+x+1)}{(x-1)(x^2+x+1)} = \frac{A(x-1)(x^2+x+1)}{x-1} + \frac{(Bx+C)(x-1)(x^2+x+1)}{x^2+x+1} \text{ OR}$$

$$\Rightarrow 12x+3 = A(x^2+x+1) + (Bx+C)(x-1)$$

if $x=1 \Rightarrow 15 = 3A \Rightarrow A = 5$

if $x=0 \Rightarrow 3 = A - C \Rightarrow C = 2$

if $x=-1 \Rightarrow -9 = A + (-B+C)(-2)$

$\Rightarrow -9 = 5 + (-B+2)(-2)$

$\Rightarrow B = -5$

$12x+3 = Ax^2 + Ax + A + Bx^2 - Bx + Cx - C$

x^2: $0 = A + B$

x: $12 = A - B + C$

Con: $3 = A - C$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 12 \\ 1 & 0 & -1 & 3 \end{array}\right] \Rightarrow \text{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right] \Rightarrow$$

$$\boxed{\text{SO } \frac{5}{x-1} + \frac{-5x+2}{x^2+x+1}}$$

Solve the problem.

22) James invested his inheritance in an account that paid 6.8% interest, compounded monthly. After 6 years, he found that he now had $48,194.12. What was the original amount of his inheritance? Round your answer to the nearest dollar.

COMPOUNDED MONTHLY $\Rightarrow A = P\left(1 + \frac{r}{n}\right)^{nt}$ & $r = .068$, $n = 12$, $t = 6$, $A = 48194.12$

$$48194.12 = P\left(1 + \frac{.068}{12}\right)^{(12 \cdot 6)} \Rightarrow 48194.12 = 1.502076296\,P$$

$$\Rightarrow P = \frac{48194.12}{1.502076296} = \boxed{\$32085.00}$$

The determinant of a 3 by 3 matrix is given below. Solve for x.

23)
$$\begin{vmatrix} 5 & -3 & 1 \\ -2 & -2 & x \\ 8 & 2 & -1 \end{vmatrix} = 28$$

DOWNS – UPS

$-16 + 10x - 6$

$$\left|\begin{matrix} 5 & -3 & 1 \\ -2 & -2 & x \\ 8 & 2 & -1 \end{matrix}\right| \begin{matrix} 5 & -3 \\ -2 & -2 \\ 8 & 2 \end{matrix}$$

$10 - 24x - 4$

$\Rightarrow 10 - 24x - 4 - (-16 + 10x - 6) = 28$

$\Rightarrow 10 - 24x - 4 + 16 - 10x + 6 = 28$

$\Rightarrow -34x + 28 = 28$

$ -28 \quad -28$

$\Rightarrow \dfrac{-34x}{-34} = \dfrac{0}{-34} \Rightarrow \boxed{x = 0}$

Solve the equation.

24) $\log_3(x - 5) + \log_3(x + 3) = 2$

$3^{\log_3[(x-5)(x+3)]} = 3^2 \Rightarrow [(x-5)(x+3)] = 9$

FOIL

$\Rightarrow x^2 - 2x - 15 = 9 \Rightarrow x^2 - 2x - 24 = 0$

$ -9 \quad -9$

$\Rightarrow (x - 6)(x + 4) = 0 \Rightarrow x = 6,\ x = -4$

but $x = -4 \Rightarrow \log_3(-4 - 5) = \log_3(-9)$

IS UNDEFINED SO ONLY $\boxed{x = 6}$ WORKS

Determine whether the infinite geometric series converges or diverges. If it converges, find its sum.

25) $1 - \frac{1}{4} + \frac{1}{16} - \cdots$ $\quad a_1 = 1,\ a_2 = -\frac{1}{4},\ a_3 = \frac{1}{16}$

$a_2 \div a_1 = -\frac{1}{4} \div 1 = -1/4$ & $a_3 \div a_2 = \frac{1}{16} \div -\frac{1}{4} = \frac{1}{6} \cdot -4 = -1/4$

SO GEOMETRIC WITH COMMON RATIO $r = -\frac{1}{4}$.

SINCE $|r| = |-\frac{1}{4}| = \frac{1}{4} < 1$. $\boxed{\text{SERIES CONVERGES}}$ &

$\boxed{S_\infty = \dfrac{a_1}{1 - r}} = \dfrac{1}{1 - (-1/4)} = \dfrac{1}{1 + 1/4} = \dfrac{1}{5/4} = \boxed{\dfrac{4}{5}}$

Solve the equation. Give an exact solution and also an approximate solution rounded to the nearest thousandth.

26) $7^{(1 + 2x)} = 5^{4x}$ $\Rightarrow \ln 7^{(1+2x)} = \ln 5^{4x} \Rightarrow (1+2x)\ln 7 = 4x\ln 5$

$\Rightarrow \ln 7 + 2x\ln 7 = 4x\ln 5 \Rightarrow \ln 7 = 4x\ln 5 - 2x\ln 7$

$-2x\ln 7$ $\quad -2x\ln 7$

$\Rightarrow \ln 7 = x(4\ln 5 - 2\ln 7) \Rightarrow \boxed{x = \dfrac{\ln 7}{4\ln 5 - 2\ln 7} \approx 0.764}$

$\dfrac{\ln 7}{4\ln 5 - 2\ln 7} = \dfrac{x(4\ln 5 - 2\ln 7)}{4\ln 5 - 2\ln 7}$

Find the center, transverse axis, vertices, and foci of the hyperbola. Sketch the graph. Your graph should include the asymptotes.

27) $\frac{y^2}{4} - \frac{x^2}{16} = 1$

$c^2 = 4 + 16 = 20$

$c = \sqrt{20} = 2\sqrt{5}$

center: $(0,0)$

transverse axis: $x = 0$

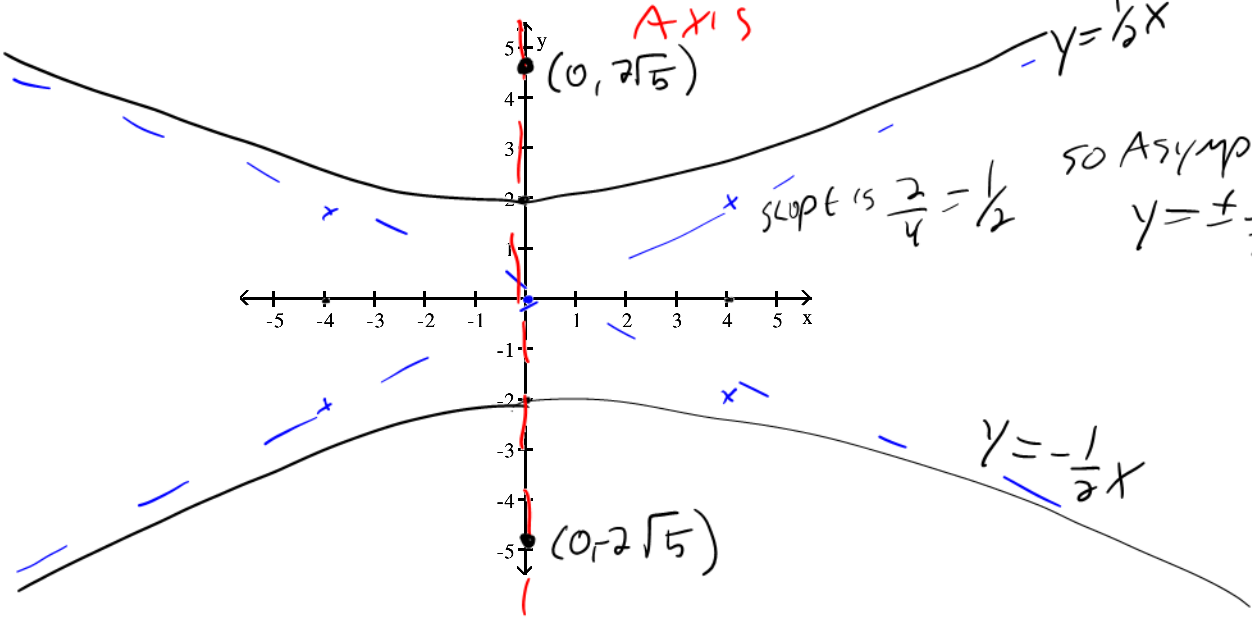
vertices: $(0,2), (0,-2)$

foci: $(0, 2\sqrt{5}), (0, -2\sqrt{5})$

TRANSVERSE AXIS

sketch:

← TRANSVERSE AXIS



so Asymptotes $y = \pm\frac{1}{2}x$

Form a polynomial f(x) with real coefficients having the given degree and zeros. Write the polynomial in standard form; do not leave it in factored form.

28) Degree: 3; zeros: –2 and 1 – 2i. , $1+2i$

$x = -2$, $x = 1 - 2i$, $x = 1 + 2i$ $\Rightarrow x + 2 = 0$, $x - 1 + 2i = 0$, $x - 1 - 2i = 0$

$\Rightarrow (x+2)(x - 1 + 2i)(x - 1 - 2i) = 0$

$\Rightarrow (x+2)(x^2 - x \cancel{- 2ix} - x + 1 \cancel{+ 2i} \cancel{+ 2ix} \cancel{- 2i} \overset{+4}{\cancel{-4i^2}}) = 0 \Rightarrow (x+2)(x^2 - 2x + 5) = 0$

$\Rightarrow x^3 \cancel{- 2x^2} + 5x \cancel{+ 2x^2} - 4x + 10 = \boxed{x^3 + x + 10 = 0}$

The sequence is defined recursively. Write the first five terms.

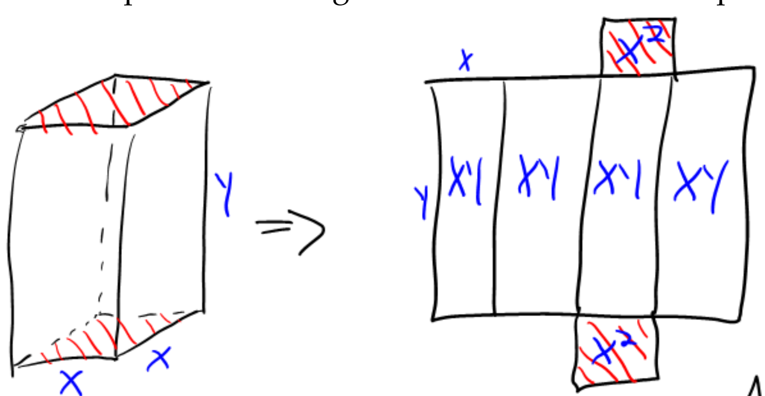
29) $a_1 = 2, \; a_2 = 5; \; a_n = a_{n-2} - 3a_{n-1}$

n	$a_n = a_{n-2} - 3a_{n-1}$
1	2
2	5
3	$a_3 = a_1 - 3a_2 = 2 - 3 \cdot 5 = -13$
4	$a_4 = a_2 - 3a_3 = 5 - 3 \cdot (-13) = 44$
5	$a_5 = a_3 - 3a_4 = -13 - 3(44) = -145$

so $\boxed{\{2, 5, -13, 44, -145, \ldots\}}$

Solve the problem.

30) A rectangular box with volume 311 cubic feet is built with a square base and top. The cost is \$1.50 per square foot for the top and the bottom and \$2.00 per square foot for the sides. Let x represent the length of a side of the base. Express the cost the box as a function of x.



AREA of top & BOTTOM

$= 2x^2$

Cost is $1.5 \cdot 2x^2 = 3x^2$

Area of sides

$= 4xy$

$\text{cost} = 2 \cdot 4xy = 8xy$

$V = x^2 y$

$311 = x^2 y \Rightarrow y = \frac{311}{x^2}$

TOTAL COST $C = 3x^2 + 8xy$

$= 3x^2 + 8\cancel{x}\left(\frac{311}{x^{\cancel{2}}}\right)$

$\boxed{\text{so } C(x) = 3x^2 + \frac{2488}{x}}$

Answer Key
Testname: AB SS F09 V2

FORM A self select

21) $\dfrac{5}{x - 1} + \dfrac{-5x + 2}{x^2 + x + 1}$

22) $32,085

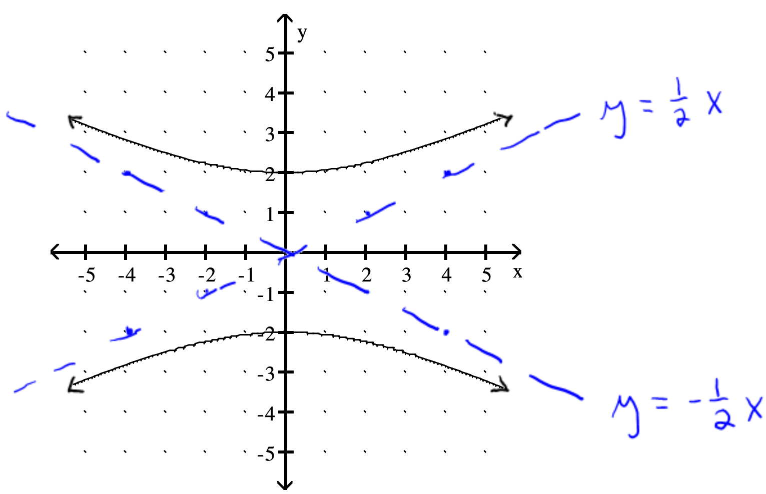
23) 0

24) x = 6, with x = −4 extraneous

25) Converges; $\dfrac{4}{5}$

26) $x = \dfrac{\ln 7}{4 \ln 5 - 2 \ln 7} \approx 0.764$

27) center at (0, 0)
transverse axis is y−axis
vertices: (0, −2), (0, 2)
foci: $(0, -2\sqrt{5}), (0, 2\sqrt{5})$
sketch:



28) $f(x) = x^3 + x + 10$

29) $a_1 = 2,\ a_2 = 5,\ a_3 = -13,\ a_4 = 44,\ a_5 = -145$

30) $C(x) = 3x^2 + \dfrac{2488}{x}$