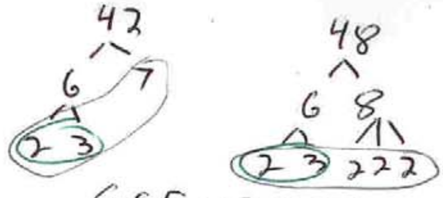
MATH 1010 TEST 3 REVIEW KEY

1. $42x^5y + 48xy^3$

$$\boxed{6xy(7x^4 + 8y^2)}$$



$GCF = 2 \cdot 3 = 6$

GCF

2. $2(y+4) - x(y+4) = \boxed{(y+4)(2-x)}$

GCF

3. $4xy + 16x + 7y + 28 =$

$= 4x(y+4) + 7(y+4) =$

$= \boxed{(y+4)(4x+7)}$

FACTOR BY GROUPING

4. $x^2 - 2x - 24 = \boxed{(x-6)(x+4)}$

BACKWARDS FOIL (BF)

5. $16y^3 - 40y^2 + 25y =$

$= y(16y^2 - 40y + 25) = y(4y-5)(4y-5) = \boxed{y(4y-5)^2}$

GCF, BF

6. $9x^4 - 9x^2 - 10$: Let $u = x^2$

THEN $9u^2 - 9u - 10 = (3u-5)(3u+2)$

$= \boxed{(3x^2-5)(3x^2+2)}$

U SUBSTITUTION, BF, BACK SUBSTITUTION

7. $10(a+6)^2 + 34(a+6) + 12$; Let $u = a+6$

THEN $10u^2 + 34u + 12 = 2(5u^2 + 17u + 6)$

$= 2(5u+2)(u+3) = 2(5(a+6)+2)((a+6)+3)$

$= 2(5a+30+2)(a+6+3) = \boxed{2(5a+32)(a+9)}$

U SUB, GCF, BF, BACK SUB, SIMPLIFY

8. $(5x+y)^2-36$; LET $u=5x+y$. USUB, DIFFERENCE OF SQUARES

THEN $u^2-36=(u+6)(u-6)$ BACKSUB

$=\boxed{(5x+y+6)(5x+y-6)}$

9. $25x^2+16$ $\boxed{\text{PRIME}}$ SUM OF SQUARES IS PRIME (IE DOESN'T FACTOR)

10. $x^3+1000=\boxed{(x+10)(x^2-10x+100)}$ SUM OF CUBES

1st 2nd

CUBEROOTS

$(1st)^2$ $(1st)(2nd)$ $(2nd)^2$

opposite sign +

11. $343x^3-1000=(7x-10)(49x^2+70x+100)$ DIFFERENCE OF CUBES

1st 2nd

CUBE ROOTS

$(1st)^2$ opp sign $(1st)(2nd)$ + $(2nd)^2$

12. $x^2+7x-60=0$ $=0$

$(x+12)(x-5)=0$ FACTOR

$x+12=0$ $x-5=0$ ZERO FACTOR PROPERTY (Z.F.)

$\boxed{x=-12,\ x=5}$

13. $^{162}\dfrac{x^2}{162}+{}^{162}\dfrac{1}{9}={}^{162}\dfrac{x}{18}$ CLEAR FRACTIONS

$x^2+18=9x$ $=0$

$-9x$ $-9x$

$x^2-9x+18=0$

$(x-6)(x-3)=0$ FACTOR

$x-6=0,\ x-3=0$ Z.F. PROPERTY

$\boxed{x=6,\ x=3}$

14. Length is 8 longer than width

$L = w + 8$

Area is 84.

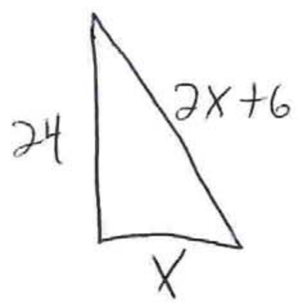
$A = L \cdot w = 84 \Rightarrow (w+8)\cdot w = 84 \Rightarrow w^2+8w-84=0$

$\Rightarrow (w+14)(w-6)=0 \Rightarrow w = \cancel{-14}, 6$ negative width is meaningless

So $w = 6$ ft & $L = w + 8 = 14$ ft $\boxed{\text{6 ft by 14 ft}}$

15. x = length of shorter leg. Longer leg is 24 meters

Hypotenuse is 6 more than twice the shorter, so hyp $= 2x+6$



Pythagorean theorem:

$x^2 + (24)^2 = (2x+6)^2$

$x^2 + 576 = 4x^2 + 24x + 36$

$-x^2 - 576 \quad -x^2 \quad -576$

$0 = 3x^2 + 24x - 540 = 0$

$3(x^2 + 8x - 180) = 0$

$3(x+18)(x-10) = 0$

$3 \neq 0 \quad x = -18 \quad x = 10$

NOT POSSIBLE (X is distance)

$\boxed{\text{10 meters = shorter leg}}$

16. DOMAIN OF A RATIONAL FUNCTION = DON'T DIVIDE BY ZERO.

$$f(x) = \frac{1-5x}{x^2-6x-16} \Rightarrow \begin{matrix} x^2-6x-16 \neq 0 \\ (x+2)(x-8) \neq 0 \end{matrix} \Rightarrow \begin{matrix} x \neq -2 \\ x \neq 8 \end{matrix} \text{ so } \boxed{\text{DOMAIN} = \{x \mid x \neq -2, x \neq 8\}}$$

17. SIMPLIFY

$$\frac{4x-8}{2-x} = \frac{4\cancel{(x-2)}}{-1\cancel{(x-2)}} = \boxed{-4}$$

18.
$$\frac{36xy^2}{x^2-49} \cdot \frac{12x-84}{3x^2y^2} = \frac{\cancel{3} \cdot 12 \cdot \cancel{x} \cdot \cancel{y^2}}{(x+7)\cancel{(x-7)}} \cdot \frac{12\cancel{(x-7)}}{\cancel{3}x \cdot \cancel{x} \cdot \cancel{y^2}} = \boxed{\frac{144}{x(x+7)}}$$

19.
$$\frac{x^2+11x+18}{x^2+12x+27} \div \frac{x^2+2x}{x^2+13x+30} = \frac{x^2+11x+18}{x^2+12x+27} \cdot \frac{x^2+13x+30}{x^2+2x} \quad (\text{changed } \div)$$

$$= \frac{\cancel{(x+9)}\cancel{(x+2)}}{\cancel{(x+9)}\cancel{(x+3)}} \cdot \frac{(x+10)\cancel{(x+3)}}{x\cancel{(x+2)}} = \boxed{\frac{x+10}{x}}$$

20.
$$\frac{11}{11} \cdot \frac{3}{7x} + \frac{6}{11x} \cdot \frac{7}{7} = \frac{33}{77x} + \frac{42}{77x} = \frac{33+42}{77x} = \boxed{\frac{75}{77x}}$$

21.
$$\frac{(m+4)}{\underset{(m+7)(m-1)}{m^2+6m-7}} \cdot \frac{(m+8)}{(m+8)} + \frac{(5m-3)}{\underset{(m+8)(m+7)}{m^2+15m+56}} \cdot \frac{(m-1)}{(m-1)} = \frac{\overset{m^2+12m+32 \quad +5m^2-8m+3}{(m+4)(m+8) + (5m-3)(m-1)}}{(m+7)(m-1)(m+8)} =$$

$$\boxed{= \frac{6m^2+4m+35}{(m+7)(m-1)(m+8)}}$$

22. $\left(\frac{2}{3}-\frac{7}{x}\right)\div\left(\frac{3}{x}+\frac{7}{2}\right) = \dfrac{2\cdot3\cdot x\,\frac{2}{3}-\frac{7}{x}\,2\cdot3\cdot x}{2\cdot3\cdot x\,\frac{3}{x}+\frac{7}{2}\,2\cdot3\cdot x} = \dfrac{4x-42}{18+21x} = \boxed{\dfrac{2(2x-21)}{3(6+7x)}}$

23. $\dfrac{x^2\cdot\frac{4}{x}+7\cdot x^2}{x^2\cdot\frac{16}{x^2}-49\cdot x^2} = \dfrac{4x+7x^2}{16-49x} = \dfrac{x\cancel{(4+7x)}}{\cancel{(4+7x)}(4-7x)} = \boxed{\dfrac{x}{4-7x}}$

24. $\dfrac{9x^{-1}+(8y)^{-1}}{x^{-2}} = \dfrac{8x^2y\,\frac{9}{x}+\frac{1}{(8y)^1}\,8x^2y}{8x^2y\,\frac{1}{x^2}} = \boxed{\dfrac{72xy+x^2}{8y}}$

25. $\dfrac{-12x^6-10x^4-4x^2}{-2x^4} = \dfrac{-12x^6}{-2x^4}\;\dfrac{-10x^4}{-2x^4}-\dfrac{4x^3}{-2x^4} =$

$\boxed{= 6x^2+5+\dfrac{2}{x^2}}$

DIVIDING BY MONOMIAL (1 TERM), SEPARATE THE FRACTION LIKE THIS

26.

$$\begin{array}{r|l} & 2x+\frac{1}{6} \\ \hline 6x^2+0x-42 & 12x^3+x^2-84x-7 \\ & -12x^3\mp0x^2\pm84x \\ \hline & \quad x^2+0x-7 \\ & -x^2\mp0x\pm7 \\ \hline & \qquad\qquad 0 \end{array}$$

$\dfrac{x^2}{6x^2}=\dfrac{1}{6}$

LONG DIVISION FOR DIVIDING BY POLYNOMIAL OF AT LEAST 2 TERMS, REMEMBER PLACE HOLDER(S)!

$\boxed{2x+\dfrac{1}{6}}$

27. SOLVE $1\cdot x^2 + \frac{1\cdot x^2}{x} = \frac{6}{x^2}\cdot x^2 \Rightarrow x^2 + x = 6 \Rightarrow x^2 + x - 6 = 0 \Rightarrow$

$\qquad\qquad\qquad\qquad -6 \quad -6$

$(x+3)(x-2) = 0 \Rightarrow \boxed{x = -3,\ x = 2}$

28. $\frac{1}{\cancel{x}}\cdot x(x-3) + \frac{1}{\cancel{x-3}}\cdot x(x-3) = \frac{(x-2)}{\cancel{x-3}}\cdot x(x-3) \Rightarrow x - 3 + x = x^2 - 2x$

$\Rightarrow 2x - 3 = x^2 - 2x \Rightarrow 0 = x^2 - 4x + 3 = 0$

$-2x \ +3 \qquad -2x \ +3$

$(x-3)(x-1) = 0$

$\Rightarrow \boxed{x = 1}, \cancel{x = 3}$ ← $x=3$ causes a $\div 0$ in the original problem, so it is extraneous.

29. $x^{-2} - 23x^{-1} + 132 = 0 \Rightarrow x^2\cdot\frac{1}{x^2} - x^2\cdot\frac{23}{x} + x^2\cdot 132 = 0$

$\Rightarrow 1 - 23x + 132x^2 = 0 \Rightarrow (1 - 12x)(1 - 11x) = 0$

$\Rightarrow 1 - 12x = 0 \qquad 1 - 11x = 0$

$1 = 12x \qquad 1 = 11x$

$\boxed{x = \frac{1}{12}, \quad x = \frac{1}{11}}$

30. $Tt\cdot\frac{PV}{\cancel{T}} = \frac{pvTt}{\cancel{t}}$ SOLVE FOR V

$\frac{\cancel{tP}V}{\cancel{tP}} = \frac{pvT}{tP}$

$\boxed{V = \frac{pvT}{tP}}$

31. TWO TIMES THE RECIPROCAL OF A NUMBER

$$2\left(\frac{1}{x}\right) = 24\left(\frac{1}{30}\right)$$

EQUALS 24 TIMES THE RECIPROCAL OF 30

$$30x\cdot\frac{2}{x} = \frac{24}{30}\cdot 30x \Rightarrow \frac{60}{24} = \frac{24x}{24} \Rightarrow x = \frac{60}{24} = \boxed{\frac{5}{2}}$$

32. Painter 6 hours , Assistant 8 hours Together T hours

$$24T\cdot\frac{1}{6} + 24T\cdot\frac{1}{8} = \frac{1}{T}\cdot 24T \Rightarrow 4T + 3T = 24 \Rightarrow \frac{7T}{7} = \frac{24}{7} \Rightarrow$$

$$T = \frac{24}{7} = \boxed{3\tfrac{3}{7}\text{ hours}}$$

33. $\frac{D}{R} = \frac{RT}{R}$ so $T = \frac{D}{R}$ Let x = speed of cyclist

$$\text{Time}_{TO} = \text{Time}_{RETURN} - 1$$

(TIME TO RETURN IS LONGER, SO SUBTRACTING 1 WILL LEAVE = TO SHORTER TIME)

$$\frac{D_{TO}}{R_{TO}} = \frac{D_{RET}}{R_{RET}} - 1$$

$$\left(T = \frac{D}{R}\right)$$

$$x\cdot\frac{17}{x} = x\cdot\frac{22}{x} - 1\cdot x$$

$$\begin{aligned} 17 &= 22 - x \\ -22 &\quad -22 \end{aligned}$$

$$\frac{-5}{-1} = \frac{-x}{-1} \Rightarrow x = \boxed{5\text{ mph}}$$