Math 1050 Final Exam Form A Solutions – Fall 2010

Multiple Choice – No partial credit

1) C

2) D

3) D

4) B

5) A

6) C

7) D

8) A

9) B

10) C

INSTRUCTIONS PART II: Questions 11 - 20, Short Response. Answer all TEN questions carefully and completely, showing your work and clearly indicating your answer.

11) Find the composite function $f \circ g$ and the domain of $f \circ g$ for

$f(x) = \dfrac{2}{x+9}$ and $g(x) = x + 4$.

composite function $f \circ g$ $\dfrac{2}{x+13}$ (simplified)

domain of $f \circ g$ $\{x \mid x \neq -13\}$ (in set notation)

$$(f \circ g)(x) = f(g(x))$$
$$= f(x+4)$$
$$= \frac{2}{(x+4)+9}$$
$$= \frac{2}{x+13}$$

domain:

$x + 4 \neq -9$

$x \neq -13$

12) Solve the system of equations using Cramer's Rule. Show your work!

$$\begin{cases} 4x - 7y = 5 \\ 2x + 5y = -3 \end{cases}$$

solution $x = \frac{2}{17},\ y = -\frac{11}{17}$

Must show determinants to get credit

$$D = \begin{vmatrix} 4 & -7 \\ 2 & 5 \end{vmatrix} = 20 + 14$$
$$\boxed{D = 34}$$

$$D_x = \begin{vmatrix} 5 & -7 \\ -3 & 5 \end{vmatrix} = 25 - 21$$
$$\boxed{D_x = 4}$$

$$D_y = \begin{vmatrix} 4 & 5 \\ 2 & -3 \end{vmatrix} = -12 - 10$$
$$\boxed{D_y = -22}$$

$$x = \frac{D_x}{D} = \frac{4}{34}$$
$$x = \frac{2}{17}$$

$$y = \frac{D_y}{D} = \frac{-22}{34}$$
$$y = -\frac{11}{17}$$

13) The sequence is defined recursively. Write the first four terms.

$a_1 = -9;\ a_n = n - a_{n-1}$

$a_1 =$ -9

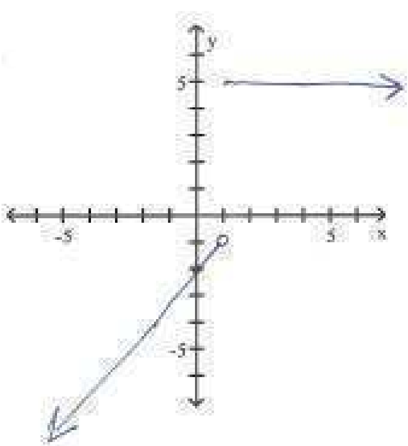
$a_2 =$ 11 $\qquad a_2 = 2 - (-9) = 11$

$a_3 =$ -8 $\qquad a_3 = 3 - 11 = -8$

$a_4 =$ 12 $\qquad a_4 = 4 - (-8) = 12$

14) Graph the function

$$f(x) = \begin{cases} x - 2 & \text{if } x < 1 \\ 5 & \text{if } x \geq 1 \end{cases}$$



15) A brick staircase has a total of 16 steps The bottom step requires 111 bricks. Each successive step requires 4 less bricks than the prior one. How many bricks are required to build the staircase? *Hint: This is an arithmetic series.*

number of bricks required to build the staircase 1296

$n = 16$

$a = 111$

$d = -4$

$a_{16} = 111 + (-4)(16-1)$

$= 51$

$S_{16} = \frac{16}{2}(111 + 51)$

$= 8(162)$

$= 1296$

16) Solve the equation $\log_3 x + \log_3(x - 24) = 4$.

solution(s) $x = 27$

$\log_3 [x(x-24)] = 4$

$3^4 = x(x-24)$

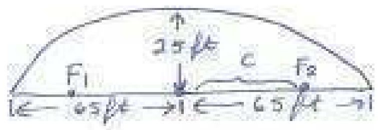
$0 = x^2 - 24x - 81$

$0 = (x-27)(x+3)$

$x = 27$ $\quad \cancel{x = -3}$

17) A hall 130 feet in length was designed as a whispering gallery. If the ceiling is 25 feet high at the center, how far from the center are the foci located? *Recall: A whispering gallery is shaped like an ellipse.*

Along with your work in solving this problem, show your drawing of the whispering gallery below, labelling as necessary.



$a = 65$

$b = 25$

$b^2 = a^2 - c^2$

$25^2 = 65^2 - c^2$

$c^2 = 65^2 - 25^2$

$c = \sqrt{3600}$

$c = 60$

distance that the foci are from the center 60 feet

18) Find the real solution(s) of the equation $x^{1/2} - 7x^{1/4} + 10 = 0$.

solution(s) $x = 16,\ x = 625$

$(x^{\frac{1}{4}})^2 - 7x^{\frac{1}{4}} + 10 = 0 \qquad u = x^{\frac{1}{4}}$

$u^2 - 7u + 10 = 0$

$(u-5)(u-2) = 0$

$(x^{\frac{1}{4}} - 5)(x^{\frac{1}{4}} - 2) = 0$

$x^{\frac{1}{4}} = 5 \qquad x^{\frac{1}{4}} = 2$

$x = 5^4 \qquad x = 2^4$

$x = 625 \qquad x = 16$

19) Find the center and radius of the circle with the equation $x^2 + y^2 - 6x - 4y = -9$. Use the center and radius to graph the circle.

$(x^2 - 6x + 9) + (y^2 - 4y + 4) = -9 + 9 + 4$

$(x-3)^2 + (y-2)^2 = 4$

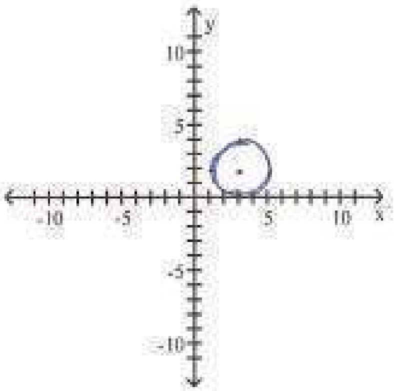
$r^2 = 4$

$r = 2$

center $(3, 2)$

radius $r = 2$

graph



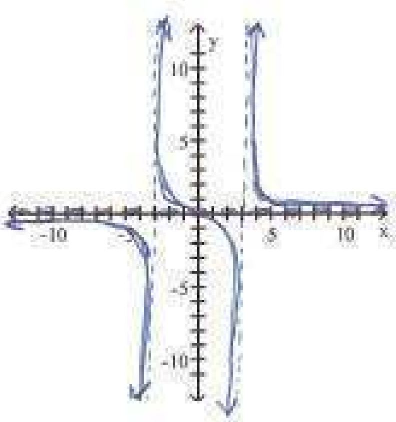
20) Find the domain, asymptotes, and intercepts. Use this information to graph the function $f(x) = \frac{x}{x^2 - 9}$.

$f(x) = \frac{x}{(x-3)(x+3)}$

domain $\{x \mid x \neq \pm 3\}$ OR $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

asymptotes $x = -3$, $x = 3$, $y = 0$

intercepts $(0, 0)$



x	y
-4	$-\frac{4}{7}$
-1	$\frac{1}{8}$
1	$-\frac{1}{8}$
4	$\frac{4}{7}$

INSTRUCTIONS PART III: Questions 21 - 30, Short Response. Answer FIVE questions only. Put an X through the 5 problems you do not want graded. If you do not cross out any problems,the first 5 problems that show any work will be the ones that are graded.

21) Randy invested his inheritance in an account that paid 6.6% interest, compounded continuously. After 5 years, he found that he now had \$44,821.17. What was the original amount of his inheritance? (Round your answer to the nearest dollar.)

original amount of inheritance \$32,223

$r = 0.066$

$t = 5$ yrs.

$A = \$44,821.17$

$44,821.17 = Pe^{(0.066)(5)}$

$44,821.17\, e^{-.33} = P$

$P = 32223.00287\ldots$

22) Write the partial fraction decomposition of the rational expression $\frac{x-8}{(x-2)(x-4)}$.

partial fraction decomposition $\frac{3}{x-2} - \frac{2}{x-4}$

$$\frac{x-8}{(x-2)(x-4)} = \frac{A}{x-2} + \frac{B}{x-4}$$

$x - 8 = A(x-4) + B(x-2)$

$\underline{x=2}$: $2-8 = A(2-4) + B(2-2)$

$-6 = -2A$

$A = 3$

$\underline{x=4}$: $4-8 = A(4-4) + B(4-2)$

$-4 = 2B$

$B = -2$

23) Find the sum of the sequence $\sum_{k=2}^{5} 8k$.

sum of the sequence 112

$$\sum_{k=2}^{5} 8k = 8(2) + 8(3) + 8(4) + 8(5)$$
$$= 16 + 24 + 32 + 40$$
$$= 112$$

24) Find all complex zeros of the function $f(x) = x^3 + 7x^2 + 16x + 10$.

complex zeros $x = -1,\ -3 - i,\ -3 + i$

potential rational zeros : $\pm 1, \pm 2, \pm 5, \pm 10$

number of positive real zeros : NONE

$$\begin{array}{r|rrrr} -1 & 1 & 7 & 16 & 10 \\ & & -1 & -6 & -10 \\ \hline & 1 & 6 & 10 & 0 \end{array}$$

$x = -1$ is a zero

$$f(x) = (x+1)(x^2 + 6x + 10)$$

$$x^2 + 6x + 10 = 0$$

$$x = \frac{-6 \pm \sqrt{36 - 40}}{2}$$

$$x = \frac{-6 \pm \sqrt{-4}}{2}$$

$$x = \frac{-6 \pm 2i}{2}$$

$$x = -3 \pm i$$

25) Solve the equation $2^{(7+3x)} = \frac{1}{4}$.

Solution(s) $x = -3$

$2^{7+3x} = \frac{1}{2^2}$

$2^{7+3x} = 2^{-2}$

$7+3x = -2$

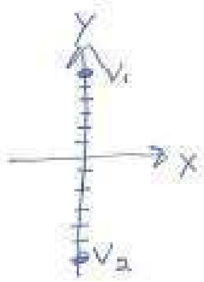
$3x = -9$

$x = -3$

26) Find an equation for the hyperbola described.

Vertices at (0, ±6); asymptote the line $y = \frac{3}{2}x$

equation $\frac{y^2}{36} - \frac{x^2}{16} = 1$



Center (0,0)

$a = 6$

$\frac{a}{b} = \frac{3}{2}$

$\frac{6}{b} = \frac{3}{2}$

$b = 4$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\frac{y^2}{6^2} - \frac{x^2}{4^2} = 1$

$\frac{y^2}{36} - \frac{x^2}{16} = 1$

27) Find the value of the determinant by hand. Do not use a calculator. Show your work!

$$\begin{vmatrix} 4 & 2 & 4 \\ 2 & 3 & 0 \\ 3 & 2 & -5 \end{vmatrix}$$

determinant -60

$-2\begin{vmatrix} 2 & 4 \\ 2 & -5 \end{vmatrix} + 3\begin{vmatrix} 4 & 4 \\ 3 & -5 \end{vmatrix}$

$= -2(-10-8) + 3(-20-12)$

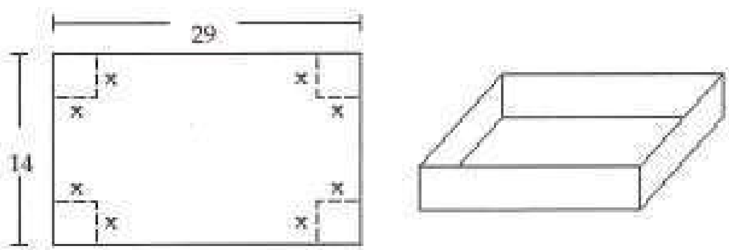
$= -2(-18) + 3(-32)$

$= 36 - 96$

$= -60$

Other methods OK but must show work to get credit.

28) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 14 inches by 29 inches by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x and state the domain.



volume as a function of x $V(x) = x(14-2x)(29-2x)$

Leave the volume function in factored form.

real world domain $\{x \mid 0 < x < 7\}$

29) Analyze the graph of the function $f(x) = -x^2(x-4)(x+3)$ as follows:

(a) Determine the end behavior: find the power function that the graph of f resembles for large values of $|x|$.

power function $y = -x^4$

(b) Find the x– and y–intercepts of the graph.

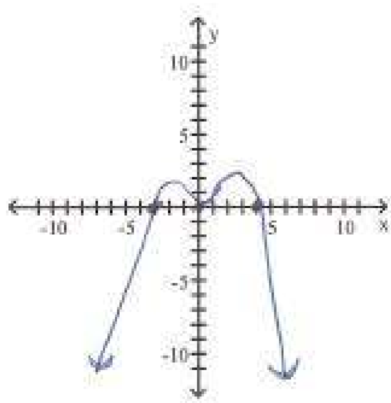
x–intercepts $(0,0)\ (4,0),\ (-3,0)$

y–intercept $(0,0)$

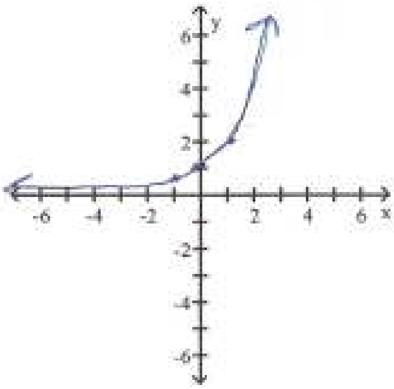
(c) Determine whether the graph crosses or touches the x–axis at each x–intercept.

list x–intercepts below	state whether graph crosses or touchees at each
$x = -3$	crosses
$x = 0$	touches
$x = 4$	crosses

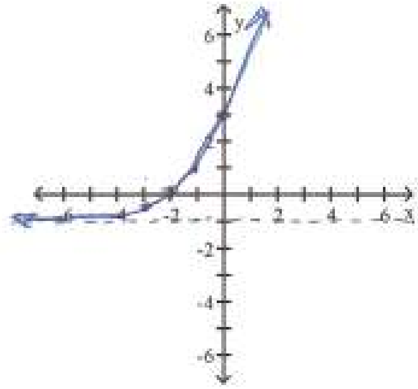
(d) Use the information obtained in (a) – (c) to sketch a graph of f by hand. Label all intercepts .



30) Graph the function $y = 2^x$. Then use transformations to graph $f(x) = 2^{(x+2)} - 1$. Label each graph.



$y = 2^x$



$f(x) = 2^{(x+2)} - 1$

3 points used in graphing $y = 2^x$ $(-1, \frac{1}{2})$, $(0, 1)$, $(1, 2)$ (OR OTHERS)

(give exact values – do not use a calculator)

domain of $f(x) = 2^{(x+2)} - 1$ $(-\infty, \infty)$ *(use interval notation)*

range of $f(x) = 2^{(x+2)} - 1$ $(-1, \infty)$ *(use interval notation)*

asymptote(s) of $f(x) = 2^{(x+2)} - 1$ $y = -1$