Math 1010 Final Exam

Form A, Fall 2008

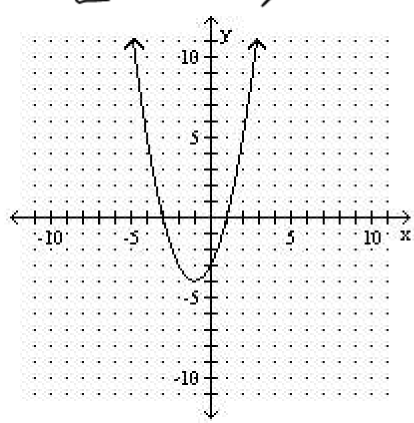
Name: KEY

Instructor: ______________________________ ID verification: ________

Each problem is equally weighted. Scientific calculators are permitted.
Time limit: Two hours.
Not allowed: notes, books, graphing/programable calculators, cell phones.

Find the domain and the range of the relation. Use the vertical line test to determine whether the graph is the graph of a function.

1) DOMAIN $= (-\infty, \infty)$



RANGE $= [-4, \infty)$

YES, FUNCTION, SINCE IT PASSES THE VERTICAL LINE TEST

A) domain: $[-4, \infty)$
range: $(-\infty, \infty)$
not a function

B) domain: $[-4, \infty)$
range: $(-\infty, \infty)$
function

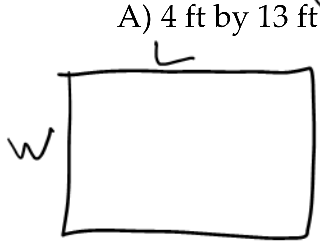
C) domain: $(-\infty, \infty)$
range: $[-4, \infty)$
not a function

D) domain: $(-\infty, \infty)$
range: $[-4, \infty)$
function

Solve the problem.

2) The length of a rectangular vegetable garden is 9 feet longer than its width. If the area of the garden is 70 square feet, find its dimensions.

A) 4 ft by 13 ft B) 7 ft by 10 ft C) 5 ft by 14 ft D) 6 ft by 15 ft



$L = (W + 9)$

$A = LW = 70$

$(w+9)w = 70$

$w^2 + 9w = 70$

$w^2 + 9w - 70 = 0$

$(w+14)(w-5) = 0$

$\cancel{w = -14}$, $\boxed{w = 5}$

$L = w + 9 = 5 + 9 = 14$

SO $\boxed{5 \text{ ft by } 14 \text{ ft}}$

Solve the equation.

3) $2^{(7 + 3x)} = \frac{1}{4} = 4^{-1} = (2^2)^{-1} = 2^{-2}$

A) $\frac{1}{2}$ B) 3 C) –3 D) 1

So $2^{7+3x} = 2^{-2} \Rightarrow 7+3x = -2 \Rightarrow 3x = -9 \Rightarrow \boxed{x = -3}$

Find the vertex of the graph of the quadratic function.

4) $f(x) = -x^2 + 6x - 3$

A) (3, 24) B) (–3, –12) C) (6, –3) D) (3, 6)

$x = \frac{-b}{2a} = \frac{-6}{-2} = 3$, $y = -x^2 + 6x - 3 = -(3)^2 + 6(3) - 3 = -9 + 18 - 3$
$= 6$ so vertex $\boxed{(3, 6)}$

Solve the problem.

5) A chemist needs 130 milliliters of a 31% solution but has only 17% (x) and 43% (y) solutions available. Find how many milliliters of each that should be mixed to get the desired solution.

A) 68 ml of 17%; 62 ml of 43% B) 65 ml of 17%; 65 ml of 43%
C) 60 ml of 17%; 70 ml of 43% D) 70 ml of 17%; 60 ml of 43%

$-.17(x + y = 130)$
$.17x + .43y = .31(130)$

$-.17x - .17y = -.17(130)$
$.17x + .43y = .31(130)$

$-.17x - .17y = -22.1$
$+\ .17x + .43y = 40.3$
$.26y = 18.2$
$\Rightarrow y = 70$

$x + y = 130$
$x + 70 = 130$
$x = 60$

so $\boxed{60 \text{ ml of } 17\%,\ 70 \text{ ml of } 43\%}$

Solve the equation.

6) $\frac{7}{x + 3} - \frac{5}{x - 3} = \frac{8}{x^2 - 9}$

A) 7 B) 44 C) –22 D) 22

$(x+3)(x-3)\frac{7}{\cancel{x+3}} - (x+3)(x-3)\frac{5}{\cancel{x-3}} = (x+3)(x-3)\frac{8}{\cancel{(x+3)(x-3)}} \Rightarrow 7(x-3) - 5(x+3) = 8 \Rightarrow 7x - 21 - 5x - 15 = 8 \Rightarrow$

$\Rightarrow 2x - 36 = 8 \Rightarrow 2x = 44 \Rightarrow \boxed{x = 22}$

IN THIS PROBLEM I WAS READY TO THROW OUT ANY ANSWERS WITH $x = 3$ or $x = -3$

Find the midpoint of the line segment whose endpoints are given.

7) $(-4, -7), (6, -8)$

A) $(-5, \frac{1}{2})$ B) $(2, -15)$ C) $(1, -\frac{15}{2})$ D) $(-10, 1)$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \Rightarrow \left(\frac{-4+6}{2}, \frac{-7+-8}{2}\right) \Rightarrow \left(\frac{2}{2}, \frac{-15}{2}\right) \Rightarrow \boxed{\left(1, \frac{-15}{2}\right)}$$

Write as an exponential equation.

8) $\log_e z = 7$

A) $7^e = z$ B) $e^z = 7$ C) $z^7 = e$ D) $e^7 = z$

$$\log_e z = 7 \Rightarrow \boxed{z = e^7}$$

Solve the system for z only.

9)
$$\begin{cases} x + y + z = -7 & R1 \\ x - y + 2z = 3 & R2 \\ 5x + y + z = -27 & R3 \end{cases}$$

A) -5 B) -4 C) 2 D) $\varnothing$

R1 + R2: $x + y + z = -7$
$+\ x - y + 2z = 3$
$2x + 3z = -4$ R4

R2+R3 $x - y + 2z = 3$
$+\ 5x + y + z = -27$
$6x + 3z = -24$ R5

$-(2x + 3z = -4)$
$6x + 3z = -24$

$-2x - 3z = 4$
$+\ 6x + 3z = -24$
$4x = -20$

$x = -5$

R4: $2x + 3z = -4 \Rightarrow 2(-5) + 3z = -4$
$\Rightarrow -10 + 3z = -4 \Rightarrow 3z = 6$
$\Rightarrow \boxed{z = 2}$

I ELIMINATED Y TWICE, THEN Z & THEN BACK SUBSTITUTED. ON MY OWN TEST, I WOULD CONTINUE & GET X, Y & Z SO I COULD CHECK IN R1, R2 & R3...

Solve the equation using the quadratic formula.

10) $8x^2 + 1 = 3x$

A) $\{-\frac{3}{16} - \frac{\sqrt{23}}{16}i, -\frac{3}{16} + \frac{\sqrt{23}}{16}i\}$ B) $\{-\frac{3}{16} - \frac{\sqrt{23}}{16}i, \frac{3}{16} + \frac{\sqrt{23}}{16}i\}$

C) $\{\frac{3}{16} - \frac{\sqrt{23}}{16}i, -\frac{3}{16} + \frac{\sqrt{23}}{16}i\}$ D) $\{\frac{3}{16} - \frac{\sqrt{23}}{16}i, \frac{3}{16} + \frac{\sqrt{23}}{16}i\}$

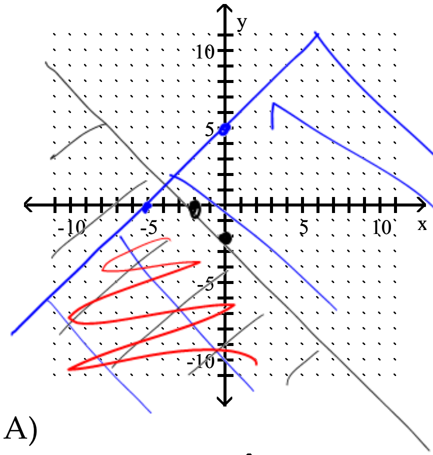
$$8x^2 + 1 = 3x \Rightarrow 8x^2 - 3x + 1 = 0$$

$$b^2 - 4ac = (-3)^2 - 4(8)(1) = 9 - 32 = -23$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{3 \pm \sqrt{-23}}{16} = \boxed{\frac{3}{16} \pm \frac{\sqrt{23}}{16} i}$$

Graph the union or intersection, as indicated.

11) The intersection of $x + y \le -2$ and $x - y \ge -5$



$x + y \le -2$

x	y
0	-2
-2	0

shades below

SOLID

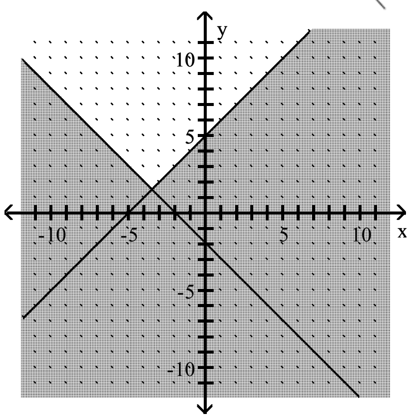
$x - y \ge -5$

x	y
0	5
-5	0

Shades Below

SOLID

A)



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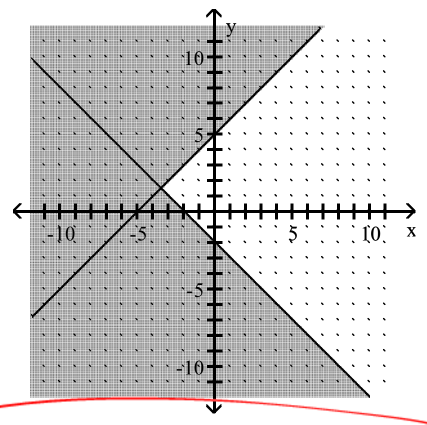

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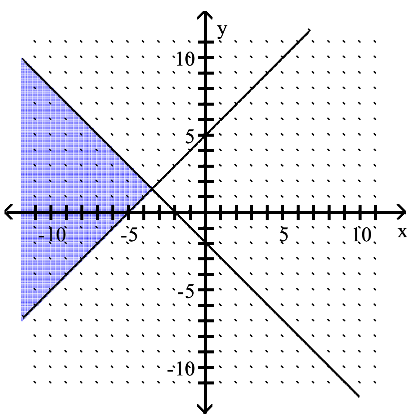
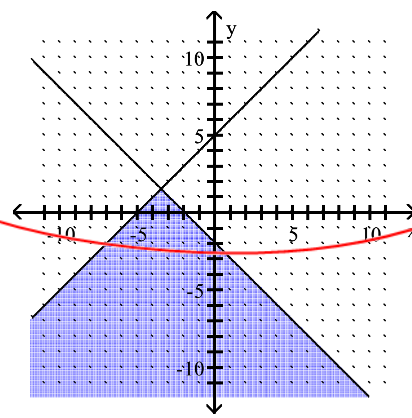


D)



Divide.

12) $(-8x^3 + 14x^2 - 15x + 3) \div (-4x + 1)$

A) $2x^2 - 3x + 3$ B) $x^2 - 3x + 3$ C) $2x^2 + 3$ D) $x^2 + 3x - 3$

$$
\begin{array}{r l}
 & \boxed{2x^2 - 3x + 3} \\
-4x+1 \,\big) & -8x^3 + 14x^2 - 15x + 3 \\
 & \pm 8x^3 \mp 2x^2 \\
\hline
 & 12x^2 - 15x \\
 & -12x^2 \pm 3x \\
\hline
 & -12x + 3 \\
 & \pm 12x \mp 3 \\
\hline
 & 0
\end{array}
$$

Solve the problem.

13) The number of mosquitoes M(x), in millions, in a certain area depends on the June rainfall x, in inches: $M(x) = 16x - x^2$. What rainfall produces the maximum number of mosquitoes?

A) 0 in. B) 8 in. C) 16 in. D) 64 in.

MAXIMUM = VERTEX

$x = \frac{-b}{2a} = \frac{-16}{-2} = \boxed{8}$

Solve the equation.

14) $\sqrt{2x - 3} = 3 - x$

A) 2, 6 B) 2 C) 6 D) $\varnothing$

$(\sqrt{2x-3})^2 = (3-x)^2 \Rightarrow 2x - 3 = 9 - 6x + x^2 \Rightarrow x^2 - 8x + 12 = 0$

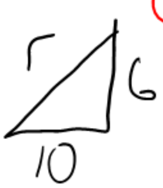
$\Rightarrow (x-6)(x-2) = 0 \Rightarrow x = 2, 6$ BUT $x = 6$ GIVES $\sqrt{9} = 3 - 6$ FALSE.

SO $\boxed{x = 2}$

Solve the problem.

15) Scott set up a volleyball net in his backyard. One of the poles, which forms a right angle with the ground, is 6 feet high. To secure the pole, he attached a rope from the top of the pole to a stake 10 feet from the bottom of the pole. To the nearest tenth of a foot, find the length of the rope.

A) 11.7 ft. B) 4.0 ft. C) 8.0 ft. D) 17.1 ft.



$r^2 = 6^2 + 10^2 \Rightarrow r^2 = 136 \Rightarrow r = \sqrt{136} \approx \boxed{11.7}$

Multiply or divide as indicated. Simplify completely.

16) $\frac{x^2 + 11x + 24}{x^2 + 14x + 48} \cdot \frac{x^2 + 6x}{x^2 - 2x - 15}$

A) $\frac{x}{x - 5}$ B) $\frac{x^2 + 6x}{x - 5}$ C) $\frac{x}{x^2 + 14x + 48}$ D) $\frac{1}{x - 5}$

$\Rightarrow \frac{(\cancel{x+8})(\cancel{x+3})}{(\cancel{x+6})(\cancel{x+8})} \cdot \frac{x(\cancel{x+6})}{(x-5)(\cancel{x+3})} \Rightarrow \frac{x}{x-5}$

Write the expression in the standard form a + bi.

17) $(4 + 4i)(3 + 5i)$

A) $20i^2 + 32i + 12$ B) $-8 - 32i$ C) $\boxed{-8 + 32i}$ D) $32 - 8i$

FOIL: $12 + \underbrace{20i + 12i}_{32i} + \underbrace{20i^2}_{\substack{20(-1)\\-20}} \Rightarrow 12 + 32i - 20 \Rightarrow -8 + 32i$

Solve the problem.

18) One pump can drain a pool in 7 minutes. When a second pump is also used, the pool only takes 5 minutes to drain. How long would it take the second pump to drain the pool if it were the only pump in use? Round your answer to the nearest tenth.

A) 24.1 minute B) 17.5 minutes C) 33.0 minutes D) 2.9 minutes

$7.5 \cdot T \quad 7.5 \cdot T \quad 7.5 \cdot T$

$\frac{1}{7} + \frac{1}{T} = \frac{1}{5} \Rightarrow 5T + 35 = 7T \Rightarrow 35 = 2T$

$\Rightarrow T = \frac{35}{2} = 17.5$

Factor the polynomial completely.

19) $8y^3z - z$

A) $z(2y - 1)(4y^2 + 1)$ B) $z(2y + 1)(4y^2 - 2y + 1)$

C) $z(2y - 1)(4y^2 + 2y + 1)$ D) $z(8y - 1)(y^2 + 2y + 1)$

GCF: $z(8y^3 - 1) \Rightarrow z(2y - 1)(4y^2 + 2y + 1)$

Simplify the expression. Express the answer using only positive exponents. Assume that all variables are positive.

20) $\dfrac{(-2x^{4/3})^3}{x^{-4/3}}$

A) $-2x^{16/3}$ B) $-8x^{8/3}$ C) $-2x^{8/3}$ D) $-8x^{16/3}$

$\Rightarrow \dfrac{(-2)^3 x^{\frac{4}{3} \cdot 3}}{x^{-4/3}} \Rightarrow \dfrac{-8x^4}{x^{-4/3}} \Rightarrow -8x^4 \cdot x^{4/3} \Rightarrow -8x^{4\frac{4}{3}}$

$\Rightarrow \boxed{-8x^{16/3}}$

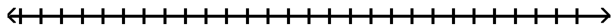
Solve the inequality. Graph the solution set.

21) $|x + 3| - 1 \geq 3$

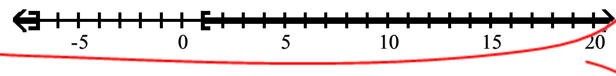
$\Rightarrow |x+3| \geq 4$

$x + 3 = 4$ $\qquad$ $x + 3 = -4$

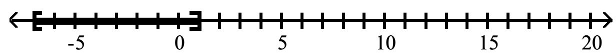
$x = 1$ $\qquad$ $x = -7$



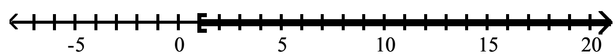
A) $(-\infty, -7] \cup [1, \infty)$



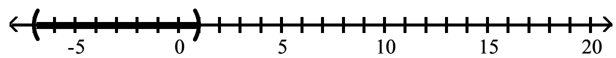
B) $[-7, 1]$

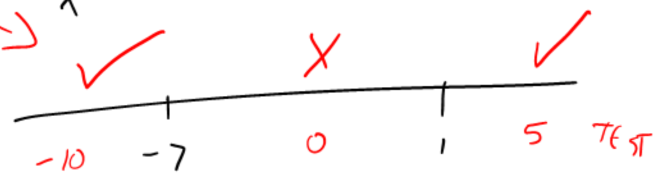


C) $[1, \infty)$



D) $(-7, 1)$





$|-10+3| - 1 \geq 3$

$|-7| - 1 \geq 3$

$6 \geq 3$

TRUE

$|0+3| - 1 \geq 3$

$|3| - 1 \geq 3$

$2 \geq 3$

FALSE

$|5+3| - 1 \geq 3$

$|8| - 1 \geq 3$

$7 \geq 3$

TRUE

Simplify the expression. Assume that all variables are positive.

22) $\sqrt{2x^2} + 5\sqrt{18x^2} + 2\sqrt{8x^2}$

A) $7x\sqrt{28}$ $\qquad$ B) $20x\sqrt{2}$ $\qquad$ C) $28x\sqrt{2}$ $\qquad$ D) $8x\sqrt{2}$

$\sqrt{2\cdot x\cdot x} + 5\sqrt{2\cdot 3\cdot 3\cdot x\cdot x} + 2\sqrt{2\cdot 2\cdot 2\cdot x\cdot x} \Rightarrow x\sqrt{2} + 15x\sqrt{2} + 4x\sqrt{2} \Rightarrow \boxed{20x\sqrt{2}}$

List the intercepts for the graph of the equation.

23) $y = x^2 + 14x + 48$

A) $(0, -6), (0, -8), (48, 0)$ $\qquad$ B) $(-6, 0), (-8, 0), (0, 48)$

C) $(0, 6), (0, 8), (48, 0)$ $\qquad$ D) $(6, 0), (8, 0), (0, 48)$

x int $\Rightarrow y = 0$

so $x^2 + 14x + 48 = 0$

$(x+6)(x+8) = 0$

$x = -6, -8$

$\Rightarrow \boxed{(-6,0), (-8,0), (0,48)}$

y int $x = 0$

so $y = 0^2 + 14\cdot 0 + 48$

$y = 48$

Rationalize the denominator.

24) $\dfrac{\sqrt{5}}{\sqrt{3} + 4}$

A) $\dfrac{\sqrt{15} - 4\sqrt{5}}{7}$ $\qquad$ B) $\dfrac{\sqrt{15} + 4\sqrt{5}}{-13}$ $\qquad$ C) $\dfrac{\sqrt{15} - 4\sqrt{5}}{-13}$ $\qquad$ D) $\dfrac{3\sqrt{15} + 3\sqrt{5}}{12}$

$\dfrac{\sqrt{5}}{\sqrt{3}+4} \cdot \dfrac{\sqrt{3}-4}{\sqrt{3}-4} \Rightarrow \dfrac{\sqrt{15} - 4\sqrt{5}}{3 - 16} \Rightarrow \dfrac{\sqrt{15} - 4\sqrt{5}}{-13}$

Find an equation of the line. Write the equation in standard form.

25) Through (−4, −7) and (4, 6).

A) $-3x + 2y = -24$ B) $3x - 2y = -24$ C) $13x - 8y = 4$ D) $-13x - 8y = 4$

$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{6 - -7}{4 - -4} \Rightarrow m = \frac{13}{8}$

$y - y_1 = m(x - x_1) \Rightarrow 8 \cdot y - 6 = \frac{13}{8}(x-4) \cdot 8$

$\Rightarrow 8y - 48 = 13(x-4) \Rightarrow 8y - 48 = 13x - 52 \Rightarrow \boxed{13x - 8y = 4}$

Find the center and the radius of the circle.

26) $x^2 + y^2 + 16x - 4y - 13 = 0$

A) center (−8, 2), radius = 81 B) center (−2, 8), radius = 9

C) center (−8, 2), radius = 9 D) center (8, −2), radius = 81

$x^2 + 16x + 64 + y^2 - 4y + 4 = 13 + 64 + 4$

$(x+8)^2 + (y-2)^2 = 81 \Rightarrow \boxed{\text{center}(-8, 2)}, \boxed{\text{radius} = \sqrt{81} = 9}$

Find an equation of the line. Write the equation using function notation.

27) Through (3, −4); perpendicular to $x + 5y = -5$

A) $f(x) = 5x - 19$ B) $f(x) = 5x - 11$

C) $f(x) = \frac{1}{5}x - \frac{23}{5}$ D) $f(x) = -\frac{1}{5}x - \frac{17}{5}$

$x + 5y = -5$

$5y = -x - 5$

$y = -\frac{1}{5}x - 1$

Perpendicular is opposite Reciprocal, so our slope is $m = 5$

$y - y_1 = m(x - x_1)$

$y + 4 = 5(x - 3)$

$y + 4 = 5x - 15$

$y = 5x - 19$

$\boxed{f(x) = 5x - 19}$

Perform the indicated operation. Simplify if possible.

28) $\frac{12}{x^2 + 4x} + \frac{5}{x} + \frac{3}{x+4}$

A) $\frac{15}{x}$ B) $\frac{5}{x}$ C) $\frac{8}{x}$ D) $\frac{3}{x}$

$\Rightarrow \frac{12}{x(x+4)} + \frac{5}{x} \cdot \frac{(x+4)}{(x+4)} + \frac{3}{x+4} \cdot \frac{x}{x} \Rightarrow \frac{12 + 5x + 20 + 3x}{x(x+4)} \Rightarrow$

$\Rightarrow \frac{8x + 32}{x(x+4)} \Rightarrow \frac{8\cancel{(x+4)}}{x\cancel{(x+4)}} \Rightarrow \boxed{\frac{8}{x}}$

Write the solution set using interval notation.

29) $5(x + 3) \leq 6(x - 8)$

A) $(-\infty, -33]$ B) $(-\infty, 63]$ C) $[63, \infty)$ D) $[-33, \infty)$

$\Rightarrow 5x + 15 \leq 6x - 48 \Rightarrow 63 \leq x \Rightarrow x \geq 63 \Rightarrow \boxed{[63, \infty)}$

Simplify.

30)

$$\frac{\frac{3}{x} + \frac{2}{x^2}}{\frac{9}{x^2} - \frac{4}{x}}$$

A) $\frac{1}{3 - 2x}$ B) $\frac{3x^2 + 2}{9 - 4x}$ C) $\frac{1}{3x - 2}$ D) $\boxed{\frac{3x + 2}{9 - 4x}}$

$$\frac{x^2 \cdot \frac{3}{\cancel{x}} + \frac{2}{\cancel{x^2}} \cdot \cancel{x^2}}{\cancel{x^2} \cdot \frac{9}{\cancel{x^2}} - \frac{4}{\cancel{x}} \cdot x^2} \Rightarrow \boxed{\frac{3x + 2}{9 - 4x}}$$