# Math 1050 TEST 4 REVIEW KEY

1. Standard form $(x-h)^2+(y-k)^2=r^2$

$r=8$, $(h,k)=(-2,7)$ so $(x--2)^2+(y-7)^2=8^2$

$$\boxed{(x+2)^2+(y-7)^2=64}$$

2. $(x-3)^2+y^2=144$ center $(3,0)$, $r=12$

$h=3$ $k=0$ $r=\sqrt{144}=12$

3. $x^2+y^2-2x+12y=12 \Rightarrow x^2-2x+1+y^2+12y+36=12+1+36$

$\Rightarrow (x-1)^2+(y+6)^2=49 \Rightarrow \boxed{\text{center } (1,-6),\ r=7}$

4. $(x-h)^2+(y-k)^2=r^2$ center $(2,-3)$ contains $(5,-3)$

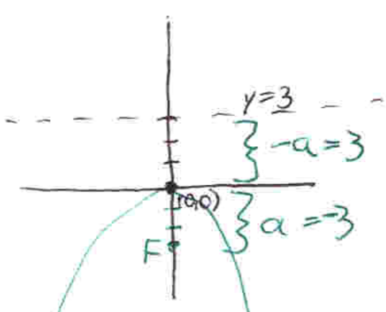
$h$ $k$ $x$ $y$

$(5-2)^2+(-3--3)^2=r^2$

$\Rightarrow 3^2=r^2 \Rightarrow r^2=9$ so $\boxed{(x-2)^2+(y+3)^2=9}$

General form: $x^2-4x+4+y^2+6y+9=9 \Rightarrow \boxed{x^2+y^2-4x+6y+4=0}$

5.

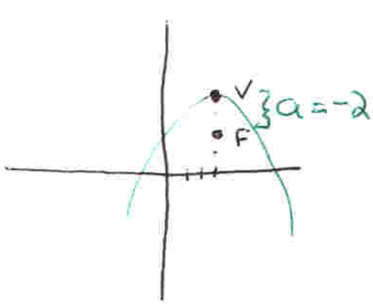



This is an $x^2$ parabola so $(x-h)^2=4a(y-k)$.

vertex $(0,0)$, so $x^2=4ay$.

Distance from vertex to directrix is $-a$, so $a=-3$ $x^2=4(-3)y \Rightarrow \boxed{x^2=-12y}$

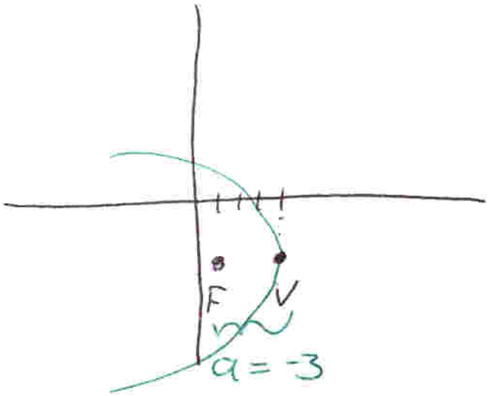
6.




$x^2$ parabola with vertex $(3,4)$, $a=-2$

$(x-3)^2=4(-2)(y-4) \Rightarrow \boxed{(x-3)^2=-8(y-4)}$

7. vertex $(4,-3)$ ; Focus $(1,-3)$




This is a $y^2$ parabola, so

$(y+3)^2 = 4(-3)(x-4) \Rightarrow \boxed{(y+3)^2 = -12(x-4)}$

8. $x^2 - 8x = 8y - 40$

$x^2 - 8x + 16 = 8y - 40 + 16$

$(x-4)^2 = 8y - 24$

$(x-4)^2 = 8(y-3)$

$\rightarrow$ Vertex $(4,3)$ ; $a = \frac{8}{4} = 2$ ; $x^2$ parabola (up/down)


Focus $(4,5)$

Directrix $y=1$

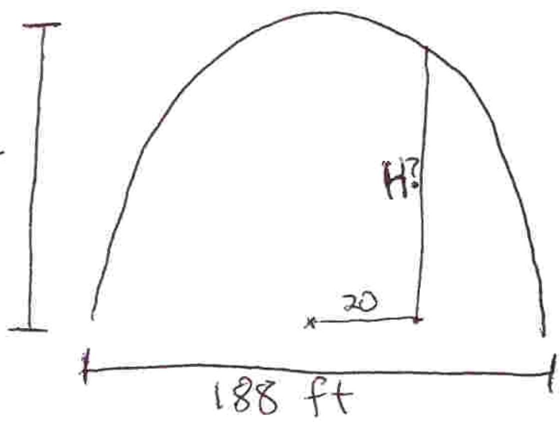
9. $x^2$ parabola, vertex $(3,-2)$, point $(5,-3)$

$(x-h)^2 = 4a(y-k)$ $\quad (5-3)^2 = 4a(-3 - -2) \Rightarrow 2^2 = 4a(-1)$

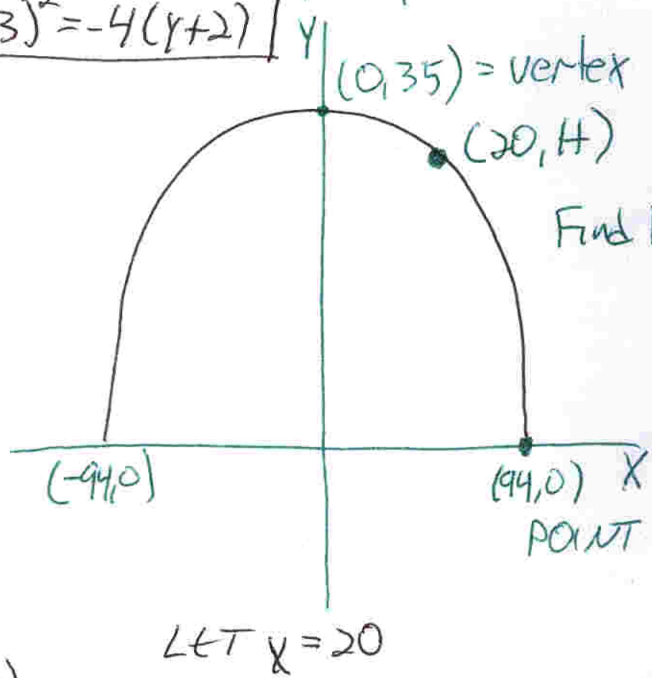
$\Rightarrow \frac{4}{-4} = \frac{-4a}{-4} \Rightarrow a = -1$

so $(x-3)^2 = 4(-1)(y+2) \Rightarrow \boxed{(x-3)^2 = -4(y+2)}$

10.




$\Rightarrow$




Find H.

$x^2$ parabola, vertex $(0,35)$, point $(94,0)$

$\Rightarrow (94-0)^2 = 4a(0-35) \Rightarrow 94^2 = 4a(-35)$

$\Rightarrow 4a = \frac{-94^2}{+35}$ so $(x)^2 = \frac{-(94)^2}{35}(y-35)$ $\nearrow$

LET $x = 20$

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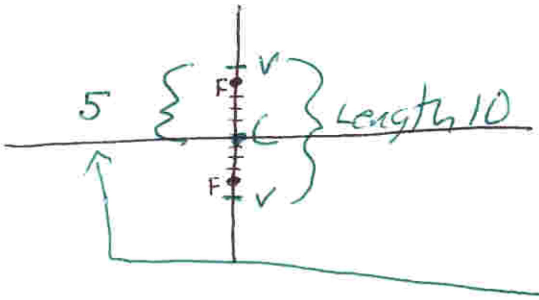
Let $y=20$ in $x^2 = -\frac{(94)^2}{35}(y-35) \Rightarrow \frac{35}{-(94)^2} \cdot 400 = -\frac{(94)^2}{35}(y-35) \cdot \frac{35}{-(94)^2}$

$\Rightarrow y-35 = -\frac{35}{(94)^2} \cdot 400 \Rightarrow y = -\frac{35}{(94)^2} \cdot 400 + 35 = \boxed{33.4156\ ft}$

$+35 \qquad +35$

11. ELLIPSE, FOCI $(0, \pm 4)$, length of major axis $= 10$

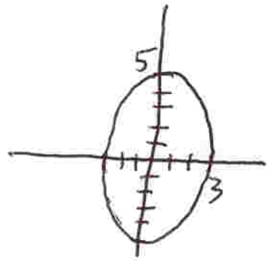



center in between Foci, so $(0,0)$

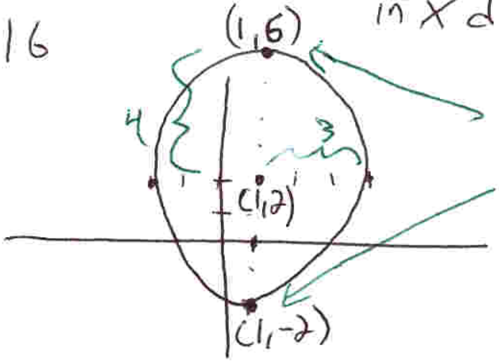
$\frac{x^2}{?} + \frac{y^2}{5^2} = 1$

DISTANCE FROM center to Focus is $c = 4$, $c^2 = L - S = 25 - ? = 4^2$

$? = 25 - 16 = 9$ so $\boxed{\frac{x^2}{9} + \frac{y^2}{25} = 1}$




12. $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{16} = 1 \Rightarrow$ center $(1,2)$, distance from center in $x$ direction $= \sqrt{9}$, DISTANCE IN $y = \sqrt{16}$




vertices $(1,6), (1,-2)$

13. $2x^2 + 5y^2 - 20x + 50y + 165 = 0$

$2x^2 - 20x \quad + 5y^2 + 50y = -165$

$2(x^2 - 10x + 25) + 5(y^2 + 10y + 25) = -165 + 25(2) + 25(5)$

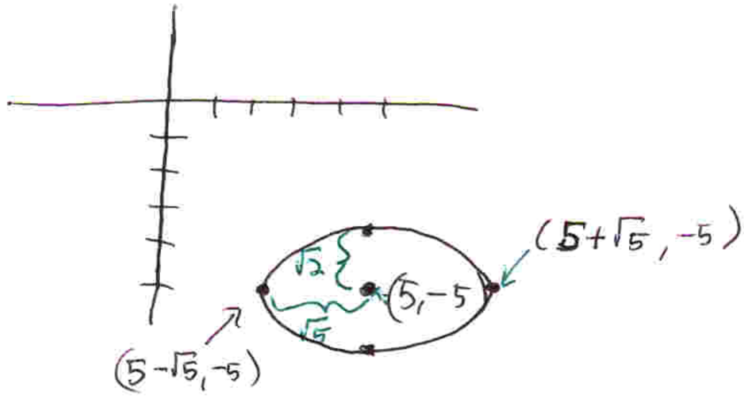
$\frac{2(x-5)^2}{10} + \frac{5(y+5)^2}{10} = \frac{10}{10} \Rightarrow \frac{(x-5)^2}{5} + \frac{(y+5)^2}{2} = 1$ STANDARD FORM

OVER

$$\frac{(x-5)^2}{5} + \frac{(y+5)^2}{2} = 1 \Rightarrow \text{center } (5,-5)$$

Focus: $c^2 = L - S = 5-2 = 3$

$c = \sqrt{3}$




From center $= (5,-5)$

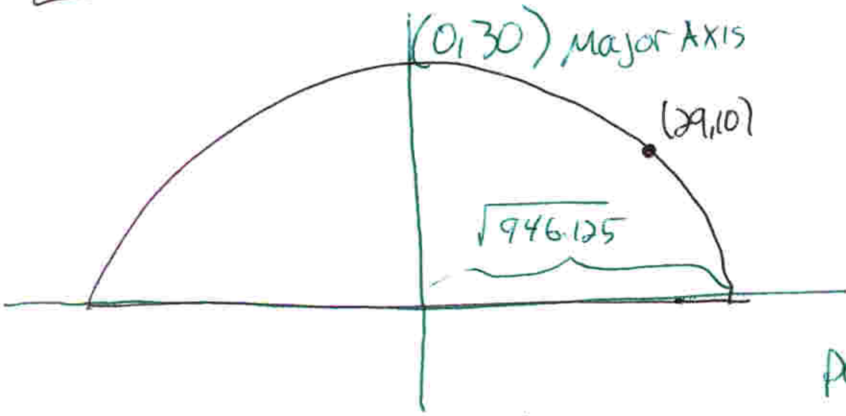
Focus $= (5 \pm \sqrt{3}, -5)$

Vertices $= (5 \pm \sqrt{5}, -5)$

14. Center $(-2,-1)$ OVER 4 in x direction, up 3 in y direction TO THE ELLIPSE, SO

$$\boxed{\frac{(x+2)^2}{16} + \frac{(y+1)^2}{9} = 1}$$

15.




Center $(0,0)$.

$$\frac{x^2}{?} + \frac{y^2}{30^2} = 1$$

TO FIND ?, we use the point $(29,10)$ TO get

$$\frac{29^2}{?} + \frac{10^2}{30^2} = 1 \Rightarrow \frac{29^2}{?} + \frac{100}{900} = 1 \Rightarrow$$
$$-\tfrac{1}{9} \qquad -\tfrac{1}{9}$$

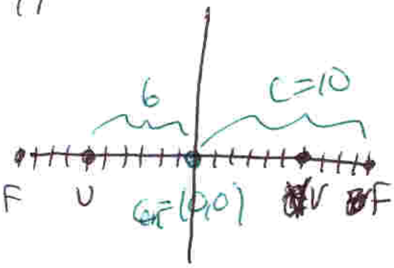
$$\frac{29^2}{?} = \frac{8}{9} \cdot 9? \Rightarrow \frac{9 \cdot 29^2}{8} = \frac{8?}{8} \Rightarrow ? = \frac{9 \cdot 29^2}{8} \approx 946.125$$

$$\Rightarrow \frac{x^2}{946.125} + \frac{y^2}{900} = 1$$

$\sqrt{946.05} = 30.76 = \frac{1}{2}$ the span SO

span $= 2(30.76) = \boxed{61.52 \text{ ft}}$

16. Hyperbola



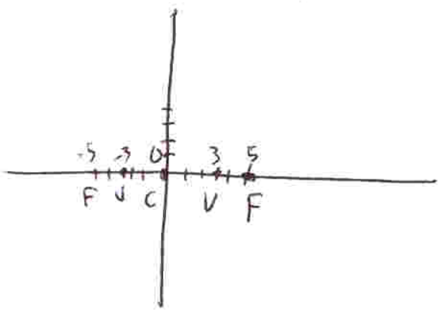

$x^2$ hyperbola, so $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

center $(0,0)$ (½ way between foci, or vert)

$$\frac{x^2}{6^2} - \frac{y^2}{?} = 1$$

$c^2 = a^2 + b^2 \Rightarrow 10^2 = 6^2 + ? \Rightarrow 100 - 36 = ? = 64$ $\Rightarrow \boxed{\dfrac{x^2}{36} - \dfrac{y^2}{64} = 1}$

17. Cen $(0,0)$, Focus $(5,0)$, vert $(3,0)$



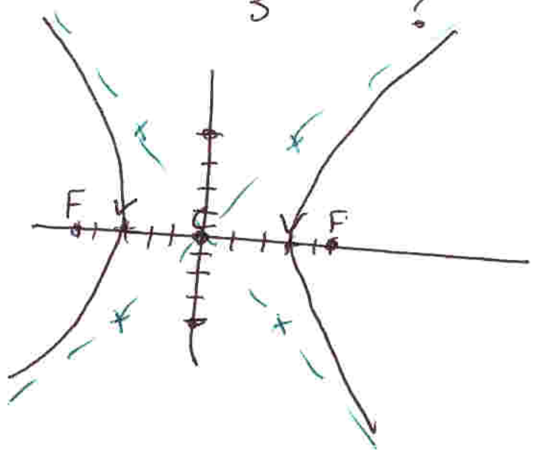

$x^2$ Hyperbola

$$\frac{x^2}{3^2} - \frac{y^2}{?} = 1$$

$c^2 = a^2 + b^2 \Rightarrow 5^2 = 3^2 + ?$

$? = 16$

$\Rightarrow \boxed{\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1}$




18. $x^2 - 4y^2 - 4x + 8y - 4 = 0 \Rightarrow x^2 - 4x - 4y^2 - 8y = 4$

$\Rightarrow (x^2 - 4x + 4) - 4(y^2 - 2y + 1) = 4 + 4 + 1(-4) \Rightarrow \dfrac{(x-2)^2}{4} - \dfrac{4(y+1)^2}{4} = \dfrac{4}{4}$

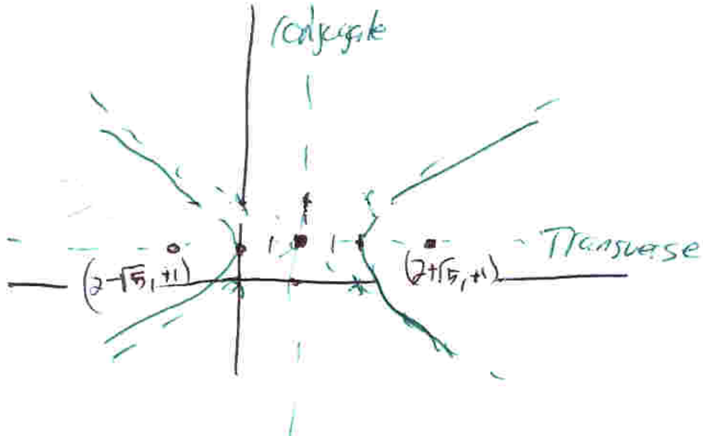
$\Rightarrow \dfrac{(x-2)^2}{4} - \dfrac{(y+1)^2}{1} = 1 \Rightarrow \boxed{\text{cen } (2,+1)}$, $\boxed{\text{vert } (4,+1), (0,+1)}$

$c^2 = 4 + 1 = 5 \quad c = \sqrt{5}$ $\boxed{\text{Foci } (2 \pm \sqrt{5}, +1)}$

Slope of asymptotes $= \dfrac{y}{x} = \dfrac{\sqrt{1}}{\sqrt{4}} = \dfrac{1}{2}$

$\boxed{\text{so asymptotes } y - 1 = \pm \tfrac{1}{2}(x - 2)}$

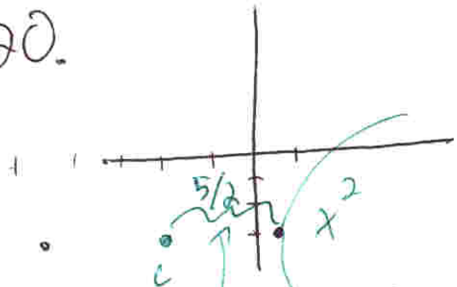
$\boxed{\text{transverse axis } y = +1}$



19. $\frac{x^2}{9} - \frac{y^2}{25} = 1$ Slope $= \frac{y}{x} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3}$, center $(0,0)$ so $y-0 = \pm\frac{5}{3}(x-0)$

$$\boxed{y = \pm\frac{5}{3}x}$$

20.




$\frac{\frac{1}{2} + \frac{-9}{2}}{2} = \frac{-\frac{8}{2}}{2} = \frac{-4}{2} = -2$ midpoint = x of center

center $(-2, -3)$ check: Asymptotes $y+3 = \pm\frac{6}{5}(x+2)$

$(-2, -3)$ ✓

slope $= \frac{y}{x} = \frac{6}{5}$. $x = \frac{5}{2}$

so $\frac{y}{\frac{5}{2}} = \frac{6}{5} \Rightarrow 5y = \frac{5}{2}\cdot 6 = 15$

$y = 3,\ y^2 = 9$

$$\boxed{\frac{(x+2)^2}{(\frac{5}{2})^2} - \frac{(y+3)^2}{9} = 1}$$