Math 1010 Final Exam
Form B, Fall 2009

Name: KEY

Instructor: ______________________________ ID verification: ________

Each problem is equally weighted. Scientific calculators are permitted.
Time limit: Two hours.
Not allowed: notes, books, graphing/programable calculators, cell phones or other hand held devices.

Find the exact solution. If no solution exists, state this.

1) $2^{(2x + 1)} = 32$

A) $x = 4$ B) $x = 2$ C) $x = 3$ D) $x = 16$

$\log_2 2^{2x+1} = \log_2 2^5$

$2x+1 = 5$

$-1 \quad -1$

$\frac{2x}{2} = \frac{4}{2}$

$\boxed{x = 2}$

Simplify. Write the answer using positive exponents only. Leave the answer in exponential notation.

2) $\left(\frac{2x^3y^{-3}}{x^{-3}y^4}\right)^{-5} = \frac{2^{-5}x^{-15}y^{15}}{x^{15}y^{-20}} = \frac{y^{15}y^{20}}{2^5x^{15}x^{15}} = \boxed{\frac{y^{35}}{32x^{30}}}$

A) $\frac{-8x^{30}}{y^{35}}$ B) $\frac{y^{35}}{2x^6}$ C) $\frac{y^{35}}{2x^{30}}$ D) $\frac{y^{35}}{32x^{30}}$

Solve.

3) A helicopter goes 270 miles with the wind in the same time it can go 180 miles against the wind. The speed of the wind is 6 miles per hour. Find the speed of the helicopter with no wind.

R

A) 45 mph B) 30 mph C) 36 mph D) 24 mph

	D	R	T
with	270	R+6	T
Against	180	R−6	T

$270 = (R+6)T \Rightarrow T = \frac{270}{R+6}$

$180 = (R-6)T \Rightarrow (R+6)180 = (R-6)\left(\frac{270}{\cancel{R+6}}\right)\cancel{(R+6)}$

$180R + 1080 = 270R - 1620$

$-180R \quad +1620 \quad -180R \quad +1620$

$\frac{2700}{90} = \frac{90R}{90}$

$\boxed{R = 30 \text{ mph}}$

Solve for m.

4) $3m^2 + 8m + 1 = 0$

A) $m = \frac{-4 \pm \sqrt{13}}{6}$ B) $m = \frac{-4 \pm \sqrt{19}}{3}$ C) $m = \frac{-8 \pm \sqrt{13}}{3}$ D) $m = \frac{-4 \pm \sqrt{13}}{3}$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-8 \pm \sqrt{52}}{6} = \frac{\cancel{-8}^{-4} \pm \cancel{2}^{1}\sqrt{13}}{\cancel{6}_{3}} = \boxed{\frac{-4 \pm \sqrt{13}}{3}}$

$b^2 - 4ac = 8^2 - 4(3)(1)$
$= 64 - 12 = 52$

Solve.

5) $\sqrt{5q + 6} = 6$

A) $q = \frac{42}{5}$ B) $q = 36$ C) $q = 6$ D) $q = \frac{36}{5}$

$(\sqrt{5q+6})^2 = (6)^2$

$5q + 6 = 36$
$\quad -6 \quad -6$

$\boxed{q = 6}$

$\frac{5q}{5} = \frac{30}{5}$

6) John owns a hotdog stand. He has found that his profit is represented by the equation $P = -x^2 + 64x + 82$, with P being the profit in dollars, and x the number of hotdogs sold. How many hotdogs must he sell to earn the most profit?

A) 25 hotdogs B) 32 hotdogs C) 33 hotdogs D) 50 hotdogs

most profit ⇒ MAX = VERTEX

$x = \frac{-b}{2a} = \frac{-64}{2(-1)} = \boxed{32}$

Find the exact solution. If no solution exists, state this.

7) $\log_2 (3x - 3) = 1$

A) $x = 2$ B) $x = \frac{5}{3}$ C) $x = \frac{5}{4}$ D) No solution

$2^{\cancel{\log_2}(3x-3)} = 2^1$

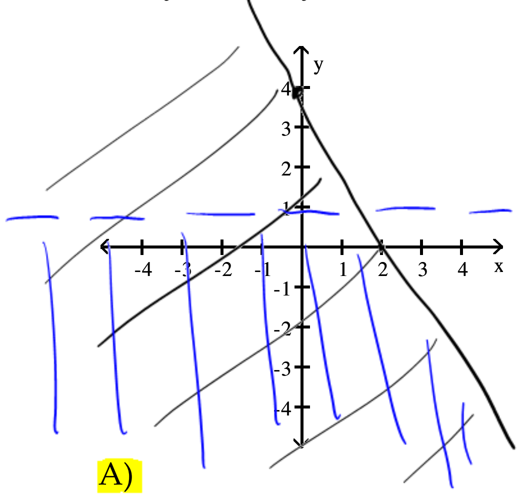
$3x - 3 = 2$
$\quad +3 \quad +3$

$\boxed{x = \frac{5}{3}}$

$\frac{3x}{3} = \frac{5}{3}$

Graph the system of linear inequalities.

8) $2x + y \le 4$ and $y - 1 < 0$



$2x+y \le 4$

x	y
0	4
2	0

shade below

SOLID

$y-1<0$

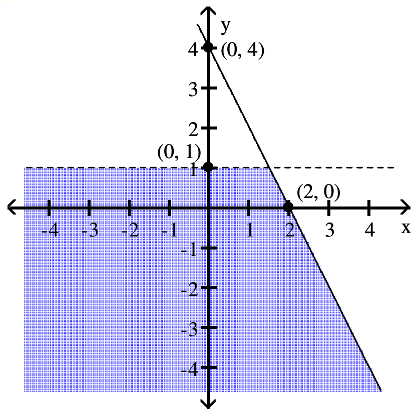
$y<1$

CROSSES y => HORIZONTAL

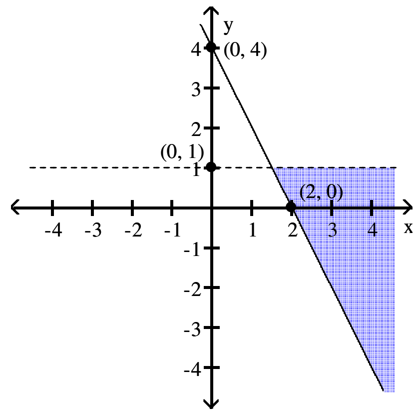
SHADES BELOW.

DASHED

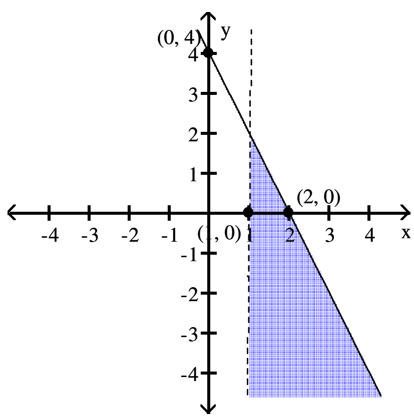
A)



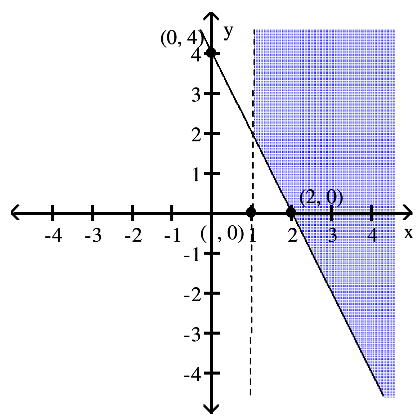
B)



C)



D)



Perform the indicated operation and simplify. Write the answer in the form a + bi.

9) $3i(6 - 2i)$

A) $18i - 6i^2$ B) $18i + 6i^2$ C) $-6 + 18i$ D) $6 + 18i$

$3i(6-2i)$

$18i - 6i^2$

$18i - 6(-1)$

$18i + 6$

$\boxed{6+18i}$

Find the center and the radius of the circle.

10) $x^2 + y^2 + 6x - 40 = 0$

A) $(-3, 0)$, $r = 7$ B) $(-3, 0)$, $r = 49$ C) $(3, 0)$, $r = 7$ D) $(3, 0)$, $r = 49$

$x^2 + 6x + 9 + y^2 = 40 + 9$

$(x+3)^2 + y^2 = 49$

cen $(-3, 0)$

$r = \sqrt{49} = 7$

Find the distance between the pair of points. Give your answer in exact form.

11) $(-3, 2)$ and $(1, -4)$

A) $2\sqrt{13}$ B) $20\sqrt{5}$ C) -2 D) 10

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(1-3)^2 + (-4-2)^2}$

$d = \sqrt{(4)^2 + (-6)^2}$

$= \sqrt{16+36}$

$d = \sqrt{52}$

$4 \cdot 13$

$\boxed{d = 2\sqrt{13}}$

Solve the equation.

12) $x^3 + 10 = 10x^2 + x$

A) $\{10, 1, -10\}$ B) $\{-1, 1, -10\}$ C) $\{10, 1\}$ D) $\{10, -1, 1\}$

$x^3 + 10 = \cancel{10x^2} + \cancel{x}$

$-10x^2 - x \quad -10x^2 \quad -x$

$\underbrace{x^3 - 10x^2}\ \underbrace{-x + 10} = 0$

$x^2(x-10) - 1(x-10) = 0$

$(x-10)(x^2-1) = 0$

$(x-10)(x+1)(x-1) = 0$

$x = 10, -1, 1$

13) $|5x + 8| = |x - 1|$

A) $\left\{-\frac{9}{4}, -\frac{7}{6}\right\}$ B) $\varnothing$ C) $\left\{\frac{9}{4}, \frac{7}{6}\right\}$ D) $\left\{-\frac{9}{4}\right\}$

2 pieces

$5x + 8 = x - 1$

$-x \quad -8 \quad -x \quad -8$

$\frac{4x}{4} = \frac{-9}{4}$

$\boxed{x = \frac{-9}{4}}$

$5x + 8 = -x + 1$

$+x \quad -8 \quad +x \quad -8$

$\frac{6x}{6} = \frac{-7}{6}$

$\boxed{x = \frac{-7}{6}}$

Solve the problem.

14) The number of bacteria growing in an incubation culture increases with time according to $B(x) = 2500(3)^x$, where x is time in days.

Find the number of bacteria when $x = 0$ and $x = 4$.

Should be 4

A) $B(0) = 2500,\ B(2) = 202{,}500$ B) $B(0) = 2500,\ B(2) = 67{,}500$

C) $B(0) = 7500,\ B(2) = 202{,}500$ D) $B(0) = 2500,\ B(2) = 30{,}000$

$$B(0) = 2500(3)^0 = 2500(1) = 2500$$

$$B(4) = 2500(3)^4 = 2500(81) = 202500$$

Multiply.

15) $(2\sqrt{2} + 7\sqrt{5})(6\sqrt{2} + 5\sqrt{5})$

A) $12\sqrt{2} + 35\sqrt{5}$ B) $12\sqrt{2} + 35\sqrt{5} + 52\sqrt{10}$

C) $-151 + 52\sqrt{10}$ D) $199 + 52\sqrt{10}$

$$(2\sqrt{2} + 7\sqrt{5})(6\sqrt{2} + 5\sqrt{5})$$

$$\overset{F}{12\sqrt{4}} + \overset{O}{10\sqrt{10}} + \overset{I}{42\sqrt{10}} + \overset{L}{35\sqrt{25}}$$

$$12(2) + 52\sqrt{10} + 35(5)$$

$$24 + 52\sqrt{10} + 175$$

$$\boxed{199 + 52\sqrt{10}}$$

Find an equation of the line containing the given pair of points. Write your final answer as a linear function in slope–intercept form.

16) (5, −5) and (2, 1)

A) $f(x) = -2x + 2$ B) $f(x) = 2x + 5$ C) $f(x) = -2x + 5$ D) $f(x) = 5x - 2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - {}^-5}{2 - 5} = \frac{6}{-3} = -2$$

$$y - y_1 = m(x - x_1) \Rightarrow y - {}^-5 = -2(x - 5)$$

$$y + 5 = -2x + 10$$

$$\underline{-5} \qquad \underline{-5}$$

$$y = -2x + 5$$

$$\boxed{f(x) = -2x + 5}$$

Find the function value.

17) Find $f(-4)$ when $f(x) = x^2 - 5x + 2$.

A) $f(-4) = 38$ B) $f(-4) = -2$ C) $f(-4) = 34$ D) $f(-4) = 6$

$$f(-4) = (-4)^2 - 5(-4) + 2$$

$$= 16 + 20 + 2$$

$$= \boxed{38}$$

Rationalize the denominator. Assume all variables represent positive numbers.

18) $\dfrac{5\sqrt{x}}{\sqrt{x} + 2\sqrt{y}}$

A) $\dfrac{5x + 10\sqrt{xy}}{x + 2y}$ B) $\dfrac{5x + 10\sqrt{xy}}{x + 4y}$ C) $\dfrac{5x - 10\sqrt{xy}}{x - 2y}$ D) $\dfrac{5x - 10\sqrt{xy}}{x - 4y}$

$$\frac{5\sqrt{x}}{(\sqrt{x}+2\sqrt{y})} \cdot \frac{(\sqrt{x}-2\sqrt{y})}{(\sqrt{x}-2\sqrt{y})} = \frac{5x-10\sqrt{xy}}{x - \cancel{2\sqrt{xy}} + \cancel{2\sqrt{xy}} - 4y} = \boxed{\frac{5x-10\sqrt{xy}}{x-4y}}$$

FOIL

Solve the problem.

19) To make jewelry, Anne wishes to mix a metal alloy that is 22% copper with an alloy that is 25% copper to form 63 ounces of an alloy that is 24% copper. How many ounces of the 22% copper alloy must be used?

A) 21 ounces B) 42 ounces C) 47 ounces D) 23 ounces

x = oz of 22%

y = oz of 25%

$x + y = 63 \Rightarrow y = (63 - x)$

$.22x + .25y = .24(63)$

$.22x + .25(63-x) = 15.12$

$.22x + 15.75 - .25x = 15.12$

$-15.75 \qquad -15.75$

$-.03x = -0.63$

$\frac{-.03x}{-.03} = \frac{-0.63}{-.03}$

$\boxed{x = 21\text{ oz}}$

Multiply or divide as indicated. Simplify completely.

20) $\dfrac{x^3 + 1}{x^3 - x^2 + x} \div \dfrac{-12x - 12}{6x}$

A) $-\dfrac{x^2 + 1}{2}$ B) $-\dfrac{1}{2}$ C) $-\dfrac{x^3 + 1}{2(x + 1)}$ D) $\dfrac{x + 1}{2(-x - 1)}$

$$\frac{x^3+1}{x^3-x^2+x} \div \frac{-12x-12}{6x} \Rightarrow \frac{x^3+1}{x^3-x^2+x} \cdot \frac{6x}{-12x-12} \Rightarrow \frac{\cancel{(x+1)}\cancel{(x^2-x+1)}}{\cancel{x}\cancel{(x^2-x+1)}} \cdot \frac{\cancel{6x}}{\underset{-2}{\cancel{-12}}\cancel{(x+1)}}$$

$$\Rightarrow \boxed{\frac{1}{-2}}$$

Solve the system for z.

21) $4x - y + 3z = 12$
$2x + 9z = -5$
$x + 4y + 6z = -32$

A) $z = 1$ B) $z = -1$ C) $z = 2$ D) $z = -2$

$4R1 + R3$

$16x - 4y + 12z = 48$
$+\; x + 4y + 6z = -32$
$17x + 18z = 16$

$17x + 18z = 16$
$(2x + 9z = -5)\,{-2}$

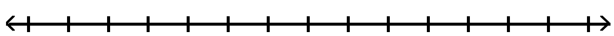
$17x + 18z = 16$
$+\; -4x - 18z = 10$
$13x = 26$
$\frac{13x}{13} = \frac{26}{13}$
$x = 2$

$2x + 9z = -5$
$2(2) + 9z = -5$

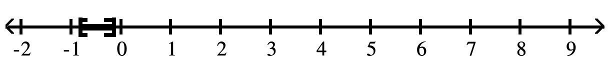
$4 + 9z = -5$
$-4 \qquad -4$
$9z = -9$
$\frac{9z}{9} = \frac{-9}{9}$
$\boxed{z = -1}$

Solve and graph.

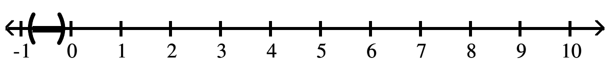
22) $|6k + 3| \le 2$



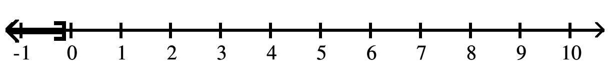
A) $\left[-\frac{5}{6}, -\frac{1}{6}\right]$



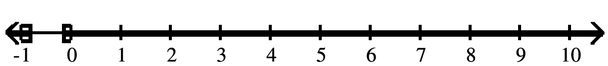
B) $\left(-\frac{5}{6}, -\frac{1}{6}\right)$



C) $\left(-\infty, -\frac{1}{6}\right]$



D) $\left(-\infty, -\frac{5}{6}\right] \cup \left[-\frac{1}{6}, \infty\right)$

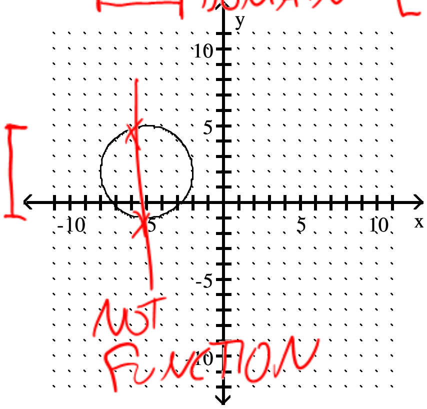


$-2 \le 6k + 3 \le 2$
$-3 \qquad -3 \qquad -3$
$-5 \le 6k \le -1$
$\frac{-5}{6} \le \frac{6k}{6} \le \frac{-1}{6}$
$-5/6 \le k \le -1/6$

$\boxed{\left[-5/6, -1/6\right]}$

Find the domain and the range of the relation. Use the vertical line test to determine whether the graph is the graph of a function.

23) -8 -2 DOMAIN [-8,-2]



RANGE 5 -1 [-1,5]

NOT FUNCTION

A) domain: [−8, −2]
range: [−1, 5]
not a function

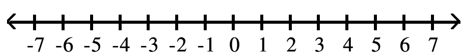
B) domain: [−1, 5]
range: [−8, −2]
function

C) domain: [−1, 5]
range: [−8, −2]
not a function

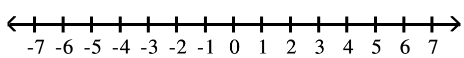
D) domain: [−8, −2]
range: [−1, 5]
function

Solve the inequality and graph the solution set.

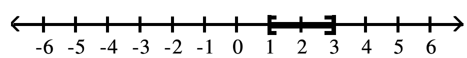
24) $12x - 8 < 4x$ or $-4x \le -12$



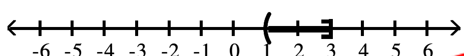
A) ∅



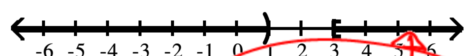
B) [1,3]



C) (1,3)



D) $(-\infty,1) \cup [3,\infty)$



$12x - 8 = 4x$

$8x = 8$

$x = 1$

$-4x = -12$

$x = 3$

TEST 0 2 5

1 3

$12(0) - 8 < 4(0)$

$-8 < 0$

Yes

$12(2) - 8 < 4(2)$

$24 - 8 < 8$ NO

$-4(2) \le -12$

$-8 \le -12$ NO

$12(5) - 8 < 4(5)$

$60 - 8 < 20$ NO

$-4(5) \le -12$

$-20 \le -12$ YES

Find the x– and y–intercepts. If no x–intercepts exist, state so.

25) $f(x) = x^2 + 12x$

A) No x–intercept; y–intercept (0,0)

B) x–intercepts (0, −12) and (−12, 0); y–intercept (0,0)

C) x–intercepts (0, 0) and (−12, 0); y–intercept (0,0)

D) x–intercepts (0, 0) and (12, 0); y–intercept (0,0)

$f(0) = 0^2 + 12(0) = 0$

$(0,0)$

$0 = x^2 + 12x$

$0 = x(x+12)$

$x = 0, -12$

$(0,0), (-12,0)$

Simplify.

26)

$$\frac{\frac{1}{x}+\frac{4}{x^2}}{x+\frac{64}{x^2}}$$

A) $\frac{1}{x^2+4x+16}$ B) $\frac{x+4}{x^2+64}$ C) $\frac{1}{x^2+16}$ D) $\frac{1}{x^2-4x+16}$

$$\frac{x^2\cdot\frac{1}{x}+\frac{4}{x^2}\cdot x^2}{x^2\cdot\frac{x}{1}+\frac{64}{x^2}\cdot x^2} \Rightarrow \frac{x+4}{x^3+64} \Rightarrow \frac{\cancel{x+4}^{\,1}}{\cancel{(x+4)}(x^2-4x+16)} \Rightarrow \frac{1}{x^2-4x+16}$$

Find the domain of the function h.

27) $h(x)=\frac{x-1}{x^2+5x-14}$

A) {x| x is a real number and $x \neq -2$ and $x \neq 0$}
B) {x| x is a real number and $x \neq -7$ and $x \neq 2$ and $x \neq 1$}
C) {x| x is a real number and $x \neq -7$ and $x \neq 2$}
D) {x| x is a real number and $x \neq 0$}
E) {x| x is a real number and $x \neq -2$ and $x \neq 0$ and $x \neq -7$ and $x \neq 0$}

Den $\neq 0$ $x^2+5x-14=0$

$(x+7)(x-2)=0$

$x=-7,\ x=2$

DOMAIN $\{x / x \neq -7, x \neq 2\}$

For the pair of functions f and g, find all values of x for which f(x) = g(x).

28) $f(x)=\frac{x-2}{28}$, $g(x)=\frac{1}{x+1}$

A) x = 2, −1 B) x = 6, −5 C) x = 27, 2 D) x = −1, 30

$f(x)=g(x)$

$28(x+1)\cdot\frac{(x-2)}{\cancel{28}}=\frac{1}{\cancel{(x+1)}}\cdot 28(\cancel{x+1})$

$(x-2)(x+1)=28$

FOIL

$x^2-x-2=28$

$-28 \quad -28$

$x^2-x-30=0$

$(x-6)(x+5)=0$

$\boxed{x=6,-5}$

Perform the indicated operation and simplify.

29) $\frac{a+b}{a-b} - \frac{3ab+3b^2}{a^2-b^2}$

A) $\frac{a-2b}{a-b}$ B) $\frac{a^2-2ab-2b^2}{a^2-b^2}$ C) $\frac{a+2b}{a-b}$ D) $\frac{a-2b}{a+b}$

$$\frac{a+b}{a-b} - \frac{3ab+3b^2}{a^2-b^2} \Rightarrow \frac{a+b}{a-b} - \frac{3b(\cancel{a+b})}{(\cancel{a+b})(a-b)} \Rightarrow \frac{a+b}{a-b} - \frac{3b}{a-b} \Rightarrow$$

$$\Rightarrow \frac{a+b-3b}{a-b} \Rightarrow \boxed{\frac{a-2b}{a-b}}$$

Find an equation for the described linear function.

30) Through $\left(0, \frac{1}{3}\right)$ and parallel to $5x - 8y = 2$

A) $y = \frac{5}{8}x + \frac{1}{3}$ B) $y = \frac{8}{5}x + \frac{1}{3}$ C) $y = -5x + \frac{1}{3}$ D) $y = -\frac{5}{8}x + \frac{1}{3}$

$5x - 8y = 2$

$-5x \qquad -5x$

$-8y = -5x + 2$

$\overline{-8} \qquad \overline{-8} \qquad \overline{-8}$

$y = \frac{5}{8}x - \frac{1}{4}$

$m = 5/8$ so parallel $m = 5/8$

(perpendicular $m = \frac{-8}{5}$)

$y - y_1 = m(x - x_1)$

$y - \frac{1}{3} = \frac{5}{8}(x - 0)$

$y - \frac{1}{3} = 5/8 x$

$+1/3 \qquad +1/3$

$\boxed{y = 5/8 x + \frac{1}{3}}$