

3.1 Use identities to find the exact value of the function for the given value.

1) $\sin \alpha = \frac{3}{4}$ and α is in quadrant II; Find $\tan \alpha$.

Simplify the expression.

2) $\cot x \sec x \sin x$

3) $\frac{1}{\cot^2 x} + \sec x \cos x$

4) $\cos x - \cos x \sin^2 x$

Determine whether the function is odd, even, or neither.

5) $f(x) = \cos x \csc x$

Simplify the expression.

6) $\frac{\cos(-x)}{\tan(-x)} - \sin x$

3.2 Multiply and simplify.

7) $(1 - \cos x)(1 + \cos x)$

Factor and simplify the expression.

8) $\sin^2 x + \sin^2 x \cot^2 x$

Identify the equation as either an identity or not.

9) $\frac{\sin x}{1 - \cos x} + \frac{\sin x}{1 + \cos x} = 2 \csc x$

10) $\frac{\cot^2 x}{\csc x - 1} = \frac{1 + \sin x}{\sin x}$

3.3 Find the exact value by using a sum or difference identity.

11) $\cos 75^\circ$

Find the exact value by using a sum or difference identity.

12) $\cos \frac{21\pi}{12}$

Use the sum/difference identities to simplify the expression. Do not use a calculator.

13) $\cos 5^\circ \cos 40^\circ - \sin 5^\circ \sin 40^\circ$

Write in terms of the cofunction of a complementary angle.

14) $\cos \frac{\pi}{24}$

15) $\cot \frac{17\pi}{18}$

Find $\cos(A + B)$.

16) $\sin A = -\frac{1}{2}$ and $\sin B = \frac{1}{4}$, with A in quadrant IV and B in quadrant II.

Find $\cos(A - B)$.

17) $\cos A = \frac{\sqrt{10}}{10}$ and $\sin B = -\frac{\sqrt{3}}{2}$, where $0^\circ < A < 90^\circ$ and $270^\circ < B < 360^\circ$.

Use the identities for the cosine of a sum or a difference to write the expression as a single function of α .

18) $\cos(\alpha + 90^\circ)$

3.4 Find the exact value by using a sum or difference identity.

19) $\sin \frac{11\pi}{12}$

Find the exact value by using a sum or difference identity.

20) $\tan 105^\circ$

Use trigonometric identities to find the exact value.

21) $\frac{\tan 5^\circ + \tan 25^\circ}{1 - \tan 5^\circ \tan 25^\circ}$

Use a sum or difference identity to find the exact value.

22) $\sin \frac{\pi}{15} \cos \frac{4\pi}{15} + \cos \frac{\pi}{15} \sin \frac{4\pi}{15}$

Solve the problem.

23) If $\cos A = \frac{1}{3}$ and $\sin B = \frac{1}{4}$, where $0 \leq A \leq \frac{\pi}{2}$ and $\frac{\pi}{2} \leq B \leq \pi$, then find $\sin(A - B)$.

Using a sum or difference identity, write the following as an expression involving functions of α .

24) $\sin(\alpha + 45^\circ)$

Decide whether the expression is or is not an identity.

25) $\sin(A + B) \sin(A - B) = \sin^2 A - \sin^2 B$

Determine if the equation is an identity.

26) $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$

3.5 Find the exact value by using a half-angle identity.

27) $\sin(22.5^\circ)$

Find the exact value by using a half-angle identity.

$$28) \cos\left(-\frac{\pi}{8}\right)$$

$$29) \tan\left(\frac{7\pi}{8}\right)$$

Determine whether the positive or negative sign makes the equation correct. Do not use a calculator.

$$30) \cos 111^\circ = \pm \sqrt{\frac{1 + \cos 222^\circ}{2}}$$

Use identities to simplify the expression. Do not use a calculator.

$$31) 2 \cos^2 22.5^\circ - 1$$

$$32) \sin 22.5^\circ \cos 22.5^\circ$$

Identify the equation as either an identity or not.

$$33) \frac{2 - \tan x}{2 + \tan x} = 1$$

$$34) \frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$35) \sin 2x = 2 \sin x$$

Decide whether the expression is or is not an identity.

$$36) \tan^2 \frac{x}{2} = \frac{1 - \cos x}{1 + \cos x}$$

$$37) \sin^2 \frac{x}{2} = \frac{\sec x + 1}{2 \sec x}$$

Solve the problem.

$$38) \text{ Find } \cos 2\theta. \sin \theta = \frac{15}{17}, \theta \text{ lies in quadrant I.}$$

$$39) \text{ Find } \sin 2\theta. \tan \theta = \frac{7}{24}, \theta \text{ lies in quadrant III.}$$

$$40) \text{ Find } \tan 2\theta. \sin \theta = \frac{20}{29}, \theta \text{ lies in quadrant II.}$$

Use the given information given to find the exact value of the trigonometric function.

$$41) \cos \theta = \frac{1}{4}, \csc \theta > 0 \quad \text{Find } \sin \frac{\theta}{2}.$$

42) $\sin \theta = -\frac{3}{5}$, θ lies in quadrant IV Find $\sin \frac{\theta}{2}$.

4.1 Find the exact value of the expression without using a calculator or table.

43) $\sin^{-1} \left(\frac{\sqrt{2}}{2} \right)$

Find the exact value of the expression without using a calculator or table.

44) $\csc^{-1} (-2)$

Find the exact value of the expression in degrees without using a calculator or table.

45) $\cos^{-1} \left(\frac{\sqrt{2}}{2} \right)$

46) $\arctan \left(-\frac{\sqrt{3}}{3} \right)$

Find the exact value of the composition.

47) $\csc \left(\sin^{-1} \left(\frac{3}{5} \right) \right)$

48) $\sin (\arctan (2))$

49) $\cos \left(\frac{1}{2} \arcsin \left(\frac{5}{13} \right) \right)$

Find an equivalent algebraic expression for the composition.

50) $\sin (\operatorname{arccot} (x))$

51) $\cot (\arctan (x))$

4.2 Find all real numbers that satisfy the equation.

52) $\sin x = -\frac{\sqrt{3}}{2}$

Find all real numbers that satisfy the equation.

53) $10 \cos x + 8\sqrt{2} = 8 \cos x + 7\sqrt{2}$

Find all angles in degrees that satisfy the equation. Round approximate answers to the nearest tenth of a degree.

54) $\tan \alpha = -2.01$

Solve the equation for $0 \leq t < 2\pi$. Approximate the solution to four decimal places.

55) $\sin t = \frac{1}{2}$

Solve the equation.

$$56) \sqrt{3} \tan(\alpha) + 1 = 0 \text{ for } -360^\circ \leq \alpha \leq 360^\circ$$

Solve the equation for x.

$$57) y = 3 \tan(2x - 1)$$

$$58) y = 3 \tan 2x - 1$$

Find the inverse of the function, and state the domain and range of the inverse function.

$$59) f(x) = 5 \sin(3x) \text{ for } -\frac{\pi}{6} \leq x \leq \frac{\pi}{6}$$

Find all real numbers that satisfy the equation. Round approximate answers to 2 decimal places.

$$60) 5 = 5 \sin(x) + 2$$

4.3 Find all real numbers that satisfy the equation.

$$61) \sqrt{3} \sec 2x = 2$$

Find all real numbers that satisfy the equation.

$$62) \cot \frac{x}{3} = 1$$

Find all values of θ in $[0^\circ, 360^\circ)$ that satisfy the equation.

$$63) \cos \theta = -\frac{\sqrt{3}}{2}$$

$$64) 2 \sin 2\theta - \sqrt{3} = 0$$

Find all real numbers in $[0, 2\pi]$ that satisfy the equation.

$$65) \sin 4x = \frac{\sqrt{3}}{2}$$

$$66) 2 \cos x + 1 = 0$$

Find all angles in degrees that satisfy the equation. Round approximate answers to the nearest tenth of a degree.

$$67) \sin 3\alpha = 0.593$$

Find all real numbers that satisfy the equation. Round approximate answers to the nearest hundredth.

$$68) 10 \cos\left(\frac{x}{2}\right) - 3 = 0$$

4.4 Find all real numbers in the interval $[0, 2\pi)$ that satisfy the equation.

$$69) \cos^2 x + 2 \cos x + 1 = 0$$

Find all real numbers in the interval $[0, 2\pi)$ that satisfy the equation.

70) $2 \sin^2 x = \sin x$

71) $\cos x = \sin x$

72) $\sin^2 x - \cos^2 x = 0$

Find all values of x in the interval $[0^\circ, 360^\circ)$ that satisfy the equation. Round approximate answers to the nearest tenth of a degree.

73) $\sin^2 x - 8 \sin x - 4 = 0$

74) $7 \cot^2 x - 5 = 0$

Solve the problem.

75) A weight is suspended on a system of springs and oscillates up and down according to

$$P = \frac{1}{10}[\sin(2t) + \sin t]$$

where P is the position in meters above or below the point of equilibrium ($P = 0$) and t is time in seconds. Find the time when the weight is at equilibrium. Find the exact values. Do not use a calculator.

Answer Key

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1) $-\frac{3\sqrt{7}}{7}$

2) 1

3) $\sec^2 x$

4) $\cos^3 x$

5) Odd

6) $-\csc x$

7) $\sin^2 x$

8) 1

9) Identity

10) Identity

11) $\frac{\sqrt{2}(\sqrt{3}-1)}{4}$

12) $\frac{\sqrt{2}}{2}$

13) $\frac{\sqrt{2}}{2}$

14) $\sin \frac{11\pi}{24}$

15) $\tan \frac{-4\pi}{9}$

16) $\frac{1-3\sqrt{5}}{8}$

17) $\frac{\sqrt{10}-3\sqrt{30}}{20}$

18) $-\sin \alpha$

19) $\frac{\sqrt{2}(\sqrt{3}-1)}{4}$

20) $-2 - \sqrt{3}$

21) $\frac{\sqrt{3}}{3}$

22) $\frac{\sqrt{3}}{2}$

23) $-\frac{2\sqrt{30}+1}{12}$

24) $\frac{\sqrt{2}}{2} \cos \alpha + \frac{\sqrt{2}}{2} \sin \alpha$

25) Identity

26) Identity

27) $\frac{1}{2} \sqrt{2-\sqrt{2}}$

28) $\frac{1}{2} \sqrt{2+\sqrt{2}}$

29) $1 - \sqrt{2}$

Answer Key

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30) Negative

31) $\frac{\sqrt{2}}{2}$

32) $\frac{\sqrt{2}}{4}$

33) Not an identity

34) Identity

35) Not an identity

36) Identity

37) Not an identity

38) $-\frac{161}{289}$

39) $\frac{336}{625}$

40) $-\frac{840}{41}$

41) $\frac{\sqrt{6}}{4}$

42) $-\frac{\sqrt{10}}{10}$

43) $\frac{\pi}{4}$

44) $\frac{-\pi}{6}$

45) 45°

46) -30°

47) $\frac{5}{3}$

48) $\frac{2\sqrt{5}}{5}$

49) $\frac{5\sqrt{26}}{26}$

50) $\frac{1}{\sqrt{x^2 + 1}}$

51) $\frac{1}{x}$

52) $\{x \mid x = -\frac{\pi}{3} + 2k\pi, x = -\frac{2\pi}{3} + 2k\pi\}$

53) $x = \frac{3\pi}{4} + 2n\pi$ or $x = \frac{5\pi}{4} + 2n\pi$

54) $\{\alpha \mid \alpha = 116.5^\circ + k180^\circ\}$

55) 0.5236, 2.6180

56) $\{-30^\circ, -210^\circ, 150^\circ, 330^\circ\}$

57) $x = \frac{1}{2} + \frac{1}{2} \arctan \frac{y}{3}$

Answer Key

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58) $x = \frac{1}{2} \arctan \frac{y+1}{3}$

59) $f^{-1}(x) = \frac{1}{3} \sin^{-1}\left(\frac{x}{5}\right)$; domain = $[-5, 5]$; range = $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

60) $0.64 + 2k\pi$ or $2.50 + 2k\pi$ where k is any integer

61) $\left\{x \mid x = \frac{\pi}{12} + k\pi \text{ or } x = \frac{11\pi}{12} + k\pi\right\}$

62) $\left\{x \mid x = \frac{3\pi}{4} + 6k\pi \text{ or } x = \frac{15\pi}{4} + 6k\pi\right\}$

63) $\{150^\circ, 210^\circ\}$

64) $\{30^\circ, 60^\circ, 210^\circ, 240^\circ\}$

65) $\frac{\pi}{12}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{12}, \frac{7\pi}{6}, \frac{13\pi}{12}, \frac{5\pi}{3}, \frac{19\pi}{12}$

66) $\frac{2\pi}{3}, \frac{4\pi}{3}$

67) $\{\alpha \mid \alpha = 12.1^\circ + k120^\circ \text{ or } \alpha = 47.9^\circ + k120^\circ\}$

68) $\{x \mid x = 2.53 + 4k\pi \text{ or } x = 10.03 + 4k\pi\}$

69) $\{\pi\}$

70) $\left\{0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$

71) $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

72) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

73) $\{208.2^\circ, 331.8^\circ\}$

74) $\{49.8^\circ, 130.2^\circ, 229.8^\circ, 310.2^\circ\}$

75) $0 \text{ sec}, \frac{2\pi}{3} \text{ sec}, \frac{4\pi}{3} \text{ sec}, \pi \text{ sec}$