Salt Lake Community College
MATH 1050 Final Exam Form B
Fall Semester 2008

Name: KEY **Instructor:** ______________

Student ID: ____________ **ID Verification:** ____________ **Section Number:** ______________

This exam has three parts: Part I -- Ten Multiple choice questions
Part II -- Ten open ended questions - You **must show** all your work.
Part III -- Choose five out of ten open ended questions - You **must show** your work and indicate which five problems are to be graded.

You are **NOT allowed** to use **books** or **notes**.

Part I
Questions 1 - 10 Multiple Choice
Answer all TEN questions and circle the most correct answer.

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1) Determine whether the infinite geometric series converges or diverges. If it converges, find its sum.

$4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \ldots$

A) Converges; 3 B) Converges; $\frac{8}{3}$

C) Converges; $-\frac{4}{3}$ D) Diverges

2) Find the amount that results from \$12,000 invested at 7% compounded quarterly after a period of 3 years.

A) \$14,523.12 B) \$14,777.27 C) \$14,700.52 D) \$2777.27

3) Write the equation that results if the square root function is shifted 7 units to the right.

A) $y = \sqrt{x} + 7$ B) $y = \sqrt{x} - 7$ C) $y = \sqrt{x + 7}$ D) $y = \sqrt{x - 7}$

1. $4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \cdots$ is an infinite sum, so $S_\infty = \frac{a_1}{1-r}$

we need r: $-\frac{4}{3} \div 4 = -\frac{1}{3}$, $\frac{4}{9} \div -\frac{4}{3} = -\frac{1}{3}$ so $r = -1/3$

& $S_\infty = \frac{4}{1 - -1/3} = \frac{4}{1+1/3} = \frac{4}{4/3} = 4 \cdot \frac{3}{4} = \boxed{3}$

2. $A = P\left(1 + \frac{r}{n}\right)^{(nt)}$ & $P = 12000$, $r = 0.07$, $n = 4$ (quarterly), $t = 3$

So $A = 12000\left(1 + \frac{.07}{4}\right)^{(4 \cdot 3)} = \boxed{\$14777.27}$

3. FROM THE MULTIPLE CHOICE,

$\sqrt{x} + 7$ is up 7

$\sqrt{x} - 7$ is down 7

$\sqrt{x+7}$ is Left 7

$\sqrt{x-7}$ is Right 7.

so $\boxed{y = \sqrt{x-7}}$

4) A projectile is thrown upward so that its distance above the ground after t seconds is $h = -11t^2 + 462t$. After how many seconds does it reach its maximum height?

A) 31.5 sec B) 21.0 sec C) 10.0 sec D) 42.0 sec

5) Form a polynomial whose zeros and degree are given. Zeros: 2, multiplicity 2; -2, multiplicity 2; degree 4

A) $f(x) = x^4 - 4x^3 + 8x^2 - 8x + 16$
B) $f(x) = x^4 - 8x^2 + 16$
C) $f(x) = x^4 + 8x^2 + 16$
D) $f(x) = x^4 + 4x^3 - 8x^2 + 8x - 16$

6) Give the equation of the oblique asymptote of $g(x) = \dfrac{x^2 + 7x - 2}{x - 2}$

A) $y = x + 9$ B) $y = x - 2$ C) $y = 1$ D) $y = x - 9$

7) Perform the indicated operations and simplify.

Let $A = \begin{bmatrix} 0 & -1 \\ 2 & 0 \\ 6 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 6 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} -6 & 1 & 2 \\ 0 & -6 & -1 \end{bmatrix}$. Find C(A - B).

A) $\begin{bmatrix} 14 & 6 \\ 6 & -6 \end{bmatrix}$ B) $\begin{bmatrix} 6 & -6 \\ 14 & 6 \end{bmatrix}$ C) $\begin{bmatrix} -11 & 5 \\ -6 & 6 \end{bmatrix}$ D) $\begin{bmatrix} -22 & 6 \\ 7 & 4 \end{bmatrix}$

8) Evaluate $(f \circ g)(4)$ using the values given in the table.

x	1	5	8	12
f(x)	-2	8	2	13

x	-5	-2	1	4
g(x)	1	-5	5	8

A) 8 B) 2 C) 5 D) Undefined

9) Find the vertex and axis of symmetry of the graph of $f(x) = -7x^2 - 14x - 3$.

A) (2, -59) ; x = 2
B) (-2, -17) ; x = -2
C) (-1, 4) ; x = -1
D) (-1,-3) ; x = -1

4. $h = -11t^2 + 462t$, max @ vertex.

$t = \frac{-b}{2a} = \frac{-462}{-22} = \boxed{21 \text{ seconds}}$

(had they asked for the max height, I would plug $t = 21$ into the $h =$ function)

5. $x = 2$, multiplicity 2 $\Rightarrow (x-2)^2$

$x = -2$, mult 2 $\Rightarrow (x+2)^2$

$\Rightarrow \boxed{y = (x+2)^2(x-2)^2}$ FOIL

$\Rightarrow y = (x^2+4x+4)(x^2-4x+4)$ DISTRIBUTE

$\boxed{y = x^4 - 8x^2 + 16}$

6. OBLIQUE Asymptote? $\Rightarrow$ LONG DIVISION

$$\Rightarrow \begin{array}{r|l} & x + 9 \\ \hline x-2 & x^2 + 7x - 2 \\ & -x^2 \pm 2x \\ \hline & 9x - 2 \\ & -9x \pm 18 \\ \hline & 16 \end{array}$$

16 Remainder

$\Rightarrow \boxed{y = x+9}$

7. First, $A - B = \begin{bmatrix} 0 & -1 \\ 2 & 0 \\ 6 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & -1 \\ 0 & 0 \end{bmatrix}$, so $C(A-B) = \begin{bmatrix} -6 & 1 & 2 \\ 0 & -6 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & -1 \\ 0 & 0 \end{bmatrix} \Rightarrow$

2×3 3×2 possible

$$\Rightarrow \begin{bmatrix} -6(2) + 1(1) + 2(0) & (-6)(-1) + 1(-1) + 2(0) \\ 0(2) + (-6)(1) + (-1)(0) & 0(-1) + (-6)(-1) + (-1)(0) \end{bmatrix} = \begin{bmatrix} -11 & 5 \\ -6 & 6 \end{bmatrix}$$

8. $(f \circ g)(4) = f(g(4))$. Now $g(4) = 8$ (by 2nd table)

so $f(g(4)) = f(8) = \boxed{2}$ (by 1st table)

9. vertex: $x = \frac{-b}{2a} = \frac{-(-14)}{2(-7)} = \frac{14}{-14} = -1$; $y = -7(-1)^2 - 14(-1) - 3 = -7 + 14 - 3 = 4$

so $\boxed{\text{vertex } (-1, 4)}$ & Axis of symmetry is $\boxed{x = -1}$

10) Find the domain of $f(x) = \sqrt{10 - x}$

A) $\{x|x \neq \sqrt{10}\}$ B) $\{x|x \neq 10\}$ C) $\{x|x \leq \sqrt{10}\}$ D) $\{x|x \leq 10\}$

$10 - x \geq 0 \Rightarrow 10 \geq x \Rightarrow x \leq 10 \Rightarrow \{x \mid x \leq 10\}$

Part II

Question 11 - 20 Open Ended

Answer all TEN questions. To receive full credit, you must show all your work . It must be neat and well organized. Clearly indicate your final answer.

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11) Solve the system using the inverse matrix method.

$$\begin{cases} x + 2y + 3z = -7 \\ x + y + z = 10 \\ 2x + 2y + z = 11 \end{cases}$$

$Ax = b \Rightarrow x = A^{-1} \cdot b$

The inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ is $\begin{bmatrix} -1 & 4 & -1 \\ 1 & -5 & 2 \\ 0 & 2 & -1 \end{bmatrix}$. so $x = \begin{bmatrix} -1 & 4 & -1 \\ 1 & -5 & 2 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -7 \\ 10 \\ 11 \end{bmatrix}$

$$= \begin{bmatrix} (-1)(-7) + 4(10) + (-1)(11) \\ 1(-7) + (-5)(10) + (2)(11) \\ 0(-7) + 2(10) + (-1)(11) \end{bmatrix} = \boxed{\begin{bmatrix} 36 \\ -35 \\ 9 \end{bmatrix}}$$

or $(36, -35, 9)$

12) Solve $\log_3 x + \log_3(x - 24) = 4$.

$\Rightarrow \log_3 x(x-24) = 4 \Rightarrow x(x-24) = 3^4 \Rightarrow x^2 - 24x = 81$

3 3

$\Rightarrow x^2 - 24x - 81 = 0 \Rightarrow (x - 27)(x + 3) = 0 \Rightarrow x = -3, 27$

But $x = -3$ leads to $\log_3(-3)$ which is undefined, so

$\boxed{x = 27}$

13) Write the partial fraction decomposition of $\frac{10x + 2}{(x - 1)(x^2 + x + 1)}$. $= \frac{A}{x-1} + \frac{(Bx+C)}{x^2+x+1}$

$$\Rightarrow 10x+2 = A(x^2+x+1) + (x-1)(Bx+C) \Rightarrow 10x^2+2 = Ax^2+Ax+A+Bx^2-Bx+Cx-C$$

$$\Rightarrow 10x+2 = (A+B)x^2+(A-B+C)x + (A-C)$$

$$\Rightarrow \begin{array}{l} A+B = 0 \\ A-B+C = 10 \\ A - C = 2 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 10 \\ 1 & 0 & -1 & 2 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array}\right] \Rightarrow \begin{array}{l} A=4 \\ B=-4 \\ C=2 \end{array}$$

$$\text{so } \boxed{\frac{4}{x-1} + \frac{-4x+2}{x^2+x+1}}$$

14) Find the inverse function of $f(x) = \frac{3x - 2}{x + 5}$. State the domain and range of **f** and **f-1** .

$$x = \frac{3y-2}{y+5} \Rightarrow x(y+5) = 3y-2 \Rightarrow$$

$$\Rightarrow xy+5x = 3y-2 \Rightarrow xy-3y = -5x-2$$

$$\Rightarrow y(x-3) = -5x-2 \Rightarrow \boxed{y = \frac{-5x-2}{x-3}}$$

Domain of f: $x \neq -5$

Range of f: $y \neq 3$

$f^{-1}(x) = \frac{-5x-2}{x-3}$

Domain of f^{-1}: $x \neq 3$

Range of f^{-1}: $y \neq -5$

$x^2 \Rightarrow x=0$ $\quad x^2-3=0 \Rightarrow x^2=3 \Rightarrow x=\sqrt{3},\ x=-\sqrt{3}$

15) For the polynomial, $f(x) = \frac{1}{5}x^2(x^2 - 3)(x - 3)$,list each real zero and its multiplicity.

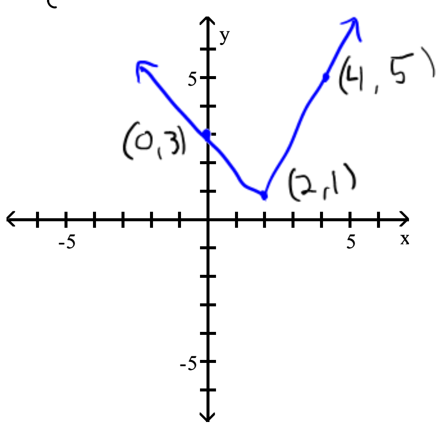
Determine whether the graph crosses or touches the x-axis at each x -intercept.

Zero	Multiplicity	Touch or Cross
0	2	T
$\sqrt{3}$	1	C
$-\sqrt{3}$	1	C
3	1	C

$x^2-3 = (x+\sqrt{3})^1(x-\sqrt{3})^1$

16) Graph the function. Label at least three points on the graph of f.

$$f(x) = \begin{cases} -x + 3 & \text{if } x < 2 \quad f1 \\ 2x - 3 & \text{if } x \geq 2 \quad f2 \end{cases}$$



x	f1	
0	3	
2	1	hollow

x	f2	
2	1	solid
4	5	

17) Find the center, foci, and vertices of the ellipse. Draw the graph.

$$\frac{(x+2)^2}{16} + \frac{(y-2)^2}{9} = 1$$

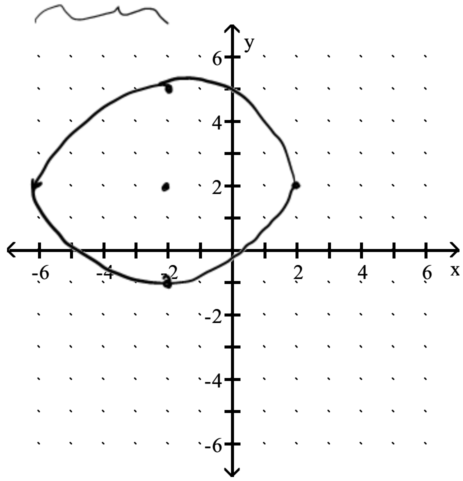
Center: $(-2,2)$

Foci: $(-2\pm\sqrt{7}, 2)$ Vertices: $(2,2), (-6,2)$

$a = 4$ $b = 3$

a

b



cen@ $(-2,2)$

$c^2 = a^2 - b^2 = 16 - 9 = 7 \Rightarrow c = \sqrt{7}$

L,R so $(-2\pm\sqrt{7}, 2)$ Focus

vertices end of major axis,
so L,R 4

18) Find the inverse of the matrix by hand. **Do not** use your calculator**.** Show your work.

$$\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

$$\begin{array}{c} 3 \\ -5 \end{array} \left[\begin{array}{cc|cc} 5 & 3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right] \Rightarrow \begin{array}{rrrr} 15 & 9 & 3 & 0 \\ +\ -15 & -10 & 0 & -5 \\ \hline 0 & -1 & 3 & -5 \end{array}$$

$$\begin{array}{c} 1 \\ 3 \end{array} \left[\begin{array}{cc|cc} 5 & 3 & 1 & 0 \\ 0 & -1 & 3 & -5 \end{array}\right] \Rightarrow \begin{array}{rrrr} 5 & 3 & 1 & 0 \\ +0 & -3 & 9 & -15 \\ \hline 5 & 0 & 10 & -15 \end{array}$$

$$\begin{array}{c} \frac{1}{5} \\ -1 \end{array} \left[\begin{array}{cc|cc} 5 & 0 & 10 & -15 \\ 0 & -1 & 3 & -5 \end{array}\right] \Rightarrow \begin{array}{rrrr} 1 & 0 & 2 & -3 \\ 0 & 1 & -3 & 5 \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & -3 & 5 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

19) Use the given zero to find the remaining zeros of the function.

$f(x) = x^3 - 2x^2 - 11x + 52$; zero: -4

	1	-2	-11	52
-4	1	-6	13	0

$\Rightarrow x^2 - 6x + 13 = 0$

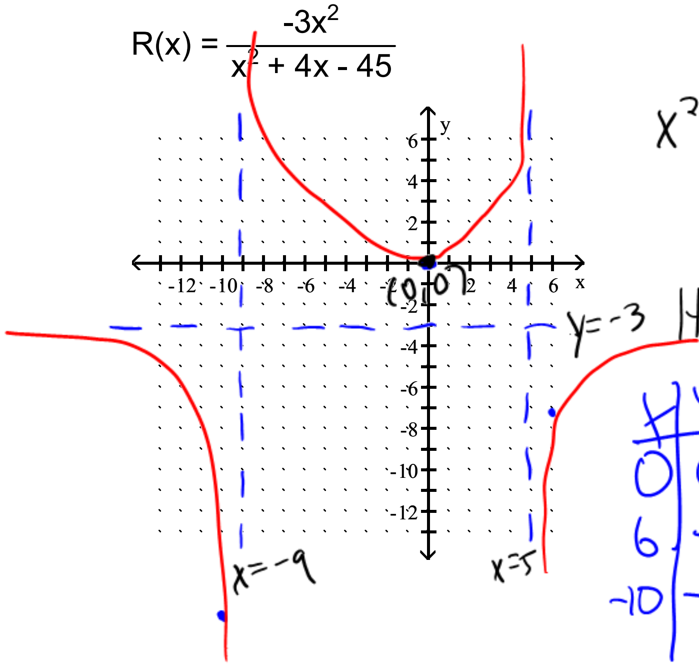
$x = \dfrac{6 \pm \sqrt{36-52}}{2} \Rightarrow x = \dfrac{6}{2} \pm \dfrac{\sqrt{-16}}{2} \Rightarrow x = 3 \pm \dfrac{4i}{2}$

$\Rightarrow x = \boxed{3 \pm 2i}$

20) Find the **domain** of the rational function. Draw the **graph**. **Label** all the intercepts and asymptotes.

$R(x) = \dfrac{-3x^2}{x^2 + 4x - 45}$

Domain: $\{x \mid x \neq -9, x \neq 5\}$



$x^2 + 4x - 45 = 0 \Rightarrow (x+9)(x-5) = 0$

$\Rightarrow \boxed{x = -9, 5}$ are Vertical Asymptotes &

Horizontal: $\dfrac{-3x^2}{x^2} = \boxed{-3 = y}$

x	y
0	0
6	-7.2
-10	-20

Part III

Questions 21 - 30 Self Select

Choose FIVE of the next TEN question to complete. To receive full credit, you must show all your work . It must be neat and well organized. Clearly indicate your final answer. CROSS OUT the problems that you do not wish to be graded.

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21) Use the graph of f to do the following:
Find the intervals on which it is increasing, decreasing, or constant.

Increasing $(-1,1)$

Decreasing $(-3,-2)\cup(2,4)$

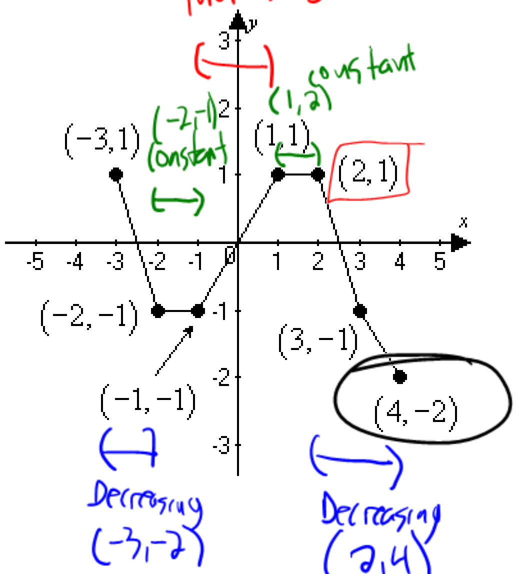
Constant $(-2,-1)\cup(1,2)$

Find f(2). $\boxed{=1}$ Increasing $(-1,1)$

Find x when f(x) = -2.

$f(4)=-2$

$\boxed{\text{so } x=4}$



22) Express as a single logarithm.

$4 \log_6 4 + \frac{1}{4} \log_6 (x - 3) - \frac{1}{2} \log_6 x \Rightarrow \log_6 4^4 + \log_6 (x-3)^{1/4} - \log_6 x^{1/2}$

$$\Rightarrow \boxed{\log_6 \frac{4^4(x-3)^{1/4}}{x^{1/2}}}$$

23) The half-life of silicon-32 is 710 years. If 70 grams is present now, how much will be present in 400 years? (Round your answer to three decimal places.)

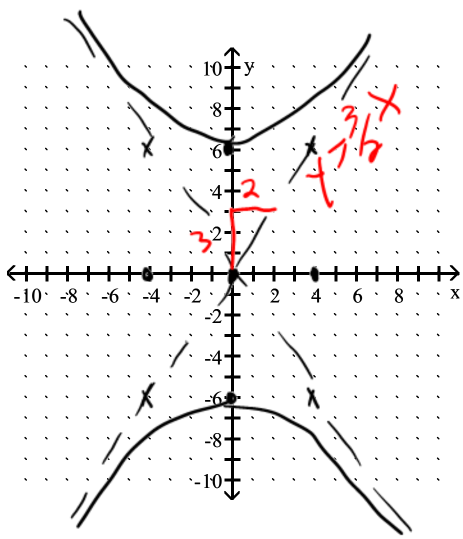
$Q = Q_0 e^{kt} \Rightarrow$ if $Q_0 = 70$, after a halflife would be $Q = 35$,

so $35 = 70e^{k(710)} \Rightarrow \frac{35}{70} = e^{710k} \Rightarrow \ln\left(\frac{35}{70}\right) = 710k \Rightarrow$

$\Rightarrow k = \frac{1}{710}\ln\left(\frac{35}{70}\right) \approx -0.0097626\,4$ (stored in calc)

400yrs: $Q = 70e^{k(400)} \Rightarrow \boxed{Q = 47.370 \text{ grams}}$ (using stored k)

24) Find an **equation** for the hyperbola with vertices at (0, ±6) and one of its asymptotes the line y = $\frac{3}{2}$x. Draw the **graph**.



cen (0,0) is ½ way between & hyperbola opens up/down, so y^2 term first, and a=6

$$\frac{y^2}{36} - \frac{x^2}{b^2} = 1$$

we also know $\frac{6}{b} = \frac{3}{2}$ from asymptote

so $b = 4$ & $b^2 = 16$

& $\boxed{\frac{y^2}{36} - \frac{x^2}{16} = 1}$

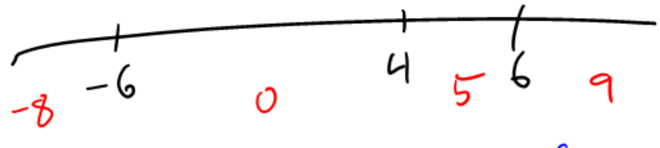
25) Solve $\dfrac{(x-4)^2}{x^2-36} > 0$. Express the solution using interval notation.

$\boxed{(-\infty,-6)\cup(6,\infty)}$

x-Int @ $x=4$

VA @ $x=6,-6$

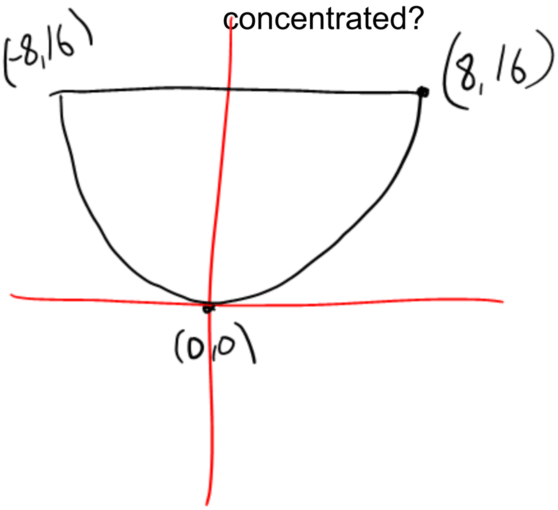
so



$(-\infty,-6)$ $(6,\infty)$

x	-8	-6	0	4	5	6	9
$\frac{(x-4)^2}{x^2-36}>0$	$\frac{144}{28}>0$ TRUE	$\frac{100}{0}>0$ FALSE	$\frac{16}{-36}>0$ False	$\frac{0}{-20}>0$ False	$\frac{1}{-11}>0$ False	$\frac{4}{0}>0$ False	$\frac{25}{45}>0$ TRUE

26) A reflecting telescope contains a mirror shaped like a paraboloid of revolution. If the mirror is 16 inches across at its opening and is 16 inches deep, where will the light be concentrated?



$(x-h)^2=4a(y-k) \Rightarrow x^2=4ay$

& (8,16) is a solution, so

$8^2=4a(16) \Rightarrow a=1$

& Light will be concentrated

$\boxed{\text{1 inch from vertex}}$

27) Find the sum. $\sum_{n=1}^{50} (4n + 2)$ $= 6 + 10 + 14 + \cdots + 202$

Arithmetic with $d = 4$, $n = 50$, $a_1 = 6$.

$a_{50} = 202$.

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{50}{2}(6 + 202)$$

$$= \boxed{5200}$$

28) Find the domain of the composite function $f \circ g$ given $f(x) = \sqrt{x - 2}$ and $g(x) = \frac{2}{x - 10}$.

$(f \circ g)(x) = f(g(x))$: Domain in 2 parts.

First, x is in g, so $x \neq 10$

2^{nd}, $g(x)$ is in f, so $g(x) \geq 2$

$$\Rightarrow \frac{2}{x-10} \geq 2 \Rightarrow \frac{2}{x-10} - 2 \geq 0 \Rightarrow \frac{2}{x-10} - \frac{2(x-10)}{x-10} \geq 0 \Rightarrow \frac{22-2x}{x-10} \geq 0$$

$\Rightarrow$ test: 9, 10, 10.5, 11, 12

x	9	10	10.5	11	12
$\frac{22-2x}{x-10} \geq 0$	$\frac{4}{-1} \geq 0$ False	$\frac{2}{0} \geq 0$ False	$\frac{1}{.5} \geq 0$ TRUE	$\frac{0}{1} \geq 0$ TRUE	$\frac{-2}{2} \geq 0$ False

SO DOMAIN IS $(10, 11]$

29) Solve $x^3 - 5x^2 + 5x = 1$. Give exact values.

$$x^3-5x^2+5x-1=0$$

$$\begin{array}{c|cccc} & 1 & -5 & 5 & -1 \\ \hline 1 & 1 & -4 & 1 & 0 \end{array} \Rightarrow x=1$$

$$x^2-4x+1=0$$

$$x=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$$

so $\boxed{1,\ 2\pm\sqrt{3}}$

30) Solve the exponential equation. (Round your answer to three decimal places.)
$4^{(1+2x)} = 17$

$$\ln 4^{(1+2x)}=\ln 17$$

$$(1+2x)\ln 4=\ln 17$$

$$1+2x=\frac{\ln 17}{\ln 4}$$

$$2x=-1+\frac{\ln 17}{\ln 4}$$

$$x=-\frac{1}{2}+\frac{1}{2}\frac{\ln 17}{\ln 4}\Rightarrow \boxed{x\approx 0.522}$$

OR $\log_4 4^{(1+2x)}=\log_4 17$

$$\Rightarrow 1+2x=\log_4 17$$

$$2x=-1+\log_4 17$$

$$x=-\frac{1}{2}+\frac{1}{2}\log_4 17$$

$$x=-\frac{1}{2}+\frac{1}{2}\frac{\ln 17}{\ln 4}$$ ((change of Base)