1050 TEST 3 REVIEW

1. $(f \circ f)(1) = f(f(1)) = f(2(1)+4) = f(6) = 2(6)+4 = 16.$

2. $(f \circ g)(x) = f(g(x)) = f(8x-8) = \sqrt{(8x-8)+4} = \sqrt{\underbrace{8x-4}_{4(2x-1)}} = 2\sqrt{2x-1}$

3. DOMAIN OF $f \circ g$. WE INHERET THE DOMAIN OF THE INNER FUNCTION G, SO WE HAVE $x \neq -2$. WE ALSO CAN'T HAVE $g(x)$ equal an x restricted from $f(x)$'s domain, so $g(x) \neq -9$.

$\frac{9}{x+2} \neq -9 \qquad 9 \neq -9(x+2) \qquad 9 \neq -9x-18 \qquad 27 \neq -9x \qquad x \neq -3$

SO DOMAIN OF $(f \circ g)(x)$ is $\{x \mid x \neq -2, -3\}$

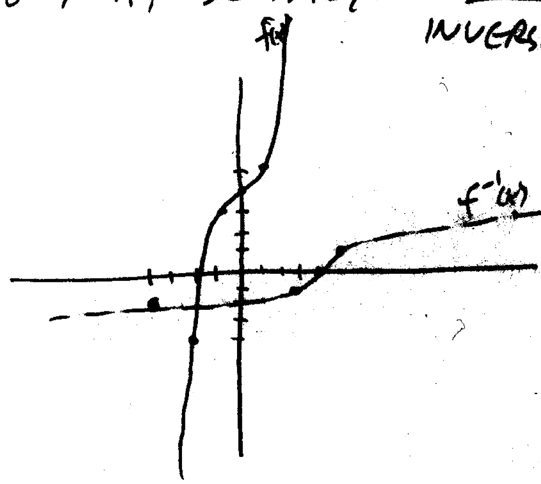
4. Is $f(x)$ and $g(x)$ inverses? We need to check that $(f \circ g)(x) = x$. $(f \circ g)(x) = f(g(x)) = f\left(\frac{x}{9}-6\right) =$

$= 9\left(\frac{x}{9}-6\right)+6 = x-54+6 = x-48 \neq x$, SO THEY ARE <u>NOT</u> INVERSES.

5. $f(x) = x^3+4$ $\qquad f^{-1}(x)$ (SWITCH x & y)

x	$f(x)$
-2	-4
-1	3
0	4
1	5
2	16

x	$f^{-1}(x)$
-4	-2
3	-1
4	0
5	1
16	2



6. $f(x) = (x+2)^3 - 8$

$y = (x+2)^3 - 8$

$x = (y+2)^3 - 8$

$x+8 = (y+2)^3$

$\sqrt[3]{x+8} = y+2$

$\sqrt[3]{x+8} - 2 = y$

$\boxed{f^{-1}(x) = \sqrt[3]{x+8} - 2}$

7. $4.4 \wedge 4.16 \approx 475.076$

8.

x	$H(x)$
-1	3
0	7
1	11
2	15
3	19

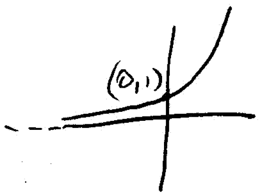
EXPONENTIAL IF y VALUES HAVE COMMON RATIO:

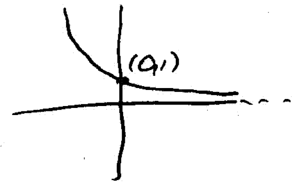
$\frac{7}{3}, \frac{11}{7}, \frac{15}{11}, \frac{19}{15}$ NOT EQUAL

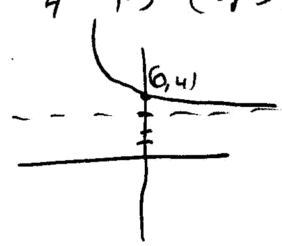
SO NOT EXPONENTIAL

($7-3 = 11-7 = 15-11 = 19-15 = 4$. THIS IS LINEAR)

9. 4^{x} $\Rightarrow$ 4^{-x} (y ref) $\Rightarrow$ $4^{-x}+3$ (up 3)







DOM $(-\infty,\infty)$
RAN $(3,\infty)$
NO VERTICAL ASYMPTOTE
HORIZONTAL ASYMPTOTE
$y=3$

10. $3^{6-3x}=\frac{1}{27}=\frac{1}{3^3}=3^{-3} \Rightarrow 3^{6-3x}=3^{-3} \Rightarrow 6-3x=-3 \Rightarrow \cancel{-3}x=-9 \Rightarrow x=3.$

11. $\log_5 5^{-3}=\log_5\frac{1}{125} \Rightarrow -3=\log_5\left(\frac{1}{125}\right)$

12. $61^{y}=\cancel{61}^{\log_{61}x} \Rightarrow 61^{y}=x$

13. $\log_5 1=0$ "5 to what power is 1?" 0

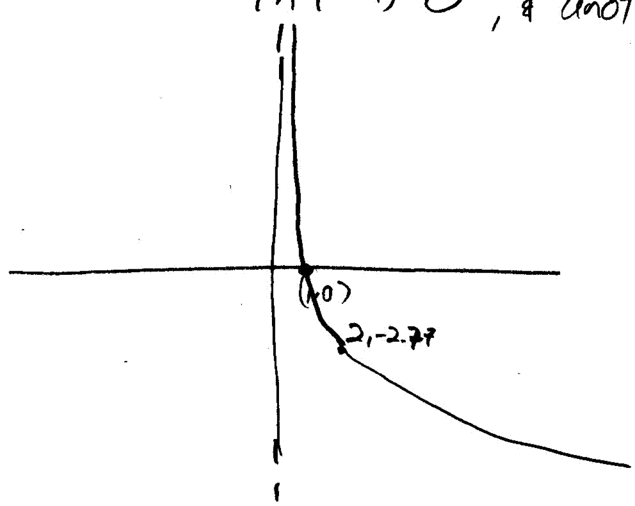
14. $\log_4\sqrt{4}=\log_4 4^{1/2}=\frac{1}{2}\log_4 4=\frac{1}{2}\cdot 1=\frac{1}{2}$

15. $\ln e=\cancel{\ln e}^{1}=1$

16. DOMAIN of $\log_6\square$ is $\square>0$. So $\log_5(x+2) \Rightarrow x+2>0$
$\boxed{x>-2 \quad (-2,\infty)}$

17. GRAPH $f(x)=-4\ln x$ Use $\ln 0$ is Vertical Asymptote, $\ln 1$ is 0, & another x.

x	y
0	VA
1	$-4\ln 1=-4(0)=0$
2	$-4\ln 2\approx -2.77$



18. $5^{\log_5 x} = 5^2 \Rightarrow x = 5^2 = 25.$

19. $\log_6 \frac{A}{B} = \log_6 A - \log_6 B = 4 - -20 = 24$

20. $\ln y = \ln 4x + \ln c \Rightarrow \ln y = \ln(4xc) \Rightarrow e^{\ln y} = e^{\ln(4xc)} \Rightarrow$

$\Rightarrow y = 4xc = 4cx$

21. $\log_3 \frac{16\sqrt{x}}{y} = \log_3 \frac{16x^{1/2}}{y} = \log_3 16 + \log_3 x^{1/2} - \log_3 y = \log_3 16 + \frac{1}{2}\log_3 x - \log_3 y.$

22. $6\log_c q - \frac{5}{6}\log_c r + \frac{1}{2}\log_c f - 5\log_c P = \log_c q^6 - \log_c r^{5/6} + \log_c f^{1/2} - \log_c P^5 = \log_c\left(\frac{q^6 f^{1/2}}{r^{5/6} p^5}\right)$

23. $\log_{6.3} 2.1 = \frac{\ln 2.1}{\ln 6.3} \left(\text{or } \frac{\log 2.1}{\log 6.3}\right) \approx 0.403$

24. By change of base, $\log_8 3x = \frac{\log_2 3x}{\log_2 8} = \frac{\log_2 3x}{3} = \frac{1}{3}\log_2 3x.$

so $\frac{1}{3}\log_2(x+6) = \frac{1}{3}\log_2 3x \Rightarrow \log_2(x+6) = \log_2 3x \Rightarrow x+6 = 3x$

$\Rightarrow 2x = 6 \Rightarrow x = 3.$

25. $\log x + \log 6 = \log 6x = 0.3031 \Rightarrow 6x = 10^{.3031} \Rightarrow x = \frac{10^{.3031}}{6} \approx .33$

26. $\pi^{x+1} = e^{2x}$

$\ln \pi^{x+1} = \ln e^{2x}$

$(x+1)\ln\pi = 2x$

$x\ln\pi + \ln\pi = 2x$

$\ln\pi = 2x - x\ln\pi$

$\ln\pi = x(2 - \ln\pi)$

$x = \frac{\ln\pi}{2 - \ln\pi}.$

27. Compounded quarterly $\Rightarrow$ $n=4$. $A=P\left(1+\frac{r}{n}\right)^{n\cdot m}$ ← using m instead of t

$r=.11$ (use decimal)

$A=480\left(1+\frac{.11}{4}\right)^{(4\cdot 6)} \approx \$920.46$

28. COMPOUNDED CONTINUOUSLY $\Rightarrow$ $A=Pe^{rt}$

$r=.091$. FOR EFFECTIVE RATE, USE $P=100$, $t=1$

SO $A=100\,e^{.091(1)} \approx \$109.527$

EFFECTIVE RATE $= 9.527\%$

29. COMPOUNDED MONTHLY $\Rightarrow$ $A=P\left(1+\frac{r}{n}\right)^{nt}$, $n=12$

SO $10000=P\left(1+\frac{.18}{12}\right)^{(12\cdot 2)}=P(1.4295\ldots)$

SO $P=\dfrac{10000}{1.42950\ldots}$ or $10000 \div$ ANS $= \$6995.44$

30. DOUBLING. USE $P=100$, $A=200$. ANNUAL RATE $\Rightarrow n=1$. $t=1$

$200=100\left(1+\frac{r}{1}\right)^{(1\cdot 9)} \Rightarrow 200=100(1+r)^9 \Rightarrow 2=(1+r)^9$

$\Rightarrow \sqrt[9]{2}=1+r \Rightarrow r=\sqrt[9]{2}-1=2\wedge(1\div 9)-1\approx 0.800597$

$= 8.006\%$

31. POPULATION FOLLOWS THE EXPONENTIAL LAW $\Rightarrow$ $A=Pe^{rt}$ (or $N=N_0e^{kt}$)

$P=120$ (initial value) $t=0$ is 1990, so 1992 is $t=2$.

$240=120e^{r(2)} \Rightarrow 2=e^{2r} \Rightarrow \ln 2=2r \Rightarrow r=\frac{1}{2}\ln 2$.

IN 8 years,

$A=120\,e^{(\frac{1}{2}\ln 2)\cdot 8}=1920$ rabbits.

32. $\frac{1}{2}$ life is 5600 years. Use $A=Pe^{rt}$ (or $Q=Q_0e^{kt}$)
with $P=100$, $A=50$.

$$50=100e^{r(5600)} \Rightarrow \frac{1}{2}=e^{5600r} \Rightarrow \ln(\tfrac{1}{2})=5600r \Rightarrow$$

$$r=\frac{\ln(\frac{1}{2})}{5600}$$

If 25% is left, $P=100$, $A=25$.

$$25=100\,e^{\left(\frac{\ln(\frac{1}{2})}{5600}\right)t} \Rightarrow \frac{1}{4}=e^{\left(\frac{\ln(\frac{1}{2})}{5600}\right)t} \Rightarrow \ln(\tfrac{1}{4})=\frac{\ln(\frac{1}{2})}{5600}t$$

$$\Rightarrow t=\frac{5600\ln(\frac{1}{4})}{\ln(\frac{1}{2})} \approx 11200 \text{ years}$$

The answer 11180 has a rounding error in it.

33. $T=38^\circ$, $u_0=79^\circ$

$$U(t)=38+(79-38)e^{kt} \Rightarrow U=38+41e^{kt}$$

$$u(10)=74 \Rightarrow 74=38+41e^{10k} \Rightarrow 36=41e^{10k} \Rightarrow \frac{36}{41}=e^{10k} \Rightarrow$$

$$\Rightarrow \ln(\tfrac{36}{41})=10k \Rightarrow k=\frac{1}{10}\ln(\tfrac{36}{41}).$$

WHEN IS IT 58°?

$$58=38+41\,e^{\left(\frac{1}{10}\ln(\frac{36}{41})\right)t} \Rightarrow 20=41\,e^{\left(\frac{1}{10}\ln(\frac{36}{41})\right)t} \Rightarrow$$

$$\Rightarrow \frac{20}{41}=e^{\left(\frac{1}{10}\ln(\frac{36}{41})\right)t} \Rightarrow \ln(\tfrac{20}{41})=\frac{1}{10}\ln(\tfrac{36}{41})\,t \Rightarrow$$

$$\Rightarrow t=\frac{10\ln(\frac{20}{41})}{\ln(\frac{36}{41})} \approx 55.196 = 55 \text{ minutes.}$$