EXAM 4 REVIEW KEY

1. $\begin{cases} 6x+y=8 \\ 9x+3y=6 \end{cases} \xrightarrow{-3R_1} \begin{cases} -18x-3y=-24 \\ 9x+3y=6 \end{cases}$

$+$

$-9x = -18$

$\frac{-9x}{-9} = \frac{-18}{-9}$

$\Rightarrow \boxed{x=2}$, so

$6(2)+y=8$

$12+y=8$

$\boxed{y=-4}$

$\boxed{(2,-4)}$

2. Plug the given points into the equation $y=ax^2+bx+c$:

$(-2,-4)$: $-4=a(-2)^2+b(-2)+c$

$\Rightarrow \underline{4a-2b+c=-4}$

$(1,-1)$: $-1=a(+1)^2+b(1)+c$

$\Rightarrow \underline{a+b+c=-1}$

$(3,-19)$: $-19=a(3)^2+b(3)+c$

$\Rightarrow \underline{9a+3b+c=-19}$

& we have the system

$$\begin{cases} 4a-2b+c=-4 \\ a+b+c=-1 \\ 9a+3b+c=-19 \end{cases}$$

NOW SOLVE IT:

WE CAN ELIMINATE C EASILY: R_1-R_2 & R_1-R_3

R_1-R_2: $4a-2b+c=-4$

$+\ -a-b-c=1$

$3a-3b=-3$

or $\underline{a-b=-1}$

R_1-R_3: $4a-2b+c=-4$

$+\ -9a-3b-c=19$

$-5a-5b=15$

or $\underline{-a-b=3}$

NOW WE HAVE

$$\begin{cases} a-b=-1 \\ -a-b=3 \end{cases}$$

$+$

$-2b=2$

$\boxed{b=-1}$

$a-(-1)=-1$

$a+1=-1$

$\boxed{a=-2}$

$a+b+c=-1$

$-2-1+c=-1$

$\boxed{c=2}$

& OUR ANSWER IS

$\boxed{y=-2x^2-x+2}$

3. $\begin{vmatrix} 3 & 4 \\ -8 & 8 \end{vmatrix} = (3)(8) - (-8)(4) = 24 + 32 = \boxed{56}$

DOWN ↘ − UP ↗

4. $\begin{vmatrix} 5 & 9 \\ -2 & x \end{vmatrix} = 8 \Rightarrow 5x - (-18) = 8 \Rightarrow \underset{-18}{5x+18} = \underset{-18}{8} \Rightarrow \frac{5x}{5} = \frac{-10}{5} \Rightarrow \boxed{x = -2}$

5. $\begin{cases} 4x - 7y = 5 \\ 2x + 5y = -3 \end{cases}$ $D = \begin{vmatrix} 4 & -7 \\ 2 & 5 \end{vmatrix} = 20 - (-14) = 34$ $D \neq 0$ SO CRAMER'S RULE <u>DOES</u> APPLY.

x COL y COL

$D_x = \begin{vmatrix} 5 & -7 \\ -3 & 5 \end{vmatrix} = 25 - 21 = 4$ $D_y = \begin{vmatrix} 4 & 5 \\ 2 & -3 \end{vmatrix} = -12 - 10 = -22$

$x = \frac{D_x}{D} = \frac{4}{34} = \frac{2}{17}$, $y = \frac{D_y}{D} = \frac{-22}{34} = \frac{-11}{17}$ $\boxed{\left(\frac{2}{17}, \frac{-11}{17}\right)}$

6. $\left|\begin{matrix} 2 & 1 & 6 \\ 1 & 2 & 2 \\ 1 & 5 & 6 \end{matrix}\right| \begin{matrix} 2 & 1 \\ 1 & 2 \\ 1 & 5 \end{matrix} = \underbrace{(2)(2)(6) + (1)(2)(1) + (6)(1)(5)}_{\text{DOWN}} - \underbrace{[(1)(2)(6) + (5)(2)(2) + (6)(1)(1)]}_{\text{UP}}$

$= 24 + 2 + 30 - (12 + 20 + 6) = 56 - 38 = \boxed{18}$

7. (work like #26)

$D = \begin{vmatrix} -2 & 0 & -7 \\ -3 & 3 & 3 \\ 2 & -2 & 0 \end{vmatrix} = -12$ $D_x = \begin{vmatrix} -81 & 0 & -7 \\ 21 & 3 & 3 \\ 4 & -2 & 0 \end{vmatrix} = -108$

$D_y = \begin{vmatrix} -2 & -81 & -7 \\ -3 & 21 & 3 \\ 2 & 4 & 0 \end{vmatrix} = -84$ $D_z = \begin{vmatrix} -2 & 0 & -81 \\ -3 & 3 & 21 \\ 2 & -2 & 4 \end{vmatrix} = -108$

$x = \frac{D_x}{D} = \frac{-108}{-12} = 9$, $y = \frac{D_y}{D} = \frac{-84}{-12} = 7$ $z = \frac{D_z}{D} = \frac{-108}{-12} = 9$ $\boxed{(9, 7, 9)}$

8. NOT POSSIBLE TO ADD MATRICES, THEY HAVE TO BE THE SAME ORDER (SIZE)

9. $\begin{bmatrix} 8 & -7 \\ -6 & -9 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 7 & 2 \\ 7 & 1 \\ 8 & -7 \end{bmatrix} = \begin{bmatrix} 8+7 & -7+2 \\ -6+7 & -9+1 \\ 5+8 & 6-7 \end{bmatrix} = \boxed{\begin{bmatrix} 15 & -5 \\ 1 & -8 \\ 13 & -1 \end{bmatrix}}$

10. $-2A+4B = -2\begin{bmatrix} 2 & -1 \\ 7 & 9 \end{bmatrix} + 4\begin{bmatrix} 5 & -3 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -14 & -18 \end{bmatrix} + \begin{bmatrix} 20 & -12 \\ 16 & 28 \end{bmatrix} = \boxed{\begin{bmatrix} 16 & -10 \\ 2 & 10 \end{bmatrix}}$

11.

$$\begin{array}{cc} & \begin{bmatrix} 2 & 3 & -8 \\ -7 & -3 & -6 \\ 4 & -3 & 8 \end{bmatrix} \\ \begin{bmatrix} -1 & -1 & 3 \\ 9 & 4 & -6 \end{bmatrix} & \boxed{\begin{bmatrix} 17 & -9 & 38 \\ -34 & 33 & -144 \end{bmatrix}} \end{array}$$

TO GET THE −144, for example, it is POSITION ROW 2, COLUMN 3.

$$[9\ 4\ -6]\begin{bmatrix} -8 \\ -6 \\ 8 \end{bmatrix}$$

THE PATTERN is $1^{st}\cdot 1^{st} + 2^{nd}\cdot 2^{nd} + 3^{rd}\cdot 3^{rd}$

$(9)(-8) + (4)(-6) + (-6)(8) = -72-24-48 = -144$

12.

$\left[\begin{array}{cc|cc} 6 & -4 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{array}\right]$ $R_1: r_2+r_1$

$$\begin{array}{r} 6\ -4\ 1\ 0 \\ +\ 0\ \ 4\ 0\ 1 \\ \hline 6\ \ 0\ 1\ 1 \end{array}$$

$\left[\begin{array}{cc|cc} 6 & 0 & 1 & 1 \\ 0 & 4 & 0 & 1 \end{array}\right]$ $R_1: \frac{1}{6}r_1$ $\frac{1}{6}(6\,0\,1\,1) = 1\ 0\ \frac{1}{6}\ \frac{1}{6}$

$R_2: \frac{1}{4}r_2$ $\frac{1}{4}(0\,4\,0\,1) = 0\ 1\ 0\ \frac{1}{4}$

$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{6} & \frac{1}{6} \\ 0 & 1 & 0 & \frac{1}{4} \end{array}\right]$ SO $A^{-1} = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{4} \end{bmatrix}$ TO CHECK, WE NEED $A^{-1}\cdot A = I$

$$\begin{array}{cc} & \begin{bmatrix} 6 & -4 \\ 0 & 4 \end{bmatrix} \\ \begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{4} \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

✓

13. $\begin{cases} x+3y=-8 \\ 21x+6y=3 \end{cases} \Rightarrow \begin{bmatrix} 1 & 3 \\ 21 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \end{bmatrix} \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow X = A^{-1}\cdot b$

A^{-1}: $\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 21 & 6 & 0 & 1 \end{array}\right]$ RREF LIKE #32 $\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -2/19 & 1/19 \\ 0 & 1 & 7/19 & -1/57 \end{array}\right]$

SO $X = A^{-1}\cdot b = \begin{bmatrix} -2/19 & 1/19 \\ 7/19 & -1/57 \end{bmatrix}\begin{bmatrix} -8 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$\left(\frac{-2}{19}\right)(-8) + \left(\frac{1}{19}\right)(3) = \frac{16}{19} + \frac{3}{19} = \frac{19}{19} = 1$

$\left(\frac{7}{19}\right)(-8) + \left(-\frac{1}{57}\right)(3) = \frac{-56}{19} - \frac{3}{57} = \frac{-56}{19} - \frac{1}{19} = \frac{-57}{19} = -3$

$\boxed{(1,-3)}$

14. $\frac{x-1}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3} \xrightarrow[(x-4)(x-3)]{\text{MULTIPLY BY LCD}} x-1 = A(x-3) + B(x-4)$

LET $x=3 \Rightarrow 2 = -B \Rightarrow B=-2$

$x=4 \Rightarrow 3 = A$

SO $\boxed{\frac{3}{x-4} + \frac{-2}{x-3}}$

15. $\frac{50-7x}{x^3-10x^2+25x} = \frac{A}{x} + \frac{B}{x-5} + \frac{C}{(x-5)^2} \xrightarrow{\text{MULTIPLY BY LCD}} 50-7x = A(x-5)^2 + Bx(x-5) + Cx$

$x(x^2-10x+25)$

$x(x-5)^2$

LET $x=5$ $\quad \frac{15}{5} = \frac{5C}{5} \Rightarrow C=3$

LET $x=0$ $\quad 50 = 25A \Rightarrow A=2$

LET $x=1$ $\quad 43 = 16A - 4B + C = 16(2) - 4B + 3$

$43 = 32 - 4B + 3 \Rightarrow -4B = 8$

$\Rightarrow B = -2$

SO $\boxed{\frac{2}{x} + \frac{-2}{x-5} + \frac{3}{(x-5)^2}} \Longleftarrow$

16. $\frac{x^2-111}{x^4-x^2-72} = \frac{A}{x+3} + \frac{B}{x-3} + \frac{Cx+D}{x^2+8} \xrightarrow{\text{LCD}} x^2-111 = A(x-3)(x^2+8) + B(x+3)(x^2+8) + (Cx+D)(x+3)(x-3)$

$(x^2-9)(x^2+8)$

$(x+3)(x-3)$

LET $x=3$ $\quad -102 = B(6)(17) \Rightarrow B=-1$

LET $x=-3$ $\quad -102 = A(-6)(17) \Rightarrow A=1$

LET $x=0$ $\quad -111 = 1(-3)(8) + -1(3)(8) + D(3)(-3) \Rightarrow D=7$

LET $x=1$ $\quad -110 = 1(-2)(9) + -1(4)(9) + (C+7)(4)(-2) \Rightarrow C=0$

SO $\boxed{\frac{1}{x+3} + \frac{-1}{x-3} + \frac{\cancel{0x}+7}{x^2+8}}$

17. $\dfrac{x^2+2x-2}{(x^2+2)^2} = \dfrac{(Ax+B)}{x^2+2} + \dfrac{(Cx+D)}{(x^2+2)^2}$ $\overset{LCD}{\Rightarrow}$ $x^2+2x-2 = (Ax+B)(x^2+2) + Cx+D$

LET $x=0$ $\quad -2 = 2B+D$ NO MORE SMART VALUES

SO INSTEAD LET'S compare coefficients:

$\underline{0x^3} + \underline{\underline{1x^2}} + 2x - 2 = \underline{Ax^3} + 2Ax + Bx^2 + 2B + Cx + D$

x^3 coefficients: $0 = A$ $\quad 2 = 0 + C \Rightarrow C = 2$

x^2 coeffs: $1 = B$

x coeffs: $2 = 2A + C$ $\quad -2 = 2(1) + D \Rightarrow D = -4$

Constants: $-2 = 2B + D$

SO $\boxed{\dfrac{0x+1}{x^2+2} + \dfrac{2x-4}{(x^2+2)^2}}$

18-21 COME FROM SECTION 8.6, NONLINEAR SYSTEMS, AND ARE OPTIONAL. CHECK WITH INSTRUCTOR

18. $\begin{cases} x^2+y^2=181 \\ x+y=-19 \end{cases}$ SUBSTITUTE $y=-19-x$ $\quad x^2+(-19-x)^2=181 \Rightarrow x^2+361+38x+x^2=181 \Rightarrow$

$\qquad \underset{-181}{} \qquad \underset{-181}{}$

$\Rightarrow 2x^2+38x+180=0 \Rightarrow 2(x^2+19x+90)=0$

$\Rightarrow 2(x+9)(x+10)=0 \Rightarrow x=-9, x=-10$

$x=-9$: $y=-19-x=-19-(-9)=-10$

$x=-10$ $\quad y=-19-x=-19-(-10)=-9$

$\boxed{(-9,-10), \; (-10,-9)}$

19. $\begin{cases} 5x^2-2y^2=2 \\ 3x^2+4y^2=48 \end{cases}$ $\quad$ 2R1 $\quad 10x^2-4y^2=4$

R2 $+ \; 3x^2+4y^2=48$

$13x^2=52$

$x^2=4$ $\quad x=\pm\sqrt{4}$ $\quad x=\pm 2$

$x=2$: $5(2)^2-2y^2=2$

$-2y^2=2-20$

$-2y^2=-18$

$y^2=9 \quad y=\pm 3$

$\boxed{(2,3),(2,-3),(-2,3),(-2,-3)}$

20. $\begin{matrix} x^2+y^2=26 \\ x+y=6 \end{matrix}$ $\Rightarrow y=6-x, \; x^2+(6-x)^2=26$

$\Rightarrow 2x^2-12x+36=26$

$\Rightarrow 2x^2-12x+10=0 \Rightarrow 2(x^2-6x+5)=0$

$\Rightarrow 2(x-5)(x-1)=0 \Rightarrow x=5,1$

$x=5$: $y=6-x=6-5=1$ $\quad \boxed{(5,1)}$

$x=1$: $y=6-x=6-1=5$ $\quad \boxed{(1,5)}$

$\boxed{1 \,\&\, 5}$

21. $\begin{cases} 2x+2y=26 & \text{Perimeter} \\ xy=36 & \text{Area} \end{cases}$

$\dfrac{2x+2y=26}{2 \quad 2 \quad 2}$

$x+y=13$

$y=13-x$

$x(13-x)=36$

$\Rightarrow 13x-x^2=36 \Rightarrow x^2-13x+36=0$

$(x-4)(x-9)=0$

$x=4$; $y=13-x=13-4=9$

$(4,9)$

$x=9, y=4$ $\quad \boxed{4m \times 9m}$

22. $\frac{6!}{4!} = \frac{6\cdot 5\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = 6\cdot 5 = \underline{30}$

OR $\frac{6!}{4!} = \frac{6\cdot 5\cdot \cancel{4!}}{\cancel{4!}} = 6\cdot 5 = 30.$

23.

n	1	2	3	4	5
S_n	$3(1)-3$ $=0$	$3(2)-3$ $=3$	$3(3)-3$ $=6$	$3(4)-3$ $=9$	$3(5)-3$ $=12$

so $\underline{S_n = \{0, 3, 6, 9, 12, \dots\}}$

24.

n	1	2	3	4	5
S_n	$(-1)^{1-1}\left(\frac{1+3}{2(1)-1}\right) = \frac{4}{1} = 4$	$(-1)^{2-1}\left(\frac{2+3}{2(2)-1}\right) = (-1)\left(\frac{5}{3}\right) = -5/3$	$(-1)^{3-1}\left(\frac{3+3}{2(3)-1}\right) = (-1)^2\left(\frac{6}{5}\right) = \frac{6}{5}$	$(-1)^{4-1}\left(\frac{4+3}{2(4)-1}\right) = (-1)^3\left(\frac{7}{7}\right) = -1$	$\frac{8}{9}$

so $\underline{S_n = \{4, -5/3, \frac{6}{5}, -1, \frac{8}{9} \dots\}}$

25. This sequence is arithmetic and because $10-4=6$, $16-10=6$, we see the common difference is $\underline{d=6}$. ONE model for an arithmetic sequence is $\{a_n\} = \{dn + C\}$.

We have $\{a_n\} = \{6n + C\}$ & if $n=1$, we should get $a_1 = 4$

so $4 = d(1) + C = 6(1) + C = 6 + C$ & $C = -2$ so $\boxed{\{a_n\} = \{6n-2\}}$

OR Model $a_n = a_1 + (n-1)d \Rightarrow a_n = 4 + (n-1)6 = 4 + 6n - 6 = 6n - 2$

so $\boxed{\{a_n\} = 6n - 2} = \underline{2(3n-1)}$

26. This sequence is geometric & since $\frac{1}{4} \div 1 = \frac{1}{4}$ & $\frac{1}{16} \div \frac{1}{4} = \frac{1}{4}$, we see $r = \frac{1}{4}$. The model is $\{a_n\} = a_1 r^{n-1}$

so $a_n = 1 \cdot \left(\frac{1}{4}\right)^{n-1}$ & $\boxed{\{a_n\} = \left\{\left(\frac{1}{4}\right)^{n-1}\right\}} = \frac{1^{n-1}}{4^{n-1}} = \underline{\frac{1}{4^{n-1}}}$

27. We start with $a_1 = 5$, $a_n = 3a_{n-1} + 1$

so if $n = 2$, $\underline{a_2} = 3a_{2-1} + 1 = 3a_1 + 1 = 3(5) + 1 = \underline{16}$

$n = 3 \Rightarrow \underline{a_3} = 3a_{3-1} + 1 = 3a_2 + 1 = 3(16) + 1 = \underline{49}$

$n = 4 \Rightarrow \underline{a_4} = 3a_{4-1} + 1 = 3a_3 + 1 = 3(49) + 1 = \underline{148}$

so $\boxed{\{a_n\} = \{5, 16, 49, 148, \cdots\}}$

28. $a_1 = -9$, $a_n = n - a_{n-1}$

$n = 2 \Rightarrow \underline{a_2} = 2 - a_{2-1} = 2 - a_1 = 2 - (-9) = \underline{11}$

$n = 3 \Rightarrow \underline{a_3} = 3 - a_{3-1} = 3 - a_2 = 3 - 11 = \underline{-8}$

$n = 4 \Rightarrow \underline{a_4} = 4 - a_{4-1} = 4 - a_3 = 4 - (-8) = \underline{12}$

so $\boxed{\{a_n\} = \{-9, 11, -8, 12, \cdots\}}$

29. $a_1 = -4$, $a_2 = -4$, $a_{n+2} = a_{n+1} - 4a_n$

To get a_3, we need $n = 1$ so $a_{n+2} = a_{1+2} = a_3$.

$n = 1 \Rightarrow a_{1+2} = \underline{a_3} = a_{1+1} - 4a_1 = a_2 - 4a_1 = -4 - 4(-4) = \underline{12}$

$n = 2 \Rightarrow a_{2+2} = \underline{a_4} = a_{2+1} - 4a_2 = a_3 - 4a_2 = 12 - 4(-4) = \underline{28}$

$n = 3 \Rightarrow a_{3+2} = \underline{a_5} = a_{3+1} - 4a_3 = a_4 - 4a_3 = 28 - 4(12) = \underline{-20}$

so $\boxed{a_5 = -20}$

30. $\sum_{k=1}^{n} (k+10)^2 = \overset{k=1}{(1+10)^2} + \overset{k=2}{(2+10)^2} + \overset{k=3}{(3+10)^2} + \cdots + \overset{k=n}{(n+10)^2} =$

$= 11^2 + 12^2 + 13^2 + \cdots + (n+10)^2 = \boxed{121 + 144 + 169 + \cdots + (n+10)^2}$

31. We need to find a general form for the sequence: We can see $4-2=2$, $6-4=2$, that this is an arithmetic sequence with $d=2$, and $a_1=2$. So

$a_n = a_1 + (n-1)d = 2 + (n-1)2 = 2n.$

If $n=6$, we get $a_6 = 2(6) = 12$, so the last term in this sum is the 6^{th}. Thus, $2+4+6+\cdots+12 = \boxed{\sum_{k=1}^{6} 2k}$

32. Since this is a general sum, by looking at the last term $\frac{7^n}{n}$, we get the general form of the sequence $\{a_n\} = \left\{\frac{7^n}{n}\right\}$ and the last term is the n^{th} term, and the first term 7 is when $n=1$.

So $7 + \frac{7^2}{2} + \frac{7^3}{3} + \cdots + \frac{7^n}{n} = \boxed{\sum_{k=1}^{n} \frac{7^k}{k}}$

33. $\sum_{k=3}^{6} (4k-4) = (4(3)-4) + (4(4)-4) + (4(5)-4) + (4(6)-4)$

$= 8 + 12 + 16 + 20 = \boxed{56}$

34. $\sum_{k=1}^{4}\left(-\frac{1}{4}\right)^k = \left(-\frac{1}{4}\right)^1 + \left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right)^3 + \left(-\frac{1}{4}\right)^4 =$

$= -\frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \frac{1}{256} = \frac{-64}{256} + \frac{16}{256} - \frac{4}{256} + \frac{1}{256} = \boxed{\frac{-51}{256}}$

35. $\{s_n\} = \{5n+6\} = \{5(1)+6,\ 5(2)+6,\ 5(3)+6,\ 5(4)+6, \dots\}$

$= \boxed{\{11, 16, 21, 26, \dots\} \text{ \& } d = 16-11 = 5}$

36. $a_1 = 9,\ d = -2$ & $a_n = a_1 + (n-1)d$

so $a_n = 9 + (n-1)(-2) = \boxed{11 - 2n = a_n}$

& $a_{14} = 9 + (14-1)(-2) = 9 - 26 = \boxed{-17 = a_{14}}$

37. Checking $6-0=6$, $12-6=6$ we get an arithmetic sequence with $a_1 = 0$, $d = 6$ so $a_n = a_1 + (n-1)d = 0 + (n-1)6$

so $\{a_n\} = \{6n-6\}$ & $\boxed{a_{19} = 6(19) - 6 = 108}$

38. For a recursive definition of an arithmetic sequence, we need a_1 & then $a_n = a_{n-1} + d$, so we need d.

$a_{15} = a_7 + 8d \Rightarrow -33 = -9 + 8d$

so $\boxed{d = -3}$

$a_7 = a_1 + 6d \Rightarrow -9 = a_1 + 6(-3)$

so $a_1 = 9$

&

OR

$a_{15} = a_1 + 14d \Rightarrow -33 = a_1 + 14d$

$a_7 = a_1 + 6d \Rightarrow -9 = a_1 + 6d$

$-24 = 8d$

$\boxed{d = -3}$

& $-9 = a_1 - 18 \Rightarrow a_1 = 9$ so

$\boxed{a_1 = 9,\ a_n = a_{n-1} - 3}$

39. What type of sequence are we summing?

$4-2=2,\ 6-4=2$: ARITHMETIC w/ $d=2$

SUM FORMULA $S_n = \frac{n}{2}(a_1 + a_n)$: we have $a_1 = 2$, $a_n = 610$

but what is n?

$a_n = a_1 + (n-1)d \Rightarrow 610 = 2 + (n-1)\cdot 2 \Rightarrow n = 305!$

so $S_{305} = \frac{305}{2}(2+610) = \boxed{93,330}$

40. Like #39, this is arithmetic. $\{4n-4\}_{n=1} = \{0, 4, 8, 12 \ldots\}$ & $d = 4$.

we need n, a_1, a_n. $n = 28$, $a_1 = 0$ & $a_n = 4(28) - 4 = 108$

so $S_{28} = \frac{28}{2}(0 + 108) = \boxed{1512}$

41. Row 1: 22

Row 2: 25

Row 3: 28

⋮

Row 21 ?

This is another arithmetic sequence sum with $a_1 = 22$, $d = 3$, but a_n?

$a_n = a_{21} = a_1 + (n-1)d = 22 + (21-1)(3) = 82$

so $S_{21} = \frac{21}{2}(22 + 82) = 1092$

so $\boxed{1092 \text{ seats}}$

42. $S_1 = 6^1 = 6$, $S_2 = 6^2 = 36$, $S_3 = 6^3 = 216$, $S_4 = 6^4 = 1296$

so $\{S_n\} = \boxed{\{6, 36, 216, 1296, \cdots\}}$, $\boxed{r = 36 \div 6 = 6}$

43. $d_1 = \frac{3^1}{18} = \frac{1}{6}$, $d_2 = \frac{3^2}{18} = \frac{1}{2}$, $d_3 = \frac{3^3}{18} = \frac{3}{2}$, $d_4 = \frac{3^4}{18} = \frac{9}{2}$

$\{d_n\} = \boxed{\{\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \cdots\}}$ & $r = \frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \cdot \frac{6}{1} = \boxed{3 = r}$

44. $\{3n - 4\} = \{3(1) - 4, 3(2) - 4, 3(3) - 4, \cdots\} = \{-1, 2, 5, \cdots\}$

& $2 - (-1) = 3$, $5 - 2 = 3$

$\boxed{\text{ARITHMETIC \& } d = 3}$

45. $\{5n^2 - 3\} = \{5(1)^2 - 3, 5(2)^2 - 3, 5(3)^2 - 3, \cdots\} = \{2, 17, 42, \cdots\}$

& $17 - 2 = 15$, $42 - 17 = 25$ NOT ARITHMETIC

$17 \div 2 = 8.5$, $42 \div 17 \approx 2.47$ NOT GEOMETRIC

$\boxed{\text{NEITHER}}$

46. For a geometric sequence, $a_n = a_1 r^{n-1}$.

we have $a_1 = a = 6$ & $r = -5$

so $\boxed{a_5 = 6(-5)^5 = -18750}$ $\boxed{\& \ a_n = 6(-5)^{n-1}}$

47. $a_1 = -1$, $2 \div -1 = -2$, $-4 \div 2 = -2$ so $r = -2$

$a_7 = a_1 r^6 = -1(-2)^6 = \boxed{-64 = a_7}$

48. $3 \div 6 = \frac{1}{2}$, $\frac{3}{2} \div 3 = \frac{1}{2}$ so $r = \frac{1}{2}$, $a_1 = 6$

& $a_n = a_1 \cdot r^{n-1} = \boxed{\left\{6\left(\frac{1}{2}\right)^{n-1}\right\} = \{a_n\}}$

49. $1 \div 2 = \frac{1}{2}$, $\frac{1}{2} \div 1 = \frac{1}{2}$ so $r = 1/2$ & $a_1 = 2$

so $a_n = a_1 \cdot r^{n-1} = \boxed{2\left(\frac{1}{2}\right)^{n-1}} = \left(\frac{1}{2}\right)^{-1}\left(\frac{1}{2}\right)^{n-1}$

$= \underline{\left(\frac{1}{2}\right)^{n-2}}$

50. If you depreciate by 25%, there will be 75% left. So after 1 year the value is 69000(.75) = \$51750. After 2 years, $69000(.75)^2$ = \$38812.50 We have $a_1 = 51750$ & $r = .75$

so $a_7 = a_1 r^6 = 51750(.75)^6 = \boxed{\$9210.39}$