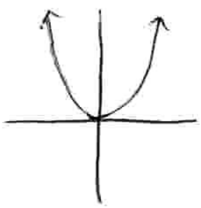
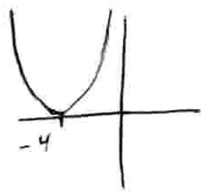
MATH 1050 EXAM 2 REVIEW KEY

1. $f(x) = x^2$



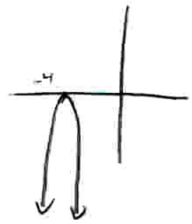
L4 →

$f(x) = (x+4)^2$



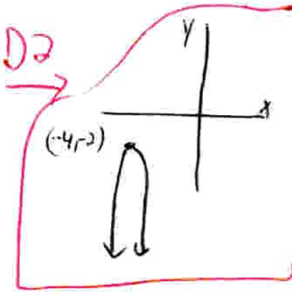
X REFLECTION & VERTICAL STRETCH →

$f(x) = -5(x+4)^2$



D2 →

$f(x) = -5(x+4)^2 - 2$



2. VERTEX is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

For $f(x) = -x^2 + 8x$, $\quad x = \frac{-b}{2a} = \frac{-8}{2(-1)} = 4$.

$y = f(4) = -(4)^2 + 8(4) = -16 + 32 = 16$

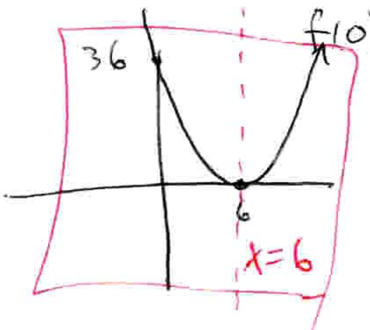
so VERTEX is $\boxed{(4, 16)}$

$\boxed{\text{Axis of symmetry } x = 4}$

3. VERTEX: $x = \frac{-b}{2a} = \frac{12}{2} = 6$, $\quad y = f(6) = (6)^2 - 12(6) + 36 = 0$ so VERTEX $\boxed{(6,0)}$

INTERCEPTS $\quad f(x) = (x-6)(x-6)$ so $x = 6$ is x intercept

$f(0) = (-6)^2 = 36$ is y intercept $\qquad$ INTERCEPTS: $\boxed{(6,0), (0,36)}$



$\boxed{x = 6 \text{ is Axis of Symmetry}}$

→ ANSWER ON ~~REV~~ REVIEW IS WRONG.

4. VERTEX: $x = \frac{-3}{4}$

$y = f\left(-\frac{3}{4}\right) = 2\left(-\frac{3}{4}\right)^2 + 3\left(-\frac{3}{4}\right) - 1$

$= \frac{-17}{8}$

VERTEX $\boxed{\left(-\frac{3}{4}, -\frac{17}{8}\right)}$

$f(x) = 2x^2 + 3x - 1 \qquad f(0) = -1 \qquad \boxed{(0,-1) \text{ y int}}$

$= \cancel{(2x-1)(x+1)}$

DOESN'T FACTOR

$\boxed{\text{Axis of symmetry } x = -\frac{3}{4}}$

X INTERCEPTS:

$0 = 2x^2 + 3x - 1$

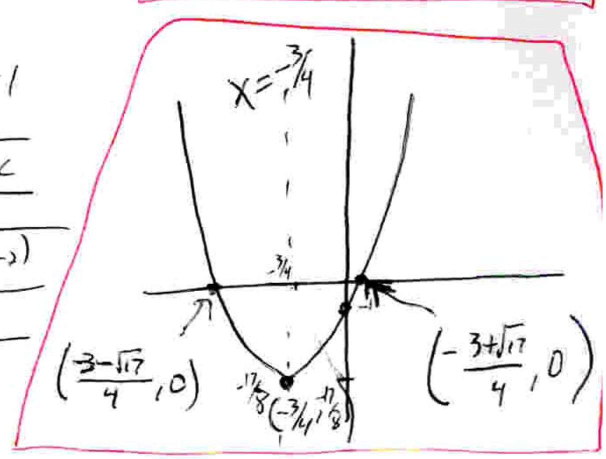
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{9 - 4(-2)}}{4}$

$= \frac{-3 \pm \sqrt{17}}{4}$

X intercepts

$\boxed{\left(\frac{-3+\sqrt{17}}{4}, 0\right), \left(\frac{-3-\sqrt{17}}{4}, 0\right)}$

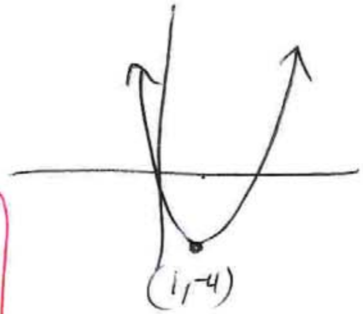


5. $f(x) = x^2 - 2x - 3$

VERTEX $x = \frac{2}{2} = 1$

$y = f(1) = 1^2 - 2(1) - 3 = -4$

PARABOLA OPENS UP



SMALLEST $x = -\infty$
LARGEST $x = \infty$

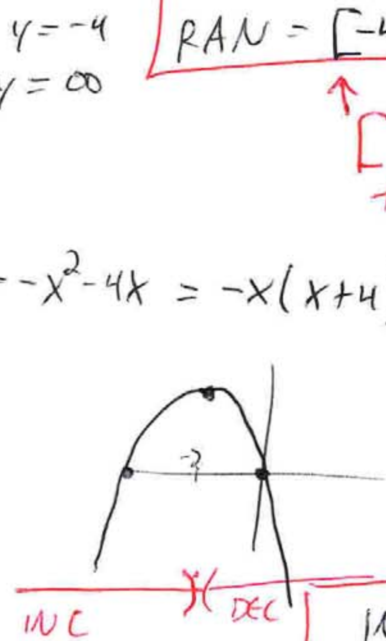
SMALLEST $y = -4$
LARGEST $y = \infty$

$\boxed{\begin{array}{l} \text{DOM} = (-\infty, \infty) \\ \text{RAN} = [-4, \infty) \end{array}}$

[is used since the point $(1,-4)$ is part of the graph, and so $y=-4$ IS in RANGE

6. $f(x) = -x^2 - 4x = -x(x+4)$ ← x intercepts are $(0,0), (-4,0)$

VERTEX is ½ way between intercepts, so $x = -2$, $y = ?$
& GRAPH OPENS DOWN



$\boxed{\begin{array}{ll} \text{INCREASING} & (-\infty, -2) \\ \text{DECREASING} & (-2, \infty) \end{array}}$

YOU CAN ALSO FIND VERTEX AS IN # 5.

7. $R(p) = -4p^2 + 1950p$ GRAPH IS A PARABOLA OPENING DOWN, SO VERTEX IS MAX POINT (Like #6)

VERTEX:

$p = \frac{-b}{2a} = \frac{-1950}{-8} = 243.75$, $R(243.75) = -4(243.75)^2 + 1950(243.75)$
$= 237656.25$

INSTEAD of $x =$

ROUND TO NEAREST \$, SO

SO $\boxed{\text{MAX REVENUE IS } \$237656}$

8. $h(t) = -16t^2 + 40t + 50$ AGAIN PARABOLA OPENING DOWN, SO VERTEX IS MAX. (HEIGHT)

$t = \frac{-b}{2a} = \frac{-40}{2(-16)} = \frac{40}{32} = \frac{5}{4}$. $h(\frac{5}{4}) = -16(\frac{5}{4})^2 + 40(\frac{5}{4}) + 50 = 75$

$\boxed{\text{MAX HEIGHT is 75 ft}}$ It TAKES $\frac{5}{4} = 1.25$ seconds to get to max & another 1.25 to come back, so

$2(1.25) = \boxed{\text{2.5 seconds to return to ground}}$

9. POLYNOMIAL of DEGREE 5 (x^{5})

10. POLYNOMIAL of DEGREE 1 $(\frac{1}{2}x^{1})$

11. NOT POLYNOMIAL: DIVIDE BY X's NOT ALLOWED.

12. NOT POLYNOMIAL: x^{fraction} not allowed.

13.

FACTOR	ZERO	MULTIPLICITY	CROSS or TOUCH
$(x+6)$	-6	1	CROSS
$(x-1)^4$	1	4	TOUCH

MULTIPLICITY of ZERO = POWER of FACTOR.
ODD MULTIPLICITY = CROSS
EVEN " = TOUCH

14. $f(x) = (x+9)^2 \Rightarrow$ zero $x = -9$ $\boxed{x \text{ int } (-9,0)}$
$f(0) = (0+9)^2 = 81 \Rightarrow \boxed{y \text{ int } (0,81)}$

15. IF WE FOILED POLYNOMIAL OUT, WE WOULD GET

$x^9 + \cdots$ so f resembles $\boxed{x^9}$ for large values of $|x|$

16. $f(x) = 4x - x^3$: DEGREE is 3 & MAX TURN POINTS = DEGREE − 1,

SO $3 - 1 = \boxed{\text{2 TURNING POINTS MAXIMUM}}$

17. x intercepts = zeros = $4, -5$.

x	y
-7	$+$
0	$+$
6	$+$

TEST ~~x~~ x's between ZEROS

$x = -7, 0, 6$

$+ \quad + \quad +$

$-7 \quad -5 \quad 0 \quad 4 \quad 6$

ALL Y VALUES OF TESTS ARE POSITIVE, SO THIS GRAPH IS ABOVE X AXIS EVERYWHERE EXCEPT AT ITS ZEROS.

$$\boxed{\begin{array}{l} \text{ABOVE: } (-\infty, -5) \cup (-5, 4) \cup (4, \infty) \\ \text{BELOW: } \varnothing \end{array}}$$

ON X AXIS $\{-5, 4\}$

18. zero -3, mult 2 $= (x+3)^2$

zero 1, mult 1 $= (x-1)$

zero 5, mult 3 $= (x-5)^3$

so $\boxed{f(x) = (x+3)^2(x-1)(x-5)^3}$

& THIS IS DEGREE 6 $(2+1+3)$

19. $f(x) = \dfrac{x+9}{x^2+64x}$

DOMAIN: DONT DIVIDE BY ZERO. SO

$x^2 + 64x \neq 0 \Rightarrow x(x+64) \neq 0$

$\Rightarrow x \neq 0, \ x \neq -64$

$$\boxed{\text{DOM} = \{x \mid x \neq 0, x \neq -64\}}$$

20. VERTICAL ASYMPTOTES HAPPEN WHEN YOU DIVIDE BY 0,

SO $x^2 + 12x + 35 = 0 \Rightarrow (x+7)(x+5) = 0$

$\Rightarrow \boxed{x = -7, \ x = -5 \text{ ARE VERTICAL ASYMPTOTES}}$

21. HORIZONTAL ASYMPTOTES: WHEN DEGREE of NUMERATOR = DEG of DEN, HA happens @ coefficients of highest powered terms:

DEGREE 2 / DEGREE 2 $\quad \dfrac{7x^2 - 2x - 3}{5x^2 - 4x + 5}$

$$\boxed{y = \frac{7}{5} \text{ is HORIZONTAL ASYMPTOTE}}$$

HORIZONTAL & OBLIQUE ASYMPTOTES ARE ALWAYS $y =$

22. WHEN DEGREE OF DENOMINATOR IS LARGER THAN DEGREE OF NUM,

$\boxed{Y=0 \text{ IS HORIZONTAL ASYMPTOTE}}$ (ALWAYS $Y=0$)

23. IF DEGREE OF NUMERATOR IS LARGER, PERFORM LONG DIVISION & DROP REMAINDER TO GET OBLIQUE ASYMPTOTE:

$$\begin{array}{r} \boxed{x \;\; -7} + \cancel{\frac{27}{x+3}} \\ x+3 \overline{) \; x^2 - 4x + 6} \\ -x^2 - 3x \\ \hline -7x + 6 \\ +7x + 21 \\ \hline 27 \end{array}$$

$\boxed{Y = X - 7 \text{ IS OBLIQUE ASYMPTOTE}}$

24. X INTERCEPTS OF A RATIONAL FUNCTION COME FROM WHEN NUMERATOR $=0$; SO $(x-7)(2x+3)=0 \Rightarrow x=-7, x=-\frac{3}{2}$

$\boxed{X \text{ INTERCEPTS: } (-7,0), (-\tfrac{3}{2},0)}$

25. Y INTERCEPT IS WHEN $X=0$, SO $f(0) = \frac{0-3}{0^2+12(0)-9} = \frac{-3}{-9} = \frac{1}{3}$

$\boxed{(0, \tfrac{1}{3}) \text{ IS Y INTERCEPT}}$

26. A HOLE OCCURS IF A FACTOR CANCELS IN A RATIONAL FUNCTION

$R(x) = \frac{x^2+x-56}{x^2-x-72} = \frac{\cancel{(x+8)}(x-7)}{(x-9)\cancel{(x+8)}} = \frac{x-7}{x-9} \leftarrow$ $\boxed{\text{VERTICAL ASYMPTOTE AT } x=9}$

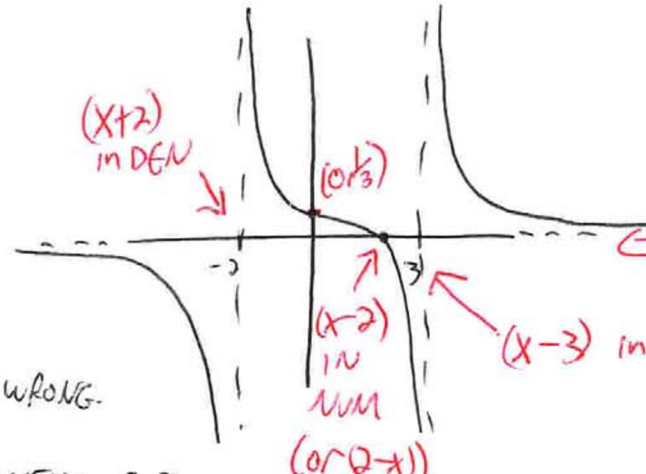
SINCE $(X+8)$ FACTORED, WE HAVE A HOLE @ $X=-8$.

TO FIND Y VALUE OF HOLE, WE COMPUTE $R(-8)$ IN ITS REDUCED FORM:

$R(-8) = \frac{-8-7}{-8-9} = \frac{-15}{-17} = \frac{15}{17}$ $\boxed{\text{HOLE @ } (-8, \tfrac{15}{17})}$

IF WE PLUG IN -8 INTO ORIGINAL FORM, WE GET $\frac{0}{0}$, undefined.

27.

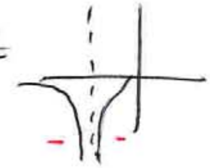


DEGREE LARGER IN DENOMINATOR.

(X-3) in DEN

D HAS WRONG FACTORS, SO IT IS WRONG.

B. HAS MULTIPLICITIES OF 2 FOR BOTH ASYMPTOTES. THIS would CAUSE GRAPH TO BE THE SAME SIGN ON BOTH SIDES OF ASYMPTOTE, LIKE



SO B is OUT

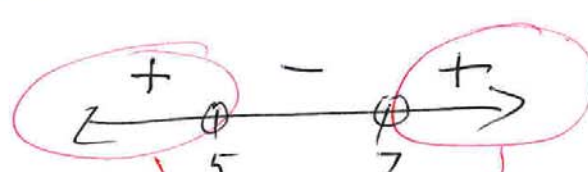
SO ANSWER IS EITHER A OR C.

WHICH HAS A POSITIVE Y INTERCEPT?

A: $R(0) = \frac{2}{2(-3)} = -\frac{1}{3}$ C: $R(0) = \frac{-2}{2(-3)} = \frac{1}{3}$

SO $\boxed{\text{C IS THE ANSWER}}$

28. $(x-5)(x-7) > 0$



zeros $x = 5, 7$ (NOT INCLUDED IN ANSWER DUE TO $>$, not $\geq$)

WE WANT $>$, SO +

TEST VALUES AROUND ZEROS, LIKE $x = 0, 6 \,\&\, 9$

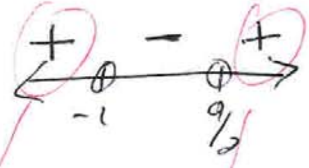
x	y
0	+
6	−
9	+

$\boxed{(-\infty, 5) \cup (7, \infty)}$

29. $2x^2-7x > 9$ $\Rightarrow$ $2x^2-7x-9>0$ $\Rightarrow$ zeros $x=\frac{9}{2}, -1$

$-9\ \ -9$ $\qquad (2x-9)(x+1)>0$

NOT INCLUDED (use ( ) instead of [ ])



TEST

x	y
-5	+
0	−
20	+

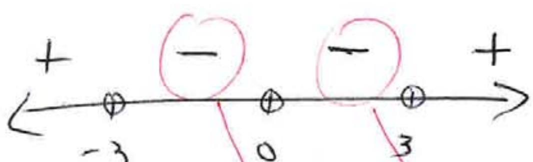
>0 is $+$, so

$\boxed{(-\infty,-1)\cup\left(\frac{9}{2},\infty\right)}$

30. $x^4 < 9x^2$ $\Rightarrow$ $x^4-9x^2<0$ $\Rightarrow$ $x^2(x+3)(x-3)<0$

$-9x^2\ \ -9x^2$ $\qquad x^2(x^2-9)<0$ $\qquad$ zeros $0, -3, 3$

NOT INCLUDED ($<$ not $\le$)



TEST

x	y
-5	+
-1	−
1	−
5	+

$<$ = −, so $\boxed{(-3,0)\cup(0,3)}$

31. $\dfrac{x-5}{x+7}<0$ $\qquad$ ZERO $x=5$

VA $x=-7$

TEST

x	y
-9	+
0	−
10	+

<0 is −, so $\boxed{(-7,5)}$

32. $\dfrac{x-2}{x} + \dfrac{2}{x-1} \le 2 \Rightarrow \dfrac{x-2}{x} + \dfrac{2}{x-1} - 2 \le 0$

$\quad\quad\quad\quad -2 \quad -2$

LCD is $x(x-1)$, so INEQUALITY'S $\dfrac{\overbrace{(x-2)(x-1)}^{\text{FOIL}} + 2x \overbrace{-2(x-1)x}^{\text{DISTRIBUTE}}}{x(x-1)} \le 0$

$\Rightarrow \dfrac{x^2-2x-x+2+2x-2x^2+2x}{x(x-1)} \le 0 \Rightarrow \dfrac{-x^2+x+2}{x(x-1)} \le 0 \Rightarrow \dfrac{-(x^2-x-2)}{x(x-1)} \le 0$

$\Rightarrow -\dfrac{(x-2)(x+1)}{x(x-1)} \le 0$ ZEROS $x = 2, -1$ INCLUDED ([,] used because $\le$)

VA = $x = 0, 1$ NOT INCLUDED ($\div 0$ NEVER!)

(VERTICAL ASYMPTOTES)

TEST

x	y
-5	−
$-\frac{1}{2}$	+
$\frac{1}{2}$	−
1.5	+
4	−

≤ 0 is −, so

$\boxed{(-\infty, -1] \cup (0, 1) \cup [2, \infty)}$

33. $x+10$ FACTOR is $x = -10$ ZERO

$1x^3 + 8x^2 - 18x + 20$

$$\begin{array}{r|rrrr} & 1 & 8 & -18 & 20 \\ \hline -10 & 1 & -2 & 2 & 0 \end{array}$$

$\leftarrow$ REMAINDER 0, $\boxed{\text{so } x+10 \text{ is A FACTOR}}$

34. $f(x) = 5x^3 - x^2 + 2$

$\pm \dfrac{\text{FACTORS of } 2}{\text{FACTORS of } 5} = \pm \dfrac{1, 2}{1, 5} = \boxed{\pm 1, 2, \tfrac{1}{5}, \tfrac{2}{5}}$

35. $f(x) = x^3 + 3x^2 - 4x - 12$

RATIONAL ZEROS $= \dfrac{1,2,3,4,6,12}{1} = \pm 1,2,3,4,6,12$

	1	3	−4	−12	
1	1	4	0	−12	NOPE
$\boxed{2}$	1	5	6	0	YES!

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x = -2, -3$

SO ZEROS

$x = 2, -2, -3$

FACTORED FORM

$f(x) = (x-2)(x+2)(x+3)$

36. RATIONAL ZEROS:

$\dfrac{1,3}{1,3} = \boxed{\pm 1, 3, \frac{1}{3}}$

	3	−1	−9	3	
1	3	2	−7	−4	NOPE
$\boxed{1/3}$	3	0	−9	0	YES!

$3x^2 - 9 = 0$

$3(x^2 - 3) = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

SOLUTIONS $\boxed{x = 1/3, \sqrt{3}, -\sqrt{3}}$

ALTERNATE METHOD:
(IF YOU CAN FACTOR)

$f(x) = x^3 + 3x^2 - 4x - 12$

$= x^2(x+3) - 4(x+3)$

$= (x+3)(x^2-4) =$

$\boxed{f(x) = (x+3)(x-2)(x+2)}$

$\boxed{\text{Zeros } x = -3, 2, -2}$

ALTERNATE

$3x^3 - x^2 - 9x + 3 = 0$

$x^2(3x-1) - 3(3x-1) = 0$

$(3x-1)(x^2-3) = 0$

$\boxed{x = 1/3}$ $\quad x^2 = 3,$

$\boxed{x = \pm\sqrt{3}}$

37.

	2	-10	2	-9
4	2	-2	-6	-33
5	2	0	2	1

Since $f(4) = -33 < 0$ & $f(5) = 1 > 0$,
There is an x between 4 & 5 where $f(x) = 0$

38.

Zero	FACTOR
3	$(x-3)$
$2-i$	$(x-2+i)$
$2+i$	$(x-2-i)$

SINCE WE ARE AFTER REAL COEFFICIENTS, THE ZERO $\boxed{2-i}$ MUST HAVE ITS CONJUGATE $\boxed{2+i}$

THIS IS NOT NECESSARY

THIS IS THE ONLY ANSWER

$$\text{So } f(x) = (x-3)(x-2+i)(x-2-i)$$

← DOESN'T HAVE ALL REAL COEFFS.

$$= (x-3)(x^2-2x-\cancel{ix}-2x+4+\cancel{2i}+\cancel{ix}-\cancel{2i}-i^2)$$

$+1$

$$= (x-3)(x^2-4x+5)$$

$$= x^3-4x^2+5x-3x^2+12x-15$$

$$\boxed{f(x) = x^3-7x^2+17x-15}$$

← HAS ALL REAL COEFFICIENTS. & DEGREE 3

39. ZERO FACTOR

ZERO	FACTOR
1	$x-1$
$3+i$	$x-3-i$
$3-i$	$x-3+i$

GIVEN DUE TO REAL COEFFICIENTS INSTRUCTIONS

$$\Rightarrow (x-1)(x-3-i)(x-3+i) =$$
$$= (x-1)(x^2-6x+10)$$
$$\boxed{f(x) = x^3-7x^2+16x-10}$$

DEGREE 3

40. ZERO $-2i$; THIS POLYNOMIAL HAS REAL COEFFICIENTS, SO $2i$ is also a zero $\therefore -2i, 2i \Rightarrow (x+2i)(x-2i) = x^2+4$.

SO x^2+4 IS A FACTOR:

$$\begin{array}{r} x^2 - 9 \\ x^2+0x+4 \overline{\big)\, x^4 - 5x^2 - 36} \\ \underline{-x^4 \mp 4x^2} \\ -9x^2 - 36 \\ \underline{\pm 9x^2 \pm 36} \\ 0 \end{array}$$

THE OTHER FACTOR IS
$x^2-9 = (x+3)(x-3)$

SO ALL SOLUTIONS: $\boxed{x = 2i, -2i, 3, -3}$

41. RATIONAL ZEROS

$\frac{1, 5, 7, 35, 49, 245}{1} = \pm 1, 5, 7, 35, 49, 245$

	1	-4	-44	196	-245
1	1	-3	-47	149	-96 NO
5	1	1	-39	1	-240 NO
$\boxed{7}$	1	3	-23	35	0 YES
7	1	10	47	364	NOT DOUBLE ROOT
$\boxed{-7}$	1	-4	5	0	YES

SO SOLUTIONS

$$\boxed{x = 7, -7, 2+i, 2-i}$$

$x^2-4x+5=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$$