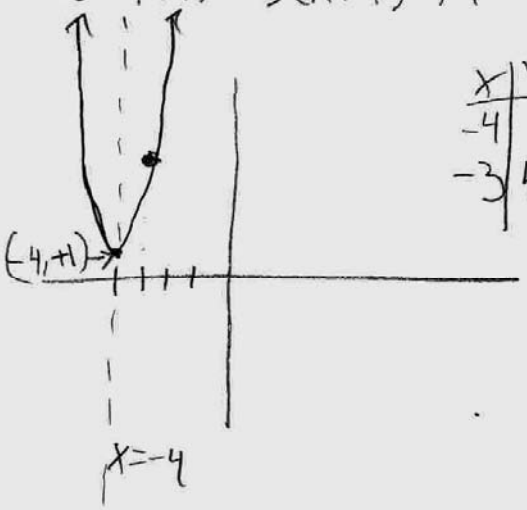
# MATH 1010 TEST 5 REVIEW KEY

1. $f(x) = 3(x+4)^2 + 1$

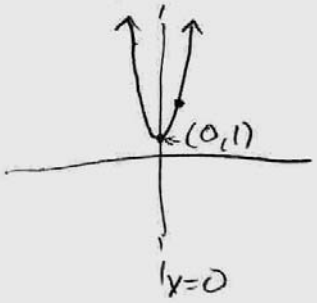



| x | y |
|---|---|
| -4 | 1 |
| -3 | 4 |

TO FIND VERTEX WHEN THE FUNCTION IS IN STANDARD FORM, FIND WHAT X VALUE MAKES THE SQUARED TERM ZERO. HERE, $x=-4$ MAKES $3(x+4)^2$ ZERO, AND $y=1$ IS ALL THAT IS LEFT SO **VERTEX $(-4,1)$.**

THE X VALUE **$x=-4$ IS ALSO THE EQUATION OF THE AXIS OF SYMMETRY.**

2. $f(x) = 2x^2 + 1$

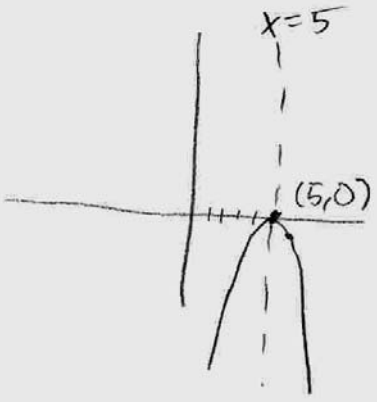



| x | y |
|---|---|
| 0 | 1 |
| 1 | 3 |

$x=0$ MAKES $2x^2$ ZERO, & 1 IS WHAT IS LEFT, SO

**VERTEX IS $(0,1)$**
**AXIS OF SYMMETRY IS $x=0$**

3. $f(x) = -(x-5)^2$




$x=5$ MAKES $-(x-5)^2$ ZERO, & 0 IS WHAT IS LEFT. SO

**VERTEX IS $(5,0)$**
**AXIS OF SYMMETRY IS $x=5$**

| x | y |
|---|---|
| 5 | 0 |
| 6 | -1 |

4. $f(x) = 5x^2 + 10x - 6$

① $f(x) = 5(x^2 + 2x \quad ) - 6$

② $f(x) = 5(x^2 + 2x + 1) - 6 - 5(1)$

$f(x) = 5(x+1)^2 - 11$

WE NEED TO COMPLETE THE SQUARE TO WRITE $f(x)$ IN STANDARD FORM.

① COEFFICIENT OF $x^2$ MUST BE 1, SO WE FACTOR OUT THE 5

② NEXT, WE ADD $\left(\frac{+2}{2}\right)^2 = 1$ TO THE X TERMS, & SUBTRACT TO KEEP THE EQUATION BALANCED.

IN THIS CASE, WE ADDED 1 & SUBTRACTED $-5(1)$. THE 5 IS DUE TO THE FACT THAT WE ADDED 1 INSIDE A SET OF PARANTHESES WITH 5 OUTSIDE.

5. $f(x) = x^2 - 6x + 3$

WE CAN COMPLETE THE SQUARE:

$f(x) = x^2 - 6x + 9 + 3 - 9$

$f(x) = (x-3)^2 - 6$

SO VERTEX IS $(3, -6)$

OR:

FIND X VALUE OF VERTEX WITH $x = \frac{-b}{2a} = \frac{-6}{2(1)} = \frac{6}{2} = 3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

FIND Y VALUE OF VERTEX BY EVALUATING FUNCTION @ X VALUE, SO

$y = f(3) = 3^2 - 6(3) + 3 = -6$

SO VERTEX IS $(3, -6)$

6. AREA AS LARGE AS POSSIBLE ⇒ THIS IS A MAX/MIN PROBLEM, THAT IS A VERTEX OF PARABOLA.

$A = LW$. SUM OF L & W IS 76

SO $L + w = 76$, $-w$ $-w$ $\Rightarrow$ $L = 76 - w$

$A = LW = (76-w)w = 76w - w^2$

SO $A = -w^2 + 76w$. THIS IS A PARABOLA LIKE $\cap$.

MAX IS AT VERTEX.

VERTEX: $w = -\frac{b}{2a} = \frac{-76}{2(-1)} = 38$.

$L = 76 - w = 76 - 38 = 38$, SO **38 ft by 38 ft**

7. MAX HEIGHT ⇒ VERTEX OF PARABOLA.

~~$h(x)$~~ $= h(t) = -16t^2 + 96t$

↑ x should be t ↑

MISTAKE IN REVIEW. SORRY.

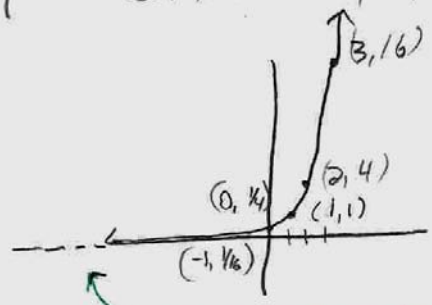
$t = \frac{-b}{2a} = \frac{-96}{2(-16)} = 3$

$h(3) = -16(3)^2 + 96(3) = 144$.

SO at 3 seconds, **the max height is 144 feet**

8. $f(x) = 4^{x-1}$ GRAPH BY PLOTTING POINTS:

| x | y |
|---|---|
| 3 | $4^2 = 16$ |
| 2 | $4^1 = 4$ |
| 1 | $4^0 = 1$ |
| 0 | $4^{-1} = 1/4$ |
| −1 | $4^{-2} = 1/16$ |




THIS DOTTED LINE IS A HORIZONTAL ASYMPTOTE, @ $y=0$, & SIGNIFIES THAT THE GRAPH WILL NEVER TOUCH HERE.

9. $2^{(5+3x)} = \frac{1}{16}$ ① GET SAME BASE

$2^{(5+3x)} = \frac{1}{2^4} = 2^{-4}$ so $2^{(5+3x)} = 2^{-4}$

NOW, HOW DO YOU UNDO AN EXPONENTIAL, BASE 2?

WITH A $\log_2$.

so $\log_2 2^{(5+3x)} = \log_2 2^{-4} \Rightarrow 5+3x = -4 \Rightarrow 3x = -9 \Rightarrow \boxed{x=-3}$

$-5 \quad -5 \qquad \frac{3x}{3} = \frac{-9}{3}$

10. FIND y WHEN YEAR IS 2000. SINCE $t=0$ IS 1995, $t=5$ IS 2000

so $y = 2889000(2.7)^{0.003(5)} = 2932364.805\ldots$

TO NEAREST 10000, $\boxed{2{,}930{,}000 \text{ PEOPLE}}$

11. $3^{\log_3\left(\frac{1}{9}\right)} = -2$ $\quad 3$

$\boxed{\frac{1}{9} = 3^{-2}}$

TO "UNDO" A $\log_3$, USE EXPONENTIAL BASE 3 (BIG 3)

YOU CAN ALSO DO THIS BY PATTERNS. THE BASE STAYS THE SAME & THE REST SWITCH PLACES.

12. $2^{\log_2 2^{-3}} = \log_2\left(\frac{1}{8}\right) \Rightarrow \boxed{-3 = \log_2\left(\frac{1}{8}\right)}$ DITTO

13. $\log_2\left(\frac{1}{4}\right)$ THE QUESTION TO ASK is "base" TO WHAT POWER is "ARGUEMENT" OR IN THIS PROBLEM, 2 TO WHAT POWER is $\frac{1}{4}$? IF YOUR BRAIN KNOWS, GREAT! THIS ANSWER is $\boxed{-2}$

IF YOUR BRAIN DOESNT KNOW, HERE ARE SOME STEPS:

$\log_2\left(\frac{1}{4}\right) = x \Rightarrow 2^{\log_2\left(\frac{1}{4}\right)} = 2^x = \frac{1}{4} = 2^x.$

$\frac{1}{4} = 2^{+x} \Rightarrow \frac{1}{2^2} = 2^x \Rightarrow 2^{-2} = 2^x$ NOW, LIKE PROB # 9.

$\Rightarrow \log_2 2^{-2} = \log_2 2^x \Rightarrow \boxed{-2} = x$

14. $9^{\log_9 20} = \boxed{20}$

15. $3^{\log_3 \left(\frac{1}{27}\right)} = 3^x$

3 TO WHAT POWER IS $\frac{1}{27}$? $\boxed{-3}$

$\frac{1}{27} = 3^x \Rightarrow \frac{1}{3^3} = 3^x \Rightarrow \log_3 3^{-3} = \log_3 3^x \Rightarrow x = \boxed{-3}$

16. $3^{\log_3 x} = 3^2 \Rightarrow x = 3^2 = \boxed{9}$

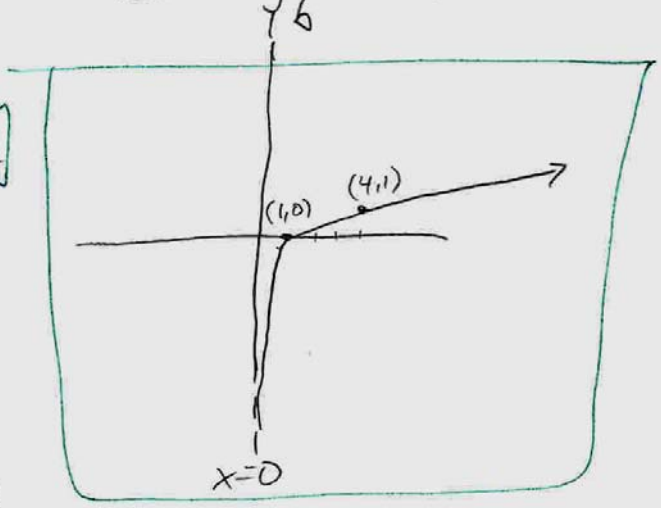
17. GRAPH $y = \log_4 x$.

| X | Y |
|---|---|
| 0 | $\log_4 0 \Rightarrow$ VA @ $x=0$ |
| 1 | $\log_4 1 = 0 \Rightarrow (1,0)$ |
| 4 | $\log_4 4 = 1 \Rightarrow (4,1)$ |

FACTS WE NEED: ① $\log_b 0$ IS UNDEFINED SO A VERTICAL ASYMPTOTE.

② $\log_b 1 = 0$

③ $\log_b b = 1$




18. $\log .001 \Rightarrow \log_{10} 10^{-2} = \boxed{-2}$

OR USE CALCULATOR.

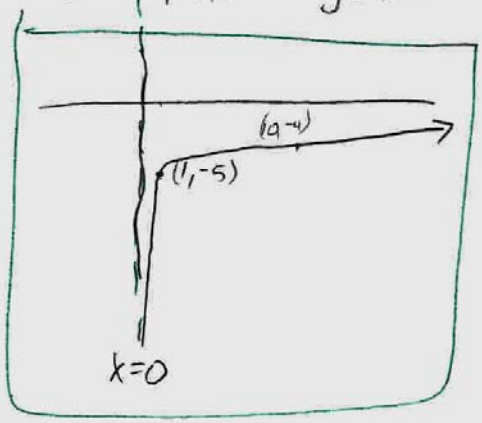
19. $\ln e^8 = \boxed{8}$

Recall that log is $\log_{10}$ and ln is $\log_e$

20. $\log_2\left(\frac{1}{5}\right) = \frac{\ln\left(\frac{1}{5}\right)}{\ln(2)}$ or $\frac{\log\left(\frac{1}{5}\right)}{\log(2)} \approx \boxed{-2.3219}$

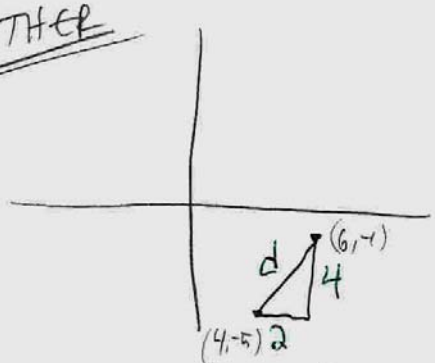
21. USING THE 3 FACTS FROM #17, $f(x) = \log(x) - 5$

| x | y |
|---|---|
| 0 | $\log 0 - 5$ UNDEFINED, so VA @ $x=0$ |
| 1 | $\log(1) - 5 = 0 - 5 = -5$ so $(1, -5)$ |
| 10 | $\log(10) - 5 = 1 - 5 = -4$ so $(10, -4)$ |




22. DISTANCE BETWEEN $(6,-1)$ & $(4,-5)$

EITHER




$d^2 = 2^2 + 4^2 = 4 + 16 = 20$

$d = \sqrt{20} = \boxed{2\sqrt{5}}$

OR $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-6)^2 + (-5-(-1))^2}$

$= \sqrt{(-2)^2 + (-4)^2}$

$= \sqrt{4 + 16}$

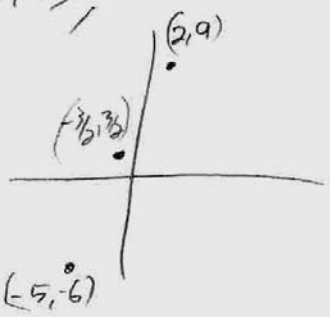
$= \sqrt{20} = \sqrt{2 \cdot 2 \cdot 5}$

$= \boxed{2\sqrt{5}}$

23. MIDPOINT IS JUST AVERAGE, SO MIDPOINT IS (AVE of X, AVE of Y)

OR $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ FOR $(-5,-6), (2,9)$,

MIDPOINT IS $\left(\frac{-5+2}{2}, \frac{-6+9}{2}\right) = \boxed{\left(\frac{-3}{2}, \frac{3}{2}\right)}$



24. $(x-2)^2 + (y-1)^2 = 16$

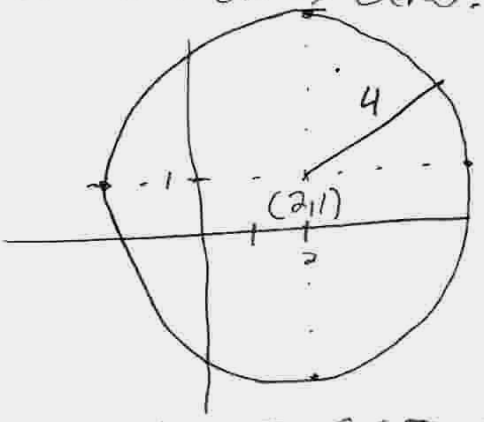
$x=2$, $y=1$ MAKE THESE ZERO.

FOR CENTER OF A CIRCLE IN STANDARD FORM, WHAT X & Y VALUES MAKE SQUARE TERMS ZERO?

SO CENTER IS $(2,1)$

$16 = r^2$, SO $r = \sqrt{16} = 4$, SO

RADIUS $r = 4$



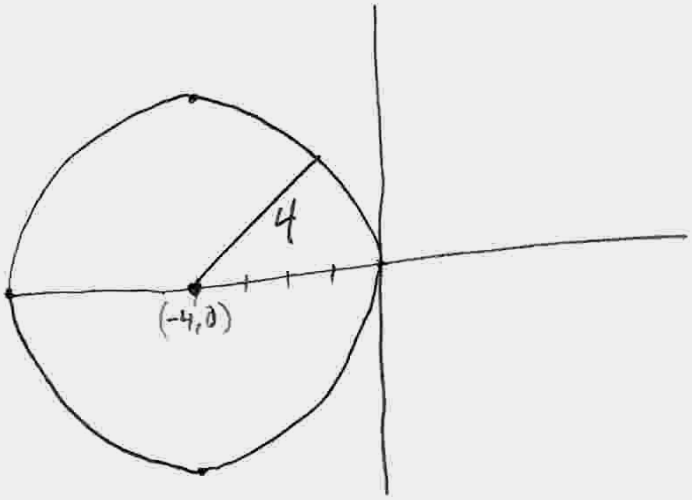

25. $y^2 + x^2 + 8x = 0$. COMPLETE SQUARES TO GET THIS CIRCLE IN STANDARD FORM.

$x^2 + 8x + 16 + y^2 = 0 + 16$

$(x+4)^2 + y^2 = 16$

CENTER IS $(-4,0)$

$r = 4$



26. CENTER $(9,-5)$ RADIUS 11

$$(x-9)^2+(y-(-5))^2=11^2$$

or $\boxed{(x-9)^2+(y+5)^2=121}$

27. $x^2+y^2-8x+2y+17=25$ . LIKE PROBLEM # 25

$-17 \quad -17$

$x^2-8x+16+y^2+2y+1=8+16+1$

$(x-4)^2+(y+1)^2=25$

CENTER $(4,-1)$

RADIUS $r=5$