Name: $\qquad$ Instructor: $\qquad$
Student ID: $\qquad$ ID Verification: $\qquad$ Section Number: $\qquad$

This exam has three parts: Part I - Ten multiple choice questions
Part II - Ten open ended questions - You MUST show all your work Part III - Choose FIVE out of ten open ended questions - You MUST show your work and indicate which five problems are to be graded

Students are NOT allowed to use books or notes.

## Questions 1-10 Multiple Choice.

Answer all TEN questions and circle the correct answer.

1) For $f(x)=x^{2}-1$, find $\frac{f(x+h)-f(x)}{h}$; simplify your answer.
A) $x+2 h$
B) $2 x+h$
C) $\frac{2 x h+h^{2}-2}{h}$
D) $2 x-2 h$
2) Consider the demand equation

$$
p=-\frac{1}{15} x+30 ; \quad 0 \leq x \leq 450
$$

where $p$ represents the price and $x$ the number of units sold.
What price should the company charge to maximize the revenue? [Hint: Remember that Revenue $=$ (quantity sold) $\cdot($ price $)$.
A) $\$ 225$
B) $\$ 30$
C) $\$ 450$
D) $\$ 15$
3) The sequence is defined recursively. What is the fourth term of this sequence?

$$
\mathrm{a}_{1}=-2, \mathrm{a}_{2}=5 \text { and } \mathrm{a}_{n}=3 \mathrm{a}_{n-1}+\mathrm{a}_{n-2} \text { for } \mathrm{n} \geq 3
$$

A) 135
B) 44
C) 27
D) 8
4) Find the oblique asymptotes, if any, of the graph of $f(x)=\frac{2 x^{3}-3 x^{2}+9 x-11}{x^{2}-5}$
A) $y=2 x$
B) $y=2 x-3$
C) $y=2 x+7$
D) No oblique asymptotes
5) Form a polynomial $f(x)$ with real coefficients having the given degree and zeros.

Degree: 3; zeros: 3 and $-2 i$
A) $f(x)=x^{3}-3 x^{2}-4 x+12$
B) $f(x)=x^{3}-3 x^{2}+4 x-12$
C) $f(x)=x^{3}+3 x^{2}+4 x+12$
D) $f(x)=x^{3}+4 x^{2}-4 x-12$
6) Let $f(x)=\left\{\begin{array}{l}x^{2} \quad \text { if } x<-1 \\ x+1 \text { if }-1 \leq x \leq 3\end{array}\right.$

Graph the function. State the domain and range.

A)

Domain: $x \leq 3$
Range: $y \geq 0$
C)


Domain: All Real Numbers
Range: $y \geq 0$
B)


Domain: $x \leq 3$
Range: $y \geq 1$
D)


Domain: All Real Numbers
Range: All Real Number
7) The graph of an exponential function is shown ( $a>1$ ). Select the function which matches the graph.
A) $f(x)=-a^{x}+2$
B) $f(x)=a^{-x}+2$
C) $f(x)=-a^{(x-2)}$
D) $f(x)=a^{(x-2)}$

8) The half-life of radioactive iodine is 8 days. If 20 grams are present now, how much will be present in 13 days? Round your answer to three decimal places. The decay follows the law of uninhibited radioactive decay.
A) 12.308 grams
B) 6.484 grams
C) 5.651 grams
D) 1.363 grams
9) Solve the inequality. $\quad x^{4}<9 x^{2}$
A) $(-3,0) \cup(0,3)$
B) $(-3,0) \cup(3, \infty)$
C) $(-\infty,-3) \cup(0,3)$
D) $(-\infty,-3) \cup(3, \infty)$
10) Find the sum.
$\sum_{k=1}^{\infty} 5\left(\frac{2}{3}\right)^{k-1}$
A) 3
B) 5
C) $\frac{10}{3}$
D) 15

Part II: Questions 11 - 20, Open ended
Answer all TEN questions. You must show your work in a clear and logical progression and clearly indicate your answer to receive full credit.
11) The graph of a function $f$ is illustrated. Use the graph of $f$ as a first step toward graphing the following function. Label the $x$ - and $y$-coordinates of the points corresponding to the labeled points of $f(x)$.
$f(x)$

$$
H(x)=f(x-1)-3
$$



12) Use the graph of the function $f$ given below to answer parts (a) - (g)

a) Find $f(3)$
b) What is the domain of $f$ ?
c) What is the range of $f$ ?
d) List the $x$-value(s) at which $f$ has a local maximum.
e) What are the local maxima?
f) Identify the intervals (if any) on which it is

Increasing: $\qquad$
Decreasing: $\qquad$
Constant: $\qquad$
g) Determine if the function is even, odd, or neither.
13)The function $f$ is one-to-one. Find its inverse. State the domain and range of $f$ and $f^{-1}$.
$f(x)=\frac{2 x}{3 x-1}$

Domain of $f$ : $\qquad$
Range of $f$ : $\qquad$

Domain of $f^{-1}$ :
Range of $f^{-1}$ :
14) Solve the equation.
$\log _{3} x+\log _{3}(x-8)=2$
15) For the polynomial $f(x)=x(x+3)^{2}(x-2)^{3}$
a) Find the zeros of $f(x)$ and their multiplicities and determine whether the graph of $f(x)$ crosses or touches the $x$-axis at each zero.

| Zero | Multiplicity | Touch/Cross |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

b) Find the power function that the graph of $f(x)$ resembles for large values of $|x|$. (This is the end behavior.)
c) Sketch the function. Label all intercepts:

16) Given that $D$, the determinant of the coefficient matrix is 23 , use Cramer's Rule to show that $\mathbf{x}=-\mathbf{2}$ in the solution of the system of equations. You must do this problem by hand.
Only show that $x=-2$. Do not solve for $y$ and $z$.

$$
\left\{\begin{array}{c}
3 x-2 y+3 z=-3 \\
5 x+3 y+8 z=-2 \\
x+y+3 z=1
\end{array}\right.
$$

17) Graph the equation. Find the center, vertices, and foci and graph the asymptotes.

$$
\frac{x^{2}}{16}-\frac{(y+2)^{2}}{9}=1
$$

Center $\qquad$

Vertices $\qquad$

Foci $\qquad$

18) Find the following sum. Assume the terms are from an arithmetic sequence.
$-1+3+7+\ldots+195$
19) Graph $\quad f(x)=\log _{2}(x+4)$ and find any intercepts. Determine the domain, range, and vertical asymptote of $f$.

Domain: $\qquad$

Range: $\qquad$
Vertical asymptote: $\qquad$
$x$-intercept(s): $\qquad$
$y$-intercept: $\qquad$

20) Find the vertex and focus of the parabola. Show your work.

$$
y^{2}-6 y=x
$$

Vertex $\qquad$

Focus $\qquad$

Part III: Questions 21 - 30, Self select
Choose FIVE out of the next TEN questions to complete. You must show all your work and clearly indicate your answer for full credit. CROSS OUT the problems that you do not want graded.
21) Find the real solutions of the equation.

$$
x^{2 / 3}+7 x^{1 / 3}+12=0
$$

22) How long does it take $\$ 1000$ to grow to $\$ 2500$ if it is invested at $6 \%$ interest compounded monthly? Round your answer to the nearest tenth of a year.
23) Let $f(x)=\frac{1}{x} ; g(x)=\frac{x+2}{x-3}$.

Find $f \circ g$ and state the domain of the composite function. Simplify your answer for $f \circ g$.

Domain of $f \circ g$ : $\qquad$
24) Find the domain of $f(x)=\sqrt{\frac{x}{x-3}}$.
25) Use the following matrices to compute the given expression. Show your work.
$A=\left[\begin{array}{cc}3 & -2 \\ 1 & 0 \\ 1 & 2\end{array}\right], \quad B=\left[\begin{array}{ccc}-3 & 2 & 1 \\ 0 & -1 & 4\end{array}\right], \quad I_{2}$ is the $2 \times 2$ Identity matrix

## $B A+2 I_{2}$

26) Let $f(x)=\frac{x^{2}-3 x-4}{x^{2}+x-2}$
a) Find the domain of $f$.
b) Find the equations of the vertical asymptotes, if any, of the graph of $f$.
c) Find the $x$ - and $y$ - intercepts, if any, of the graph of $f$. (Express your answers as ordered pairs. That is, give both the $x$ and $y$ coordinates of the points.)
x-intercept(s): $\qquad$
y-intercept: $\qquad$
d) Find the equations of any horizontal or oblique asymptotes of the graph of $f$.
e) Graph the rational function $f$.

27) Solve the following system using the inverse of the coefficient matrix (given below). Show your work.
$\left\{\begin{aligned} 3 x-3 y+z & =5 \\ -2 x+2 y-z & =-2 \\ -4 x+5 y-2 z & =4\end{aligned}\right.$
The inverse of $\left[\begin{array}{ccc}3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2\end{array}\right]$ is $\left[\begin{array}{ccc}1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0\end{array}\right]$
28) Find an equation for the ellipse. Graph the equation. Show your work for full credit.

Foci at $(-4,0)$ and $(4,0)$; vertex at $(6,0)$

29) Find the inverse of the following matrix. Use row operations and show your work.
$A=\left[\begin{array}{ll}2 & 1 \\ 5 & 2\end{array}\right]$

$$
A^{-1}=
$$

30) Write the partial fraction decomposition of the rational expression. Solve for the constants (A, B, etc.).
$\frac{5 x+13}{(x-1)(x+5)}$
