Salt Lake Community College

Math 1060 Final Exam A – Fall Semester 2010

Name:______________________ Instructor:______________________

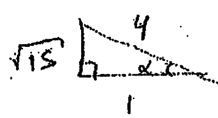
This Exam has three parts. Please read carefully the directions for each part. All problems are of equal point value. STUDENTS ARE NOT ALLOWED TO USE BOOKS OR NOTES.

PART I – You must complete this portion of the test without using a calculator. After you have finished part I, your instructor will give you the remaining parts of the exam.

When simplifying answers, it is not necessary to rationalize the denominator.

1) Find the exact values of the other five trigonometric functions for an angle α if $\cos\alpha = \frac{1}{4}$ and α is in quadrant IV.

sin α = $-\sqrt{15}/4$



tan α = $-\sqrt{15}$

csc α = $-4/\sqrt{15}$

QUAD. IV

sec α = 4

cot α = $-1/\sqrt{15}$

2) The position x, in meters, of a weight attached to a spring is given by $x = 3\sin 2t + \cos 2t$ where t is time in seconds. Find the exact position of the weight at time $t = \frac{\pi}{4}$ seconds.

$$x\left(\frac{\pi}{4}\right) = 3\sin\left(2\cdot\frac{\pi}{4}\right) + \cos\left(2\cdot\frac{\pi}{4}\right)$$

$$= 3\cdot\sin\frac{\pi}{2} + \cos\frac{\pi}{2}$$

$$= 3\cdot 1 + 0 = 3$$

3) Determine the amplitude, period, phase shift, and frequency for $y = 3\sin(2x - \pi) + 3$

Amplitude: 3

Period: π

Phase shift: $\pi/2$

Frequency: $1/\pi$

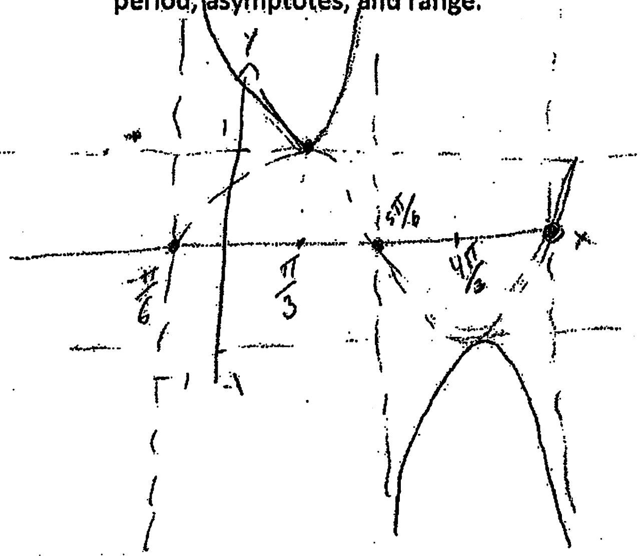
4) Find the exact value of each expression. If the expression is undefined, say so.

a) $\csc\frac{11\pi}{6} = \frac{1}{\sin\frac{11\pi}{6}} = \frac{1}{-1/2} = -2$

b) $\tan\frac{3\pi}{4} = -1$

c) $\sec(-120°) = \frac{1}{-1/2} = -2$

5) Sketch the graph of = sec $(x - \frac{\pi}{3})$. Sketch at least one full cycle and determine the period, asymptotes, and range.



PERIOD

2π

ASYMPTOTES

$x = \frac{5\pi}{6} + k\pi$

RANGE

$(-\infty, 1] \cup [1, \infty)$

6) Find the exact value of $\cos(\alpha + \beta)$ if $\sin\alpha = \frac{\sqrt{2}}{2}$ and $\cos\beta = \frac{1}{2}$ with α in quadrant II and β in quadrant IV.

$\cos\alpha = -\frac{\sqrt{2}}{2}$ $\quad \sin\beta = -\frac{\sqrt{3}}{2}$

$\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

$= -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \left(-\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{-\sqrt{2} + \sqrt{6}}{4}$

7) Use identities to simplify the following expression to a single trigonometric function.

$$\frac{1+\cos(x)\tan(x)\csc(x)}{2\csc(x)}$$

$$= \frac{1 + \cancel{\cos x}\cdot\frac{\sin x}{\cancel{\cos x}}\cdot\frac{1}{\cancel{\sin x}}}{2\cdot\frac{1}{\sin x}} = \frac{1+1}{\frac{2}{\sin x}} = \frac{\cancel{2}\cdot\sin x}{\cancel{2}}$$

$$= \boxed{\sin x}$$

8) Prove that the following equation is an identity.

$$\frac{1-sin^2(x)}{1-\sin(x)} = \frac{\csc(x)+1}{\csc(x)}$$

LHS:

$$\frac{1-\sin^2 x}{1-\sin x} = \frac{\cancel{(1-\sin x)}(1+\sin x)}{\cancel{1-\sin x}}$$

$$= 1+\sin x$$

RHS:

$$\frac{\csc x}{\csc x} + \frac{1}{\csc x}$$

$$= 1 + \sin x$$

Since both the LHS and RHS are shown equivalent to $1+\sin x$, the equation is an identity.

9) Find the exact value of the following expression. Simplify your answer.
Hint: You will need to use an identity.

$$2cos^2\left(\frac{\pi}{12}\right) - 1$$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos^2 x - (1-\cos^2 x)$

$= 2\cos^2 x - 1$

$$= \cos 2\cdot\frac{\pi}{12}$$

$$= \cos\frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{2}$$

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PART II – Work all of the following problems. Be sure to show your work where appropriate. Unsupported answers may not receive full credit. You will need a calculator for some problems. When directions specify "exact value" a calculator should not be used.

STUDENTS ARE NOT ALLOWED TO USE BOOKS OR NOTES.

When simplifying answers, it is not necessary to rationalize the denominator.

10) Two sides and an angle are given. Solve any triangle(s) that results. Please round to the nearest tenth if necessary.

$$a = 8.1, b = 10.6, \alpha = 41.2^o$$

LAW OF SINES

$\frac{\sin 41.2^\circ}{8.1} = \frac{\sin \beta}{10.6}$

$\sin \beta = \frac{10.6 \cdot \sin 41.2^\circ}{8.1}$

$\beta = 59.5^\circ$ or 120.5°

$\gamma_1 = 180^\circ - 59.5^\circ - 41.2^\circ$

$\gamma_2 = 180^\circ - 120.5^\circ - 41.2^\circ$

$\frac{\sin 41.2^\circ}{8.1} = \frac{\sin 79.3^\circ}{c_1}$

$\frac{\sin 41.2^\circ}{8.1} = \frac{\sin 18.3^\circ}{c_2}$

two triangles

$$\boxed{\begin{array}{lll} \beta_1 = 59.5^\circ & \gamma_1 = 79.3^\circ & c_1 = 12.1 \\ \beta_2 = 120.5^\circ & \gamma_2 = 18.3^\circ & c_2 = 3.9 \end{array}}$$

$h = 10.6 \sin(41.2^\circ) = 6.98$

$h < a < b \Rightarrow 2$ TRIANGLES

11) Two sides and an angle are given. Solve any triangle(s) that results. Please round to the nearest tenth if necessary.

$$a = 13, b = 14.9, \gamma = 9.8^{\circ}$$

LAW OF COSINES

$c^2 = 13^2 + 14.9^2 - 2 \cdot 13 \cdot 14.9 \cos 9.8^{\circ}$

$c = \sqrt{9.26293}$

$\boxed{c = 3.04}$

$\frac{\sin 9.8^{\circ}}{3.04} = \frac{\sin \beta}{14.9}$

$\beta = 56.5^{\circ}$ OR $\boxed{\beta = 123.5^{\circ}}$

$\boxed{\alpha = 46.7^{\circ}}$

12) At what speed in miles per hour will a bicycle travel if the rider can cause the 26-in.-diameter wheel rotate 103 revolutions per minute? Please round to the nearest tenth if necessary.

Recall: 1 mile = 5280 ft

$w = \frac{103 \text{ REV}}{1 \text{ MIN}} \cdot \frac{2\pi \text{ RAD}}{1 \text{ REV}} \cdot \frac{60 \text{ MIN}}{1 \text{ HOUR}}$

$w = 38,830.1$ RAD. PER HOUR

$V = w \cdot R$

$V = 38,830.1 \times 13 \text{ IN} \cdot \frac{1 \text{ FT}}{12 \text{ IN}} \cdot \frac{1 \text{ MI}}{5280 \text{ FT}}$

$V = 7.97$ MPH

$\boxed{V = 8.0 \text{ MPH}}$

13)

a) Use a calculator to find the value of the expression in radians rounded to the nearest hundredth.

$\csc^{-1}(5) =$ 0.20

$\sin^{-1}\left(\frac{1}{5}\right) = 0.20$ RADIANS

b) Give the exact value of the expression

$$\cos(\underbrace{\operatorname{arccot}(-1)}_{\alpha}).$$

$\cot\alpha = -1$

$\alpha = 135°$ or $\frac{3\pi}{4}$

$\cos 135° = \boxed{-\frac{\sqrt{2}}{2}}$

14) Perform the indicated operation and write the answer in the form a + bi. Give exact values, not decimal approximations.

$$\frac{6(\cos 20^{o} + i \sin 20^{0})}{12(\cos 50^{o} + i \sin 50^{0})}$$

$\frac{1}{2}\left(\cos(-30°) + i \sin(-30°)\right)$

$\frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2} i\right) = \boxed{\frac{\sqrt{3}}{4} - \frac{1}{4} i}$

15) With x in radians, find all real numbers that satisfy the equation. Please give exact answers, not decimal approximations.

$$2\sin(2x) - \sqrt{3} = 0$$

$2\sin 2x = \sqrt{3}$

$\sin 2x = \frac{\sqrt{3}}{2}$

$2x = \frac{\pi}{3} + 2k\pi$

$\boxed{x = \frac{\pi}{6} + k\pi}$

$2x = \frac{2\pi}{3} + 2k\pi$

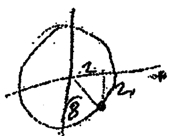
$\boxed{x = \frac{\pi}{3} + k\pi}$

16) a) Convert the complex number to trigonometric form with $0^o \leq \theta < 360^o$

2-2i

$R = \sqrt{4+4} = 2\sqrt{2}$

$\boxed{2\sqrt{2}(\cos 315^\circ + i \sin 315^\circ)}$



$\boxed{2\sqrt{2}}$

b) Simplify the following by using the trigonometric form from part (a) and DeMoivre's Theorem. Write the answer in the form a + bi.

$$(2-2i)^4$$

$315^\circ \cdot 4 = 1260^\circ - 1080^\circ = 180^\circ$

$(2\sqrt{2})^4(\cos 180^\circ + i \sin 180^\circ)$

$64(-1 + i \cdot 0) = \boxed{-64}$

17) Given $v = \langle 5,3 \rangle$ and $w = \langle -2,5 \rangle$, determine:

a) $3v - w$

$\langle 15-(-2), 9-5 \rangle$

$\boxed{\langle 17, 4 \rangle}$

b) $|w|$

$= \sqrt{4+25} = \boxed{\sqrt{29}}$

c) If v and w are parallel, perpendicular, or neither.

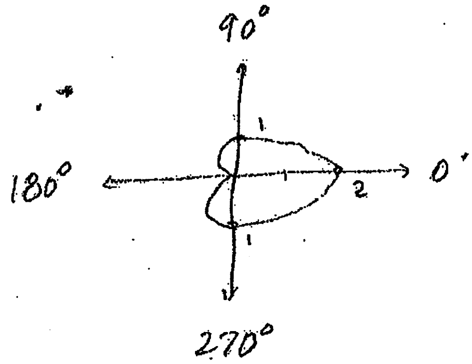
$\cos\theta = \frac{u \cdot v}{|u|\,|v|} = \frac{-10+15}{\sqrt{34}\sqrt{29}} = 0.159232$

$\theta = 80.8^\circ$

$\boxed{\text{NEITHER}}$

18) Sketch the graph of the polar equation. List at least 4 points on the curve.

$$r = 1 + \cos\theta$$

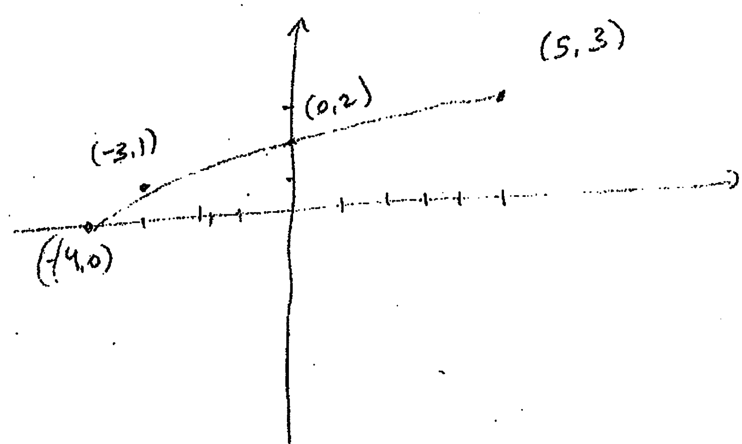


(2, 0°)
(1, 90°)
(6, 180°)
(1, 270°)

19) Graph the curve whose parametric equations are given. List at least three points on the curve.

$$x = t - 2, \quad y = \sqrt{t+2}, \quad -2 \le t \le 7$$

T	x	Y
-2	-4	0
-1	-3	1
0		
1		
2	0	2
3		
4		
5		
6		
7	5	3



$t = x + 2$

$y = \sqrt{x + 2 + 2}$

$y = \sqrt{x + 4}$

Part III – Work 3 of the problems in this section. Cross out any problems you do not want graded.

Be sure to show your work where appropriate. Unsupported answers may not receive full credit. You will need a calculator for some problems. When directions specify "exact value" a calculator should not be used.

STUDENTS ARE NOT ALLOWED TO USE BOOKS OR NOTES.

When simplifying answers, it is not necessary to rationalize the denominator.

20) Eliminate the parameter and identify the graph of the parametric equations.

$$x = 2\sin 2t, \quad y = -2\cos 2t$$

$\sin 2t = \frac{x}{2}$ $\qquad$ $\cos 2t = -y/2$

$\sin^2 2t + \cos^2 2t = 1$

$\left(\frac{x}{2}\right)^2 + \left(-\frac{y}{2}\right)^2 = 1$

$\frac{x^2}{4} + \frac{y^2}{4} = 1$

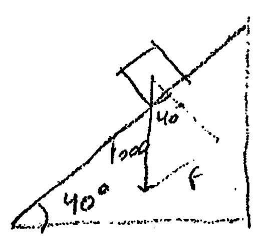
$\boxed{x^2 + y^2 = 4}$

Circle

Center $(0,0)$

Radius 2

21) Determine the amount the force required to push a 1000-lb riding lawnmower up a ramp that is inclined at a 40° ramp. Please round to the nearest tenth if necessary.



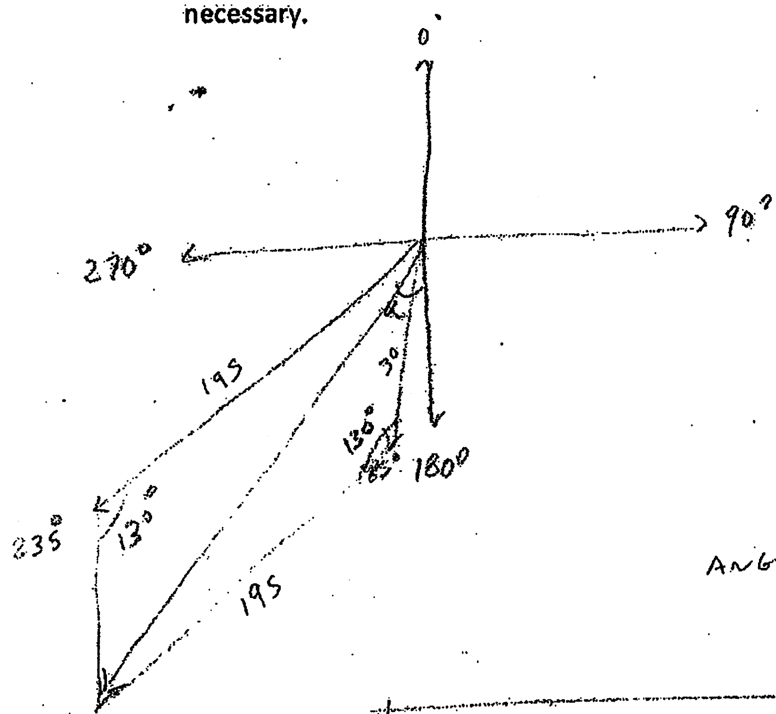
$\boxed{642.8 \text{ Lb}}$

$\sin 40° = \frac{F}{1000}$

$F = 1000 \sin 40°$

$F = 642.7876$ LB

22) The heading of a helicopter has a bearing of 235°. If the 30-mph wind has a bearing of 185° and the air speed of the helicopter is 195 mph, then what are the bearing of the course and the ground speed of the helicopter? Please round to the nearest tenth if necessary.



$x^2 = 30^2 + 195^2 - 2 \cdot 30 \cdot 195 \cos 130°$

$x = 215.5$ MPH

$\frac{\sin 130°}{215.5} = \frac{\sin \alpha}{195}$

$\alpha = 43.9°$

ANGLE $= 185° + 43.9°$

$= 228.8818°$

$\boxed{228.9°,\ 215.5 \text{ MPH}}$

23) Find all real numbers in the interval $[0, 2\pi)$ that satisfy the equation.

$$\sin\left(\frac{\pi}{6}\right)\cos x - \cos\left(\frac{\pi}{6}\right)\sin x = -\frac{1}{2}$$

$\sin\left(\frac{\pi}{6} - x\right) = -1/2$

$\frac{\pi}{6} - x = \frac{7\pi}{6}$ $\qquad$ $\frac{\pi}{6} - x = \frac{11\pi}{6}$

$-\pi = x$ $\qquad$ $\frac{\pi}{6} - \frac{11\pi}{6} = x$

$x = \pi$ $\qquad$ $x = \frac{-10\pi}{6} = -\frac{5\pi}{3} + 2\pi$

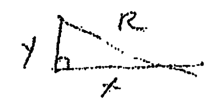
$\boxed{\frac{\pi}{3}, \pi}$ $\qquad$ $x = \frac{\pi}{3}$

24) Write an equivalent rectangular equation for the given polar equation. It is not necessary to put the equation in standard form.

$r = 2sin\theta$

$\sin\theta = \frac{y}{R}$

$\cos\theta = \frac{x}{R}$



$R^2 = 2R\sin\theta$

$$\boxed{x^2 + y^2 = 2y}$$

25) Find the cube roots of $8(\cos 30^o + i\sin 30^o)$. Express answers in trigonometric form.

$8^{1/3}\left(\cos\frac{30^\circ + 360^\circ k}{3} + i\sin\frac{30^\circ + 360^\circ k}{3}\right)$

$2\left(\cos(10^\circ + 120^\circ k) + i\sin(10^\circ + 120^\circ k)\right)$

$k = 0 \quad 2(\cos 10^\circ + i\sin 10^\circ)$

$k = 1 \quad 2(\cos 130^\circ + i\sin 130^\circ)$

$k = 2 \quad 2(\cos 250^\circ + i\sin 250^\circ)$

$$\boxed{\begin{array}{l} 2(\cos 10^\circ + i\sin 10^\circ) \\ 2(\cos 130^\circ + i\sin 130^\circ) \\ 2(\cos 250^\circ + i\sin 250^\circ) \end{array}}$$