MATH 1010 REVIEW 4 KEY

1. $(4^3)^2 = 4^{3\cdot 2} = 4^6 = \boxed{4096}$ POWER TO A POWER, MULTIPLY $(4^{3\times 2})$

2. $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \boxed{\frac{16}{81}}$ → powers (exponents) distribute to all factors

3. $(-4xy)^3 = (-4)^3x^3y^3 = \boxed{-64x^3y^3}$

4. $\sqrt{9x^{10}} = \sqrt{(3\cdot 3)(x^5\cdot x^5)} = \boxed{3x^5}$ SQUARE ROOT, LOOK FOR PAIRS

5. $\sqrt[3]{-216x^6} = \sqrt[3]{(-6)(-6)(-6)(x^2\cdot x^2\cdot x^2)} = \boxed{-6x^2}$ CUBE ROOT, LOOK FOR TRIPLES & ODD ROOT OF NEGATIVE is always Negative.

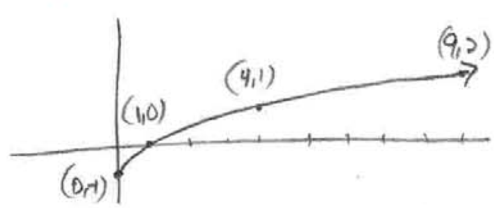
6. $\sqrt[4]{-81}$ $\boxed{\text{NOT REAL}}$ EVEN ROOTS OF NEGATIVES ARE NOT REAL (COMPLEX)

7. $f(x) = \sqrt{2x+3}$ $\quad f(39) = \sqrt{2(39)+3} = \sqrt{81} = \boxed{9}$

8. $f(x) = \sqrt{x} - 1$ DOMAIN OF EVEN ROOT: $\sqrt{\square}$, $\square \geq 0$

so $\boxed{x \geq 0 \text{ or } [0,\infty)}$

x	y
0	$\sqrt{0}-1=-1$
1	$\sqrt{1}-1=0$
4	$\sqrt{4}-1=1$
9	$\sqrt{9}-1=2$



9. $16^{\frac{1}{4}} = (16)^{1/4} = \sqrt[4]{16} = \sqrt[4]{2\cdot 2\cdot 2\cdot 2} = \boxed{2}$

10. $3x^{\frac{1}{4}} = 3(x)^{1/4} = \boxed{3\sqrt[4]{x}}$

11. $16^{5/4} = (16^{\frac{1}{4}})^5 = (\sqrt[4]{16})^5 = 2^5 = \boxed{32}$

12. $(6x)^{2/3} = (\sqrt[3]{6x})^2$ or $\sqrt[3]{(6x)^2} = \boxed{\sqrt[3]{36x^2}}$

13. $8^{-4/3} = \dfrac{1}{8^{4/3}} = \dfrac{1}{(\sqrt[3]{8})^4} = \dfrac{1}{2^4} = \boxed{\dfrac{1}{16}}$

14. $\dfrac{y^{3/4}}{y^{\frac{1}{4}}} = y^{\frac{3}{4}-\frac{1}{4}} = y^{\frac{2}{4}} = \boxed{y^{\frac{1}{2}}}$ (or $\sqrt{y}$)

IN GENERAL, IF A PROBLEM STARTS WITH RADICALS, THE ANSWER WILL BE WRITTEN WITH RADICALS. SAME FOR RATIONAL EXPONENTS.

15. $y^{\frac{5}{9}}\left(y^{\frac{3}{9}} - 5y^{\frac{2}{9}}\right) = y^{\frac{5}{9}+\frac{3}{9}} - 5y^{\frac{5}{9}+\frac{2}{9}} = \boxed{y^{\frac{8}{9}} - 5y^{\frac{7}{9}}}$

16. $\sqrt[6]{x^3} = x^{\frac{3}{6}} = x^{\frac{1}{2}} = \boxed{\sqrt{x}}$

17. $\sqrt[3]{5} \cdot \sqrt{3} = 5^{\frac{1}{3}\cdot\frac{2}{2}} \cdot 3^{\frac{1}{2}\cdot\frac{3}{3}} = 5^{\frac{2}{6}} \cdot 3^{\frac{3}{6}} = \left(5^2 \cdot 3^3\right)^{\frac{1}{6}} = (675)^{\frac{1}{6}} = \boxed{\sqrt[6]{675}}$

18. $\sqrt{15} \cdot \sqrt{15} = \sqrt{225} = \boxed{15}$ or $\sqrt{15} \cdot \sqrt{15} = \sqrt{15^2} = 15$

19. $\sqrt{150} \cdot \sqrt{6} = \sqrt{150 \cdot 6} = \sqrt{15 \cdot 10 \cdot 2 \cdot 3} = \sqrt{\underbrace{3 \cdot 5 \cdot 2 \cdot 5 \cdot 2 \cdot 3}_{\text{pairs}}} = 3 \cdot 2 \cdot 5 = \boxed{30}$

20. $\sqrt{\dfrac{20}{49}} = \dfrac{\sqrt{20}}{\sqrt{49}} = \dfrac{\sqrt{2 \cdot 2 \cdot 5}}{\sqrt{7 \cdot 7}} = \boxed{\dfrac{2\sqrt{5}}{7}}$

21. $\sqrt{\dfrac{75r^2y}{x^4}} = \dfrac{\sqrt{3 \cdot 5 \cdot 5 \cdot r \cdot r \cdot y}}{\sqrt{x^2 \cdot x^2}} = \boxed{\dfrac{5r\sqrt{3y}}{x^2}}$

22. $\sqrt{6} = \sqrt{2 \cdot 3} = \boxed{\sqrt{6}}$ NO PAIRS SO THIS DOESN'T SIMPLIFY

23. $\sqrt[7]{x^{72}}$ $\quad 7\overline{)72}$ = 10 R 2

7 GOES INTO 72 10 TIMES, WITH 2 REMAINDER, SO

$\boxed{x^{10}\sqrt[7]{x^{2}}}$

24. $\sqrt{100} + \sqrt{32} + \sqrt{64} + \sqrt{98} = 10 + 4\sqrt{2} + 8 + 7\sqrt{2}$

$100 = 10 \cdot 10$, $32 = 2 \cdot 4 \cdot 4$, $64 = 8 \cdot 8$, $98 = 2 \cdot 7 \cdot 7$

$= \boxed{18 + 11\sqrt{2}}$

25. $8\sqrt[3]{3}+2\sqrt[3]{3}=\boxed{10\sqrt[3]{3}}$ $\sqrt[3]{3}$ are like terms.

OR $8\sqrt[3]{3}+2\sqrt[3]{3}=\sqrt[3]{3}(8+2)=10\sqrt[3]{3}$

26. $\boxed{2\sqrt[4]{6}+7\sqrt[3]{6}}$ $\sqrt[4]{\ }$ & $\sqrt[3]{\ }$ are not like terms, so these don't simplify.

27. $\sqrt{5}\left(\sqrt{3}+\sqrt{7}\right)=\sqrt{5}\cdot\sqrt{3}+\sqrt{5}\cdot\sqrt{7}=\sqrt{5\cdot 3}+\sqrt{5\cdot 7}=\boxed{\sqrt{15}+\sqrt{35}}$

28. $(\sqrt{8}+3)(\sqrt{8}-3)=\overset{F}{\sqrt{8}\cdot\sqrt{8}}\ \overset{O}{\cancel{-3\sqrt{8}}}\ \overset{I}{\cancel{+3\sqrt{8}}}\ \overset{L}{-9}=8-9=\boxed{-1}$

29. $(9\sqrt{3}+8)^2=(9\sqrt{3}+8)(9\sqrt{3}+8)=\overset{F}{9\sqrt{3}\cdot 9\sqrt{3}}+\overset{O}{9\sqrt{3}\cdot 8}+\overset{I}{8\cdot 9\sqrt{3}}+\overset{L}{8\cdot 8}=$

$=9\cdot 9\cdot\sqrt{3\cdot 3}+9\cdot 8\cdot\sqrt{3}+8\cdot 9\cdot\sqrt{3}+64=81\cdot 3+72\sqrt{3}+72\sqrt{3}+64=$

$=243+144\sqrt{3}+64=\boxed{307+144\sqrt{3}}$

30. $\sqrt{\frac{1}{2}}=\frac{\sqrt{1}}{\sqrt{2}}=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{\sqrt{2\cdot 2}}=\boxed{\frac{\sqrt{2}}{2}}$

31. $\sqrt[4]{\frac{625}{9x^{19}}}=\frac{\sqrt[4]{625}}{\sqrt[4]{9x^{19}}}=\frac{\sqrt[4]{5\cdot 5\cdot 5\cdot 5}}{\sqrt[4]{3\cdot 3\cdot x^4\cdot x^4\cdot x^4\cdot x^4\cdot x^3}}=\frac{5}{x^4\sqrt[4]{3\cdot 3\cdot x\cdot x\cdot x}}\cdot\frac{\sqrt[4]{3\cdot 3\cdot x}}{\sqrt[4]{3\cdot 3\cdot x}}=$

$=\frac{5\sqrt[4]{9x}}{x^4\cdot 3\cdot x}=\boxed{\frac{5\sqrt[4]{9x}}{3x^5}}$

32. $\frac{6}{(8-\sqrt{3})}\cdot\frac{(8+\sqrt{3})}{(8+\sqrt{3})}=\frac{6(8+\sqrt{3})}{64\cancel{-8\sqrt{3}}\cancel{+8\sqrt{3}}\underbrace{-\sqrt{3\cdot 3}}_{-3}}=\boxed{\frac{6(8+\sqrt{3})}{61}\text{ or }\frac{48+6\sqrt{3}}{61}}$

33. $\frac{\sqrt[3]{3}}{\sqrt[3]{5}}\cdot\frac{\sqrt[3]{3\cdot 3}}{\sqrt[3]{3\cdot 3}}=\frac{\sqrt[3]{3\cdot 3\cdot 3}}{\sqrt[3]{5\cdot 3\cdot 3}}=\boxed{\frac{3}{\sqrt[3]{45}}}$

34. $\sqrt{x+5}-3=0$ $\Rightarrow (\sqrt{x+5})^2=(3)^2 \Rightarrow x+5=9 \Rightarrow \boxed{x=4}$?
$\quad\quad\;\; +3 \;\; +3 \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad -5 \;\; -5$

you must $\boxed{\text{ALWAYS}}$ check your answers in even root equations FOR EXTRANEOUS SOLUTIONS!!

check $\sqrt{4+5}-3=\sqrt{9}-3=3-3=0$ ✓ so $\boxed{x=4}$

35. $(\sqrt{6x-11})^2=(3-x)^2 \Rightarrow 6x-11=(3-x)(3-x) \Rightarrow 6x-11=\overset{\text{FOIL}}{9-6x+x^2} \Rightarrow$
$\quad -6x+11 \quad +11 -6x$

$x^2-12x+20=0 \Rightarrow (x-10)(x-2)=0 \Rightarrow x=10,\ x=2.$

CHECK $\sqrt{6(10)-11}=3-10 \Rightarrow \sqrt{49}=3-10 \Rightarrow 7=-7$ NO!
$\sqrt{6(2)-11}=3-2 \Rightarrow \sqrt{12-11}=1 \Rightarrow \sqrt{1}=1$ YES!
SO $\boxed{x=2}$ ONLY. $x=10$ WAS EXTRANEOUS!

36. $(\sqrt{4x+3})^2=(\sqrt{2x-3}-3)^2 \Rightarrow 4x+3=\overset{F}{2x-3}\overset{O\,I}{-6\sqrt{2x-3}}\overset{L}{+9} \Rightarrow$

$\Rightarrow 4x+3=2x+6-6\sqrt{2x-3} \Rightarrow (2x-3)^2=(-6\sqrt{2x-3})^2 \Rightarrow$
$\quad -2x-6 \;\; -2x-6$

$\Rightarrow \underset{\text{FOIL}}{4x^2-12x+9} = \underset{\text{Square both factors!}}{36(2x-3)} \Rightarrow 4x^2-12x+9=72x-108$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad -72x+108 \;\; -72x+108$

$\Rightarrow 4x^2-84x+117=0 \Rightarrow (2x-3)(2x-39)=0$

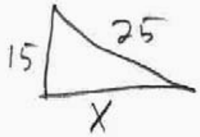
$\Rightarrow x=\frac{3}{2},\ x=\frac{39}{2}$

This Factoring is hard. The test question should be easier to factor.

Check $\sqrt{4(\frac{3}{2})+3}=\sqrt{2(\frac{3}{2})-3}-3 \Rightarrow \sqrt{6+3}=\sqrt{3-3}-3 \Rightarrow \sqrt{9}=0$ NO

$\sqrt{4(\frac{39}{2})+3}=\sqrt{2(\frac{39}{2})-3}-3 \Rightarrow \sqrt{78+3}=\sqrt{36}-3 \Rightarrow \sqrt{81}=6-3$ NO

SINCE NEITHER OF THE TWO ANSWERS WORK, THE SOLUTION IS NO SOLUTION, $\boxed{\emptyset}$ or $\{\ \}$

37. 

$x^2+15^2=25^2 \Rightarrow x^2+225=625$
$\quad\quad\quad\quad -225 \quad -225$

$\Rightarrow \sqrt{x^2}=\pm\sqrt{400} \Rightarrow x=\pm 20$ $\boxed{x=20}$

The −20 can not be correct since we are looking for distance.

38. $\sqrt{-9}=\sqrt{9}\cdot i=\boxed{3i}$ $\quad \sqrt{-1}=i$

39. $\sqrt{-2}\cdot\sqrt{-3}=\sqrt{2}\cdot i\cdot\sqrt{3}\cdot i=\sqrt{2}\cdot\sqrt{3}\cdot i\cdot i=\sqrt{6}\,i^2=\sqrt{6}(-1)=\boxed{-\sqrt{6}}$

$i^2=-1$

40. $(7+7i)-(-9+i)=7+7i+9-i=7+9+7i-i=\boxed{16+6i}$

41. $3i(2-2i)=3\cdot i\cdot 2-3\cdot i\cdot 2\cdot i=6i-6i^2=6i-6(-1)=\boxed{6+6i}$

STANDARD FORM $a+bi$

42. F O I L

$(8-5i)^2=(8-5i)(8-5i)=64-40i-40i+25i^2$

$=64-80i+25(-1)=64-80i-25=64-25-80i=\boxed{39-80i}$

43. $\frac{5}{2i}\cdot\frac{i}{i}=\frac{5i}{2i^2}=\frac{5i}{-2}=\boxed{0-\frac{5}{2}i}$

44. $\frac{6}{6-9i}\cdot\frac{6+9i}{6+9i}=\frac{6(6+9i)}{36-81i^2}=\frac{36+54i}{36+81}=\frac{36+54i}{117}=\frac{36}{117}+\frac{54}{117}i=$

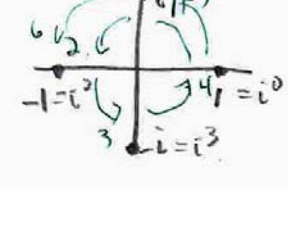
$=\boxed{\frac{4}{13}+\frac{6}{13}i}$

45. $i^4=1$, $i^3=-i$, $i^2=-1$, $i=i$

so $i^{27}=i^4\cdot i^4\cdot i^4\cdot i^4\cdot i^4\cdot i^4\cdot i^3$

$=(i^4)^6\cdot i^3=1^6\cdot i^3=i^3=\boxed{-i}$

OR $4\overline{)27}$ 6R3 so $i^3=-i$

or 

COUNTERCLOCKWISE, MOVE 27 TIMES, STARTING @ 1 & YOU END UP @ $-i$

46. $(-6i)^5 = (-6)^5 i^5 = -7776 i^4 \cdot i = \boxed{-7776i}$

$\left(\left(\frac{+14}{2}\right)^2 = 49\right)$

ALWAYS ÷ 2 & SQUARE

47. $x^2 + 14x + 13 = 0$
$\quad\quad -13 \quad -13$

$\Rightarrow x^2 + 14x + 49 = -13 + 49$

$\Rightarrow (x+7)^2 = 36$

$\uparrow$

+7 is +14 ÷ 2 ALWAYS

so $\sqrt{(x+7)^2} = \pm\sqrt{36} \Rightarrow x + 7 = \pm 6$
$\quad\quad -7 \quad -7$

$\Rightarrow x = -7 \pm 6 = -7+6 = -1$
$\quad\quad = -7-6 = -13$

$\boxed{x = -1,\ x = -13}$

48. $4x^2 + 24x + 11 = 0$
$\quad\quad -11 \quad -11$

$\Rightarrow \frac{4x^2}{4} + \frac{24x}{4} = \frac{-11}{4}$

coefficient of x^2 must be 1 before ÷ 2 & square

$\Rightarrow x^2 + 6x + 9 = -\frac{11}{4} + 9 \Rightarrow (x+3)^2 = -\frac{11}{4} + \frac{36}{4} = \frac{25}{4}$

so $\sqrt{(x+3)^2} = \pm\sqrt{\frac{25}{4}} \Rightarrow x + 3 = \pm\frac{5}{4} \Rightarrow x = -3 \pm \frac{5}{4} = -\frac{12}{4} \pm \frac{5}{4}$
$\quad\quad -3 \quad -3$

$\Rightarrow x = \frac{-12 \pm 5}{4} = \frac{-12+5}{4}, \frac{-12-5}{4} = \boxed{\frac{-7}{4}, \frac{-17}{4}}$

49. $x^2 + 16x + 53 = 0 \quad a = 1,\ b = 16,\ c = 53$

$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

so $x = \frac{-16 \pm \sqrt{16^2 - 4(1)(53)}}{2(1)} = \frac{-16 \pm \sqrt{256 - 212}}{2} = \frac{-16 \pm \sqrt{44}}{2} \Rightarrow$

$x = -\frac{16}{2} \pm \frac{\sqrt{44}}{2} = -8 \pm \frac{\sqrt{2 \cdot 2 \cdot 11}}{2} = -8 \pm \frac{2\sqrt{11}}{2} = -8 \pm \sqrt{11}$

so $\boxed{x = -8 + \sqrt{11},\ -8 - \sqrt{11}}$

50. $2x^2+10x=-5 \Rightarrow 2x^2+10x+5=0$
$+5\ \ +5$

GET IN THIS FORM $ax^2+bx+c=0$ before you get a, b & c.

so $a=2$, $b=10$ & $c=5$

$$\Rightarrow x=\frac{-10\pm\sqrt{10^2-4(2)(5)}}{2(2)}=\frac{-10\pm\sqrt{100-40}}{4}=\frac{-10\pm\sqrt{60}}{4}\Rightarrow$$

$$\Rightarrow x=\frac{-10\pm\sqrt{2\cdot3\cdot2\cdot5}}{4}=\frac{\cancel{-10}^{\,5}\pm\cancel{2}\sqrt{15}}{{}_2\cancel{4}}=\boxed{\frac{-5\pm\sqrt{15}}{2}}$$

51. $x^2-3x-1=0$ $\quad b^2-4ac=(-3)^2-4(1)(-1)=$ $\quad$ discriminant

$=9+4=13$. SINCE discriminant is positive, there will be $\boxed{\text{2 real solutions}}$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

52. $5-3x^2=-5x+4 \Rightarrow 3x^2-5x-1=0$ so $b^2-4ac=(-5)^2-4(3)(-1)=$
$-5+3x^2\quad -5+3x^2$

$=25+12=37$ positive

SO $\boxed{\text{2 REAL SOLUTIONS}}$

(IF DISCRIMINANT NEGATIVE, 2 complex solutions)
(IF DISCRIMINANT = ZERO, 1 REAL SOLUTION)

53. Area $=L\cdot W=37$. $\quad L=w+2$, SO

$$37=(w+2)\cdot w \Rightarrow \underset{-37}{37}=w^2+2w\underset{-37}{} \Rightarrow$$

$$\Rightarrow w^2+2w-37=0 \quad \text{so } w=\frac{-2\pm\sqrt{2^2-4(1)(-37)}}{2}=\frac{-2\pm\sqrt{152}}{2}=$$

$$=\frac{-2\pm\sqrt{2\cdot2\cdot2\cdot19}}{2}=\frac{\cancel{-2}^{\,1}\pm\cancel{2}^{\,1}\sqrt{38}}{{}_1\cancel{2}}=-1\pm\sqrt{38}.$$

WIDTH IS POSITIVE, so $\boxed{w=-1+\sqrt{38}}$

$$L=w+2=-1+\sqrt{38}+2=-1+2+\sqrt{38}=\boxed{1+\sqrt{38}=L}$$

so $\boxed{1+\sqrt{38}\text{ yards by }-1+\sqrt{38}\text{ yards}}$